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Common Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces using Occasionally Weakly Compatible

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Abstract

In this paper, we prove common fixed point theorems generalizing the results in [9] using the condition for continuous self mapping A, B, C, S, T and U of complete intuitionistic fuzzy metric spaces (X, M, N, *,◊) where the pairs {A, S}
{B, T} and{ C, U } are OWC and have unique common fixed point in X.

Keywords: Intuitionistic Fuzzy Metric Spaces, Compatible Mappings, Common Fixed Point, Occasionally Weakly Compatible Mapping.

Mathematics subject classifications: 45H10, 54H25

1. INTRODUCTION

The Concept of fuzzy set was introduced by Zadeh [10] in 1965 .Following the concept of fuzzy sets, Kaleva and Seikalla [4] and kramosil and Michalek [5] introduced the concept of fuzzy metric space, George and Veeramani [2] modified the concept of fuzzy metric space introduced by kramosil and Michalek [5] .

As a generalization of fuzzy sets, Atanassov [1] introduced and studied the concept of intuitionistic fuzzy sets. Using the idea of intuitionistic fuzzy sets Park [7] defined the notion of intuitionistic fuzzy metric space with the help of continuous t- norm and continuous t- conorm as a generalization of fuzzy metric space, George and

Veeramani [2] had showed that every metric induces an intuitionistic fuzzy metric and found a necessary and sufficient conditions for an intuitionistic fuzzy metric space to be complete. Kramaosil and Michalek [5] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach. Turkoglu et al [8] gave the generalization of Jungck's common fixed point theorem to intuitionistic fuzzy metric spaces.

In this paper presents common fixed point theorem which is Occasionally Weakly Compatible (OWC) in intuitionistic fuzzy metric spaces and generalization of results in [9] the condition for continuous self mapping A, B, C, S, T and U of complete fuzzy metric spaces $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ have a unique common fixed point.

2. PRELIMINARIES

Definition 2.1:

A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if * is satisfies the following conditions.

i) * is commutative and associative,

ii) * is continuous,

iii) $a * 1 = a$ for all $a \in [0,1]$,

iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2:

A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-conorm if $\Diamond$ satisfies the following conditions :

i) $\Diamond$ is commutative and associative,

ii) $\Diamond$ is continuous,

iii) $a \Diamond 0 = a$ for all $a \in [0,1]$,

iv) $a \Diamond b \le c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition 2.3:

A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-space) if X is an arbitrary set, $*$ is a continuous t-norm , $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$

(IFM-1) $M(x, y, t) + N(x, y, t) \le 1$,

(IFM-2) $M(x, y, 0) = 0$,

(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$,

(IFM-4) $M(x, y, t) = M(y, x, t)$,

(IFM-5) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,

(IFM-6) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous,

(IFM-7) $\lim_{n\to\infty} M(x, y, t) = 1$,
(IFM-8) $N(x, y, 0) = 1$,
(IFM-9) $N(x, y, t) = 0$ if and only if $x = y$,
(IFM-10) $N(x, y, t) = N(y, x, t)$,
(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$,
(IFM-12) $N(x, y, .): [0, \infty) \to [0, 1]$ is right continuous,
(IFM-13) $\lim_{n\to\infty} N(x, y, t) = 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Example 2.4:
Let (X, d) be a metric space. Denote $a * b = ab$ and $a \lozenge b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows
$M_d(x, y, t) = \frac{t}{t + d(x,y)}$, $N_d(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$

Then (M_d, N_d) is an intuitionistic fuzzy metric on X. We call this intuitionistic fuzzy metric induced by a metric the standard intuitionistic fuzzy metric.

Definition 2.5 :
(i) A sequence $\{x_n\}$ in X is said to be a Cauchy sequence iff for each $\epsilon > 0$,$t > 0$ there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ and $N(x_n, x_m, t) < \epsilon$ for all $n,m \in n_0$.
(ii) A sequence $\{x_n\}$ in X is said to be a converge to a point x in X if and only if for each $\epsilon > 0$, $t > 0$ there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1-\epsilon$ and $N(x_n, x_m, t) < \epsilon$ for all $n \geq n_0$.
(iii) A intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 2.6:
A pair of self mappings (A, S) of a intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be
i) Weakly Commuting
If $\mathcal{M}(ASx, SAx, t) \geq \mathcal{M}(Ax, Sx, t)$ and $\mathcal{N}(ASx, SAx, t) \leq \mathcal{N}(Ax, Sx, t)$ For all $x \in X$ and $t > 0$.
ii) R- Weakly Commuting
If there exist some $R > 0$ Such that $M(ASx, SAx, t) \geq M(Ax, Sx, t/R)$ and $\mathcal{N}(ASx, SAx, t) \leq \mathcal{N}(Ax, Sx, t/R)$ for all $x \in X$ and $t > 0$.

Definition 2.7:
A pair of self mappings A and S of a intuitionistic fuzzy metric space $(X, M, \mathcal{N}, *, \Diamond)$ are called reciprocally continuous on X. If $\lim_{n\to\infty} ASx_n = Ax$ and $\lim_{n\to\infty} SAx_n = Sx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.

3. OCCASIONALLY WEAKLY COMPATIBLE

Theorem 3.1:
Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, C, S, T and U be self mapping of X . Let the pairs {A, S}, {B, T}and {C, U} be occasionally weakly compatible. If there exist $q \in (0, 1)$ such that

$$\mathcal{M}(Ax, By, Cz, qt) \geq \phi\,[\min\{\mathcal{M}(Sx, Ty, Uz, t), \mathcal{M}(Sx, Ax, t), \mathcal{M}(Ty, By, t), \mathcal{M}(Uz, Cz, t)\} * \{\min\{\mathcal{M}(By, Ty, t), \mathcal{M}(Ax, Ty, t), \mathcal{M}(By, Sx, t)\}\} * \{\min\{\mathcal{M}(Cz, Uz, t), \mathcal{M}(By, Uz, t), \mathcal{M}(Cz, Ty, t)\}\}]$$ and

$$\mathcal{N}(Ax, By, Cz, qt) \leq \psi\,[\max\{\mathcal{N}(Sx, Ty, Uz, t), \mathcal{N}(Sx, Ax, t), \mathcal{N}(Ty, By, t), \mathcal{N}(Uz, Cz, t)\} * \{\max\{\mathcal{N}(By, Ty, t), \mathcal{N}(Ax, Ty, t), \mathcal{N}(By, Sx, t)\}\} * \{\max\{\mathcal{N}(Cz, Uz, t), \mathcal{N}(By, Uz, t), \mathcal{N}(Cz, Ty, t)\}\}] \qquad \textbf{(3.1.1)}$$

for all $x, y, z \in X$ and $\phi, \psi : (0, 1) \to (0, 1)$, such that $\phi(t) > t$ and $\psi(t) < t$ for all $0 < t < 1$.
Then there exist a unique common fixed point of A, B, C, S, T and U.

Proof:
Let the pairs {A, S}, {B, T} and {C, U} be occasionally weakly compatible. So there $q \in (0, 1)$ such that Ax = Sx, By = Ty and Cz = Uz we claim that Ax = By = Cz.
If not, by inequality (3.1.1)

$$\begin{aligned}\mathcal{M}(Ax, By, Cz, qt) &\geq \phi\,[\min\{\mathcal{M}(Sx, Ty, Uz, t), \mathcal{M}(Sx, Ax, t), \mathcal{M}(Ty, By, t), \mathcal{M}(Uz, Cz, t)\} \\ &\quad * \{\min\{\mathcal{M}(By, Ty, t), \mathcal{M}(Ax, Ty, t), \mathcal{M}(By, Sx, t)\} \\ &\quad * \{\min\{\mathcal{M}(Cz, Uz, t), \mathcal{M}(By, Uz, t), \mathcal{M}(Cz, Ty, t)\}\}] \\ &= \phi\,[\min\{\mathcal{M}(Ax, By, Cz, t), \mathcal{M}(Ax, Ax, t), \mathcal{M}(By, By, t), \mathcal{M}(Cz, Cz, t)\} \\ &\quad * \{\min\{\mathcal{M}(By, By, t), \mathcal{M}(Ax, By, t), \mathcal{M}(By, Ax, t)\}\} \\ &\quad * \{\min\{\mathcal{M}(Cz, Cz, t), \mathcal{M}(By, Cz, t), \mathcal{M}(Cz, By, t)\}\}] \\ &= \phi\,[\mathcal{M}(Ax, By, Cz, t)] \\ &> \mathcal{M}(Ax, By, Cz, t)\end{aligned}$$ and

$\mathcal{N}$(Ax, By, Cz, qt) ≤ ψ [max { $\mathcal{N}$(Sx, Ty, Uz, t), $\mathcal{N}$(Sx, Ax, t), $\mathcal{N}$(Ty, By, t), $\mathcal{N}$(Uz, Cz, T) }

◊ {max { $\mathcal{N}$(By, Ty, t), $\mathcal{N}$(Ax, Ty, t), $\mathcal{N}$(By, Sx, t)}}

◊ {max { $\mathcal{N}$(Cz, Uz, t), $\mathcal{N}$(By, Uz, t), $\mathcal{N}$(Cz, Ty, t)}}]

= ψ [max { $\mathcal{N}$(Ax, By, Cz, t), $\mathcal{N}$(Ax, Ax, t), $\mathcal{N}$(By, By, t), $\mathcal{N}$(Cz, Cz, t) }]

◊ {max { $\mathcal{N}$(By, By, t), $\mathcal{N}$(Ax, By, t), $\mathcal{N}$(By, Ax, t)}}

◊ {max { $\mathcal{N}$(Cz, Cz, t), $\mathcal{N}$(By, Cz, t), $\mathcal{N}$(Cz, By, t)}}]

= ψ ($\mathcal{N}$(Ax, By, Cz, t))

< $\mathcal{N}$(Ax, By, Cz, t)

Therefore Ax = By = Cz. That is Ax = Sx = By = Ty = Cz = Uz

Suppose that there is another point z_1 such that $Bz_1=Tz_1$.

Then by inequality (3.1.1) we have $Bz_1 = Tz_1 = Cz_1= Uz_1 = Az_1 = Sz_1$.

So, By = Bz_1 and v = By = Ty is the unique point of coincidence of B and T .

u is the only common fixed point of B and T.

Similarly there is a unique point $z_1 \in X$ such that $Cz_1 = Uz_1 = Az_1 = Sz_1$,

Assume that $u \neq z_1$, Then we have

$\mathcal{M}$ (u, z_1, z, qt) = $\mathcal{M}$ (Au, Bz_1, Cz, qt)

≥ ϕ [min { $\mathcal{M}$ (Su,Tz_1,Uz,t), $\mathcal{M}$ (Su, Au, t), $\mathcal{M}$ (Tz, Bz_1, t), $\mathcal{M}$ (Uz, Cz, t) }

* {min { $\mathcal{M}$ (Bz_1,Tz_1,t), $\mathcal{M}$ (Au,Tz_1,t), $\mathcal{M}$ (Bz_1,Su,t)}}

* {min { $\mathcal{M}$ (Cz, Uz, t), $\mathcal{M}$ (By, Uz, t), $\mathcal{M}$ (Cz, Tz_1, t)}}]

= ϕ [min { $\mathcal{M}$ (u, z_1, z,t), $\mathcal{M}$ (u, u, t), $\mathcal{M}$ (z_1, z, t), $\mathcal{M}$ (z, z, t) }

* {min { $\mathcal{M}$ (z_1, z_1, t), $\mathcal{M}$ (u, z_1, t), $\mathcal{M}$ (z_1, u, t)}}

* {min { $\mathcal{M}$ (z, z, t), $\mathcal{M}$ (z_1, z, t), $\mathcal{M}$ (z, z_1,t)}}]

= ϕ [$\mathcal{M}$ (u, z_1, z, t)]

> $\mathcal{M}$ (u, z_1, z, t) and

$\mathcal{N}$(u, z_1,z,qt) = $\mathcal{N}$(Au, B z_1,Cz,qt)

≤ ψ [max { $\mathcal{N}$(Su,T z_1,Uz,t), $\mathcal{N}$(Su, Au, t), $\mathcal{N}$(Tz, Bz_1, t), $\mathcal{N}$(Uz, Cz, T)}

◊ {max { $\mathcal{N}$(Bz_1, Tz_1, t), $\mathcal{N}$(Au, Tz_1, t), $\mathcal{N}$(Bz_1, Su, t)}}

◊ {max { $\mathcal{N}$(Cz, Uz, t), $\mathcal{N}$(By, Uz, t), $\mathcal{N}$(Cz, Tz_1, t)}}]

= ψ [max { $\mathcal{N}$(u, z_1, z, t), $\mathcal{N}$(u, u, t), $\mathcal{N}$(z_1, z, t), $\mathcal{N}$(z, z, t) }

◊ {max { $\mathcal{N}$(z_1, z_1, t), $\mathcal{N}$(u, z_1, t), $\mathcal{N}$(z_1, u, t)}

◊ {max { $\mathcal{N}$(z, z, t), $\mathcal{N}$(z_1, z, t), $\mathcal{N}$(z, z_1, t)}}]

= ψ [$\mathcal{N}$(u, z_1, z, t)]

< $\mathcal{N}$(u, z_1, z, t)

Therefore we have u = z_1 = z then z_1 is a common fixed point of A, B, C, S, T and U.

The uniqueness of the fixed point holds from inequality (3.1.1).

Theorem 3.2:
Let (X, $\mathcal{M}$, $\mathcal{N}$, *, ◊) be a complete intuitionistic fuzzy metric space and let A, B, C, S, T and U be self mapping of X . Let the pairs {A, S}, {B, T}and {C, U} be occasionally weakly compatible. If there exist q ∈ (0, 1) such that

$$\mathcal{M}(Ax, By, Cz, qt) \geq [\min\{\mathcal{M}(Sx, Ty, Uz, t), \mathcal{M}(Sx, Ax, t), \mathcal{M}(By, Ty, t), \mathcal{M}(Cz, Uz, t)\} * \{\min\{\mathcal{M}(Ax, Ty, t), \mathcal{M}(By, Uz, t), \mathcal{M}(By, Sx, t), \mathcal{M}(Cz, Ty, t)\}\}] \text{ and}$$
$$\mathcal{N}(Ax, By, Cz, qt) \leq [\max\{\mathcal{N}(Sx, Ty, Uz, t), \mathcal{N}(Sx, Ax, t), \mathcal{N}(By, Ty, t), \mathcal{N}(Cz, Uz, t)\} * \{\max\{\mathcal{N}(Ax, Ty, t), \mathcal{N}(By, Uz, t), \mathcal{N}(By, Sx, t), \mathcal{N}(Cz, Ty, t)\}\}] \quad \textbf{(3.2.1)}$$

for all x, y, z ∈ X and for all t > 0 then there exist a unique point z_1 ∈ X, such that $Az_1 = Sz_1 = Bz_1 = Tz_1 = Cz_1 = Uz_1 = z_1$.

Then v ∈ X is a unique point fixed such that Av = Sv = Bv = Tv = Cv = Uv = v, moreover z_1 = v so that there is a unique common fixed point of A, B, C, S, T and U.

Proof:
Let the pairs {A, S},{B, T} and {C,V} be occasionally weakly compatible. so there are points x, y, z ∈ X such that Ax = Sx, By = Ty and Cz = Uz we claim that By = Cz, If not, using inequality (3.2.1), we obtain

$$\begin{aligned}\mathcal{M}(Ax, By, Cz, qt) \geq\ & [\min\{\mathcal{M}(Sx, Ty, Uz, t), \mathcal{M}(Sx, Ax, t), \mathcal{M}(By, Ty, t), \mathcal{M}(Cz, Uz, t)\} \\ & * \{\min\{\mathcal{M}(Ax, Ty, t), \mathcal{M}(By, Uz, t), \mathcal{M}(By, Sx, t), \mathcal{M}(Cz, Ty, t)\}\}] \\ =\ & [\min\{\mathcal{M}(Ax, By, Cz, t), \mathcal{M}(Ax, Ax, t), \mathcal{M}(By, By, t), \mathcal{M}(Cz, Cz, t)\} \\ & * \{\min\{\mathcal{M}(Ax, By, t), \mathcal{M}(By, Ax, t), \mathcal{M}(By, Cz, t), \mathcal{M}(Cz, By, t)\}\}] \\ =\ & \mathcal{M}(Ax, By, Cz, t) \text{ and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(Ax, By, Cz, qt) \leq\ & [\max\{\mathcal{N}(Sx, Ty, Uz, t), \mathcal{N}(Sx, Ax, t), \mathcal{N}(By, Ty, t), \mathcal{N}(Cz, Uz, t)\} \\ & \Diamond \{\max\{\mathcal{N}(Ax, Ty, t), \mathcal{N}(By, Uz, t), \mathcal{N}(By, Sx, t), \mathcal{N}(Cz, Ty, t)\}\}] \\ =\ & [\max\{\mathcal{N}(Ax, By, Cz, t), \mathcal{N}(Ax, Ax, t), \mathcal{N}(By, By, t), \mathcal{N}(Cz, Cz, t)\} \\ & \Diamond \{\max\{\mathcal{N}(Ax, By, t), \mathcal{N}(By, Ax, t), \mathcal{N}(By, Cz, t), \mathcal{N}(Cz, By, t)\}\}] \\ =\ & (\mathcal{N}(Ax, By, Cz, t))\end{aligned}$$

Therefore Ax = By = Cz. That is Ax = Sx = By = Ty = Cz = Uz
Suppose that there is another point z_1 such that $Bz_1 = Tz_1$.
Then by inequality (3.2.1). We have $Az_1= Sz_1= Bz_1= Tz_1= Cz_1= Uz_1$.
So, By = Bz_1 and u = By = Ty is the unique point of coincidence of B and T then U is the only common fixed point of B and T.
Similarly that is a unique point z_1 ∈ X such that $z_1 = Az_1 = Sz_1 = Cz_1 = Uz_1$.
Assume that v ≠ z_1 we have

$\mathcal{M}(v, z_1, z, qt) = \mathcal{M}(Au, Bz_1, Cz, qt)$
$\geq [\min\{\mathcal{M}(Sx, Tz_1, Uz, t), \mathcal{M}(Sx, Az, t), \mathcal{M}(Bz_1, Tz_1, t), \mathcal{M}(Cz, Uz, t)\}$
$* \{\min\{\mathcal{M}(Ax, Tz_1, t), \mathcal{M}(Bz_1, Uz, t), \mathcal{M}(Bz_1, Su, t), \mathcal{M}(Cz, Tz_1, t)\}\}]$
$= [\min\{\mathcal{M}(v, z_1, z, t), \mathcal{M}(v, z_1, t), \mathcal{M}(z_1, z_1, t), \mathcal{M}(z_1, z_1, t)\})$
$* \{\min\{\mathcal{M}(v, z_1, z, t), \mathcal{M}(z_1, z, t), \mathcal{M}(z_1, v, t), \mathcal{M}(z, z_1, t)\})$
$= \mathcal{M}(v, z_1, z, t))$ and
$\mathcal{N}(u, z_1, z, qt) = \mathcal{N}(Au, Bz_1, Cz, qt)$
$\leq [\max\{\mathcal{N}(Sx, Tz_1, Uz, t), \mathcal{N}(Sx, Az, t), \mathcal{N}(Bz_1, Tz_1, t), \mathcal{N}(Cz, Uz, t)\}$
$\Diamond \{\max\{\mathcal{N}(Ax, Tz_1, t), \mathcal{N}(Bz_1, Uz, t), \mathcal{N}(Bz_1, Sv, t), \mathcal{N}(Cz, Tz_1, t)\}\}]$
$= [\max\{\mathcal{N}(v, z_1, z, t), \mathcal{N}(v, z_1, t), \mathcal{N}(z_1, z_1, t), \mathcal{N}(z_1, z_1, t)\}$
$\Diamond \{\max\{\mathcal{N}(v, z_1, z, t), \mathcal{N}(z_1, z, t), \mathcal{N}(z_1, v, t), \mathcal{N}(z, z_1, t)\}\}]$
$= \mathcal{N}(v, z_1, z, t) \Rightarrow v = z_1 = z_1$.

Theorem 3.3:
Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, C, S, T and U be self mapping of X. Let the pairs {A, S},{B, T}and {C, U} be occasionally weakly compatible. If there exist $q \in (0,1)$ such that

$\mathcal{M}(Ax, By, Cz, qt) \geq \phi\, [\mathcal{M}(Sx, Ty, Uz, t), \mathcal{M}(Sx, Bx, t), \mathcal{M}(Ty, Cz, t)\}, \mathcal{M}(By, Ty, t),$
$\mathcal{M}(Cz, Uz, t), \mathcal{M}(Ax, Ty, t), \mathcal{M}(By, Uz, t)\,]$ and
$\mathcal{N}(Ax, By, Cz, qt) \leq \psi\, [\mathcal{N}(Sx, Ty, Uz, t), \mathcal{N}(Sx, Bx, t), \mathcal{N}(Ty, Cz, t)\}, \mathcal{N}(By, Ty, t),$
$\mathcal{N}(Cz, Uz, t), \mathcal{N}(Ax, Ty, t), \mathcal{N}(By, Uz, t)\,]$ **(3.3.1)**

for all $x, y, z \in X$ and $\phi, \psi : (0, 1)^4 \to (0, 1)$ such that $\phi(t, t, 1, t) > t$ and $\psi(t, t, 0, t) < t$ for all $0 < t < 1$. Then there exist a unique common fixed point of A, B, C, S, T and U.

Proof:
Let the pairs {A, S},{B, T} and {C, U} be occasionally weakly compatible
So there are point $x, y, z \in X$ such that Ax = Sx. By = Ty and By = Uz .
we claim that By = Cz, by inequality (3.3.1) we have

$\mathcal{M}(Ax, By, Cz, qt) \geq \phi\, [\mathcal{M}(Sx, Ty, Uz, t), \mathcal{M}(Sx, By, t), \mathcal{M}(Ty, Cz, t), \mathcal{M}(By, Ty, t),$
$\mathcal{M}(Cz, Uz, t), \mathcal{M}(Ax, Ty, t), \mathcal{M}(By, Uz, t)\,]$
$= \phi\, [\mathcal{M}(Ax, By, Cz, t), \mathcal{M}(Ax, By, t), \mathcal{M}(By, Cz, t), \mathcal{M}(By, By, t),$
$\mathcal{M}(Cz, Cz, t), \mathcal{M}(Ax, By, t), \mathcal{M}(By, Cz, t)\,]$
$= \phi\, [\mathcal{M}(Ax, By, Cz, t), \mathcal{M}(Ax, By, t), \mathcal{M}(By, Cz, t), 1, 1, \mathcal{M}(Ax, By, t),$
$\mathcal{M}(By, Cz, t)]$
$> \mathcal{M}(Ax, By, Cz, t)$ and

$\mathcal{N}(Ax, By, Cz, qt) \leq \psi\, [\mathcal{N}(Sx, Ty, Uz, t), \mathcal{N}(Sx, By, t), \mathcal{N}(Ty, Cz, t), \mathcal{N}(By, Ty, t),$
$\mathcal{N}(Cz, Uz, t), \mathcal{N}(Ax, Ty, t), \mathcal{N}(By, Uz, t)]$
$= \psi\, [\mathcal{N}(Ax, By, Cz, t), \mathcal{N}(Ax, By, t), \mathcal{N}(By, Cz, t), \mathcal{N}(By, By, t),$

$\mathcal{N}$(Cz, Cz, t), $\mathcal{N}$(Ax, By, t), $\mathcal{N}$(By, Cz, t)]

= ψ [$\mathcal{N}$(Ax, By, Cz, t), $\mathcal{N}$(Ax, By, t), $\mathcal{N}$(By, Cz, t), 0, 0, $\mathcal{N}$(Ax, By, t), $\mathcal{N}$(By, Cz, t)]

< $\mathcal{N}$(Ax, By, Cz, t)

Therefore Ax = By = Cz. That is Ax = Sx = By = Ty = Cz = Uz .

Suppose that there is another point z_1 such that $Bz_1 = Tz_1$. Then by inequality (3.3.1) We have $Bz_1 = Tz_1 = Cz = Uz = Az_1 = Sz_1$, So, $Cz = Cz_1$ and v = Cz = Tz is the unique point of coincidence of C and T then v is a unique common fixed point of C and U.

Similarly there is a unique point $z_1 \in X$ such that $Z = Bz_1 = Tz_1$, thus z_1 is a common fixed point of A, B, C, S, T and U. The uniqueness of the fixed point holds from (3.3.1).

$\mathcal{M}$ (v, z_1, z, qt) = $\mathcal{M}$ (Av, Bz_1, Cz, qt)

≥ ϕ [$\mathcal{M}$ (Sv, Tz_1, Uz, t), $\mathcal{M}$ (Sv, Bz_1, t), $\mathcal{M}$ (Tz_1, Cz, t), $\mathcal{M}$ (Bz_1, Tz_1, t), $\mathcal{M}$ (Cz, Uz, t), $\mathcal{M}$ (Av, Tz_1, t), $\mathcal{M}$ (Bz, Uz, t)]

= ϕ [$\mathcal{M}$ (v, z_1, z, t), $\mathcal{M}$ (v, z_1, t), $\mathcal{M}$ (z_1, z, t), $\mathcal{M}$ (z_1, z, t), $\mathcal{M}$ (z, z, t), $\mathcal{M}$ (v, z_1, t), $\mathcal{M}$ (z_1, z, t)]

= $\mathcal{M}$(v, z_1, z, t) and

$\mathcal{N}$(v, z_1, z, qt) = $\mathcal{N}$(Av, Bz_1, Cz, qt)

≤ ψ [$\mathcal{N}$(Sv, Tz_1, Uz, t), $\mathcal{N}$(Sv, Bz_1, t), $\mathcal{N}$(Tz_1, Cz, t), $\mathcal{N}$(Bz_1, Tz_1, t), $\mathcal{N}$(Cz, Uz, t), $\mathcal{N}$(Av,Tz_1, t), $\mathcal{N}$(Bz, Uz, t)]

= ψ [$\mathcal{N}$(v, z_1, z, t), $\mathcal{N}$(v, z_1, t), $\mathcal{N}$(z_1, z, t), $\mathcal{N}$(z_1, z, t), $\mathcal{N}$(z, z, t), $\mathcal{N}$(v, z_1, t), $\mathcal{N}$(z_1, z, t)]

= $\mathcal{N}$(v, z_1, z, t) ⇒ v = z_1 = z .

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