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Fuzzy ℓ-Semigroup Via Fuzzy Partial Ordering

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Abstract

In this paper we introduce the notion of fuzzy ℓ - semigroup via fuzzy partial ordering and investigate some properties connected with fuzzy ℓ - semigroup.

Keyword: Fuzzy ℓ - semigroup

1 INTRODUCTION

The notion of a fuzzy set from the crisp set was introduced by L.A. Zadeh[6] and the study of fuzzy algebraic structures was initiated by Rosenfeld, since then various algebraic structures are converted to fuzzy algebra. The application of group theory is important in the design of fast adders and error- correcting codes, it can be used in number of applications dealing with topics such as compilation of expression in polish notation, language and grammars and for the theory of fast adders and error - detecting and correcting codes .

Lattice structure has been found to be extremely important in the areas of communication systems and information analysis. Some system models often include exessive complexity of the situation which in turn may lead to consequence where it is difficult to formulate the model or the model is too complicated to be used in practice.

Nanda. S [3] defined the notion of fuzzy lattice latter Kanakana chakraborty [2] has modified the definition for fuzzy lattice. N. K. Saha has defined the concept of Γ – semigroups and established a relation between regular Γ - semigroup and Γ – group.K.L.N. Samy[5] has investigated dually residuated lattice ordered semigroups.

In this paper we introduce the notion of fuzzy ℓ -semigroups via fuzzy partial ordering and investigate some properties connected with fuzzy lattice of fuzzy ℓ -semigroups.

2 PRELIMINARIES

Let Λ be any set and let η be a fuzzy relation defined over Λ. Then η is said to be Max-min transitive if $\eta.\eta \subseteq \eta$ or more explicitly if $\forall(a,b,c) \in \Lambda^3$ $\mu_{\eta(a,c)} \geq min\{\mu_{\eta(a,b)}, \mu_{\eta(b,c)}\}$ Reflexive if $\forall a \in \Lambda, \mu_{\eta(a,a)} = 1$

Perfect antisymmetric if $\forall(a,b) \in \Lambda^2, a \neq b, \mu_{\eta(a,b)} > 0 \Rightarrow \mu_{\eta(b,a)} = 0$, where $\mu_{\eta(a,b)}$ represent the membership value of the pair $(a,b) \in \eta$.

The fuzzy relation $\overline{P}$ defined over a set Λ is said to be fuzzy partial ordering if and only if it is reflexive, max- min transitive and perfectly antisymmetric. A set Λ along with a fuzzy partial ordering $\overline{P}$ defined on it is called a fuzzy partially ordered set.

Let Λ be a fuzzy partially ordered set with a fuzzy partial order $\overline{P}$ defined over it with each $a \in \Lambda$ we associate two fuzzy sets

The dominating class $\overline{P} \geq (a)(b) = \overline{P}(b,a)$

The dominating class $\overline{P} \leq (a)(b) = \overline{P}(a,b)$

Let ω be a non fuzzy subset of η.

Then the fuzzy upper bound of ω denoted by $U_{\phi(\omega)} = \bigcap_{a\in\omega} \overline{P} \geq (a)$.

Then the fuzzy lower bound of ω denoted by $L_{\phi(\omega)} = \bigcup_{a\in\omega} \overline{P} \leq (a)$.

Definition 2.1 *Let $\overline{\tau}$ be a fuzzy partially ordered set and let $\overline{\sigma}$ be a fuzzy subset of $\overline{\tau}$. Then $\overline{\sigma}$ is said to be a fuzzy lattice in $\overline{\tau}$ if every pair of elements in $\overline{\tau}$ has a fuzzy lower bound L_ϕ and fuzzy upper bound U_ϕ, where both L_ϕ and U_ϕ are fuzzy subsets of $\overline{\tau}$ satisfying the following two conditions:*

$$\mu_{max\{U_\phi\}(a)} \geq \mu_{\overline{\sigma}(a)}, \forall a \in \overline{\tau}$$

$$\mu_{min\{L_\phi\}(a)} \geq \mu_{\overline{\sigma}(a)}, \forall a \in \overline{\tau}$$

$\overline{\chi}$	η_1	η_2	η_3
η_1	1	.9	.8
η_2	0	1	0
η_3	0	.9	1

Example 2.1 *Let* $\bar{\tau} = \{\eta_1, \eta_2, \eta_3\}$ *be a set and* $\bar{\chi}$ *be a fuzzy partial order defined on* $\bar{\tau}$ *as below:*

Let $\bar{\sigma} = \{\eta_1/0, \eta_2/0, \eta_3/.7\}$ be any fuzzy subset of $\bar{\tau}$,

We have upper and lower bounds for $\{\eta_1, \eta_2, \eta_3\}$

$$U_{\phi(\eta_1,\eta_2)} = \{\eta_1/.9, \eta_2/0, \eta_3/0\}$$

$$L_{\phi(\eta_1,\eta_2)} = \{\eta_1/1, \eta_2/1, \eta_3/.8\}$$

$$U_{\phi(\eta_2,\eta_3)} = \{\eta_1/.8, \eta_2/0, \eta_3/.9\}$$

$$L_{\phi(\eta_2,\eta_3)} = \{\eta_1/0, \eta_2/1, \eta_3/1\}$$

$$U_{\phi(\eta_1,\eta_3)} = \{\eta_1/.8, \eta_2/0, \eta_3/0\}$$

$$L_{\phi(\eta_1,\eta_3)} = \{\eta_1/1, \eta_2/.9, \eta_3/1\}$$

Therefore, $max\{U_\phi\} = \{\eta_1/.9, \eta_2/0, \eta_3/.9\}$

$$min\{L_\phi\} = \{\eta_1/0, \eta_2/.9, \eta_3/.8$$

$$\mu_{max\{U_\phi\}(a)} \geq \mu_{\bar{\sigma}(a)}, \forall a \in \bar{\tau}$$

$$\mu_{min\{L_\phi\}(a)} \geq \mu_{\bar{\sigma}(a)}, \forall a \in \bar{\tau}$$

Thus $\bar{\sigma}$ is a fuzzy lattice. But if we consider $\bar{\sigma} = \{\eta_1/.5, \eta_2/0, \eta_3/.9\}$, then $\bar{\sigma}$ is not a fuzzy lattice on $\bar{\tau}$.

Proposition 2.1 *From the above example it is trivial that if,* $\bar{\tau} \leq \bar{\sigma}$ *then* $\bar{\tau}$ *is a fuzzy lattice.*

3 PROPERTIES OF FUZZY ℓ-SEMI-GROUP

Definition 3.1 *A set* Γ *is said to be a fuzzy* ℓ*-semigroup on* μ *if it satifies the following axioms: (i)* $\mu(xy) \geq min(\mu(x), \mu(y))$

(ii) $\mu_{max\{U_\phi\}(x)} \geq min\mu_{A(x)}$

(iii) $\mu_{min\{L_\phi\}(x)} \geq min\mu_{A(x)}$

Example 3.1 *Let* $(Z_3,*)$ *is a fuzzy* ℓ*-semigroup:*

$*$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Let $\overline{\sigma} = \{0,.6,.8\}$ be any fuzzy subset of $\overline{\tau}$,

We have upper and lower bounds for $\{0,1,2\}$

$$U_{\phi(0,1)} = \{0,0,0\}$$

$$L_{\phi(0,1)} = \{0,0,0\}$$

$$U_{\phi(1,2)} = \{0,1,1\}$$

$$L_{\phi(1,2)} = \{0,2,2\}$$

$$U_{\phi(0,2)} = \{0,0,0\}$$

$$L_{\phi(0,2)} = \{0,2,1\}$$

Therefore, $max\{U_\phi\} = \{0,1,1$

$$min\{L_\phi\} = \{0,1,1\}$$

$$\mu_{max\{U_\phi\}(a)} \geq \mu_{\overline{\sigma}(a)}, \forall a \in \overline{\tau}$$

$$\mu_{min\{L_\phi\}(a)} \geq \mu_{\overline{\sigma}(a)}, \forall a \in \overline{\tau}$$

Thus $\overline{\sigma}$ is a fuzzy ℓ- semigroup.

But if we consider $\overline{\sigma} = \{1,.8,.5\}$, then $\overline{\sigma}$ is not a fuzzy ℓ-semigroup on $\overline{\tau}$.

Theorem 3.1 *Let* Γ *and* Γ' *be two fuzzy lattice ordered semigroup and* $\mu : \Gamma \to \Gamma'$ *be a homomorphism. If* η *is a fuzzy lattice of* Γ' *then the preimage* $\Gamma^{-1}(\eta)$ *is a fuzzy lattice ordered semigroup on* Γ.

Proof: Assume that η is a fuzzy lattice on Γ.

Define $\mu_{\Gamma-1(\eta)} a = \mu_\eta \Gamma(a) = x$.

Let $x, y \in \Gamma$.

$$\mu_{\Gamma^{-1}(\eta)}(xy) = \mu_\eta \Gamma(xy)$$
$$= \mu_\eta(\Gamma(x)\Gamma(y))$$
$$\geq min\{\mu_\eta \Gamma(x), \mu_\eta \Gamma(y)\}$$
$$\geq min\{\mu_{\Gamma'(\eta)}(x), \mu_{\Gamma'(\eta)}(x)\}$$
$$\mu_{\Gamma^{-1}(\eta)}[\mu_{max\{U_\phi\}(x,y)}] = \mu_{\Gamma^{-1}(\eta)}[max(x,y)]$$
$$= \mu_\eta\{\Gamma(x \vee y)\}$$
$$\geq min\{\mu_\eta(x), \mu_\eta(y)\}$$
$$\geq min\{\mu_{\Gamma^{-1}(\eta)}(x), \mu_{\Gamma^{-1}(\eta)}(y)\}$$
$$\mu_{\Gamma^{-1}(\eta)}[\mu_{min\{L_\phi\}(x,y)}] = \mu_{\Gamma^{-1}(\eta)}[min(x,y)]$$
$$= \mu_\eta\{\Gamma(x \wedge y)\}$$
$$\geq min\{\mu_\eta(x), \mu_\eta(y)\}$$
$$\geq min\{\mu_{\Gamma^{-1}(\eta)}(x), \mu_{\Gamma^{-1}(\eta)}(y)\}$$

Therefore $\Gamma^{-1}(\eta)$ is a fuzzy lattice ordered subsemigroup.

Proposition 3.1 *If* Γ *is a fuzzy lattice ordered semigroup then,*
$\mu(a) \leq \mu(b) \Rightarrow \mu(a-c) \leq \mu(b-c)$ and $\mu(c-b) \leq \mu(c-a)$, for all a,b,c in Γ.

Proposition 3.2 *If* Γ *is a fuzzy lattice ordered semigroup then,*
$max(\mu(a), \mu(b)) - \mu(c) = max(\mu(a-c), \mu(b-c)$, for all a,b,c in Γ.

Proposition 3.3 *If* Γ *is a fuzzy lattice ordered semigroup then,*
$\mu(a) - max(\mu(b), \mu(c)) = min(\mu(a-b), \mu(a-c)$, for all a,b,c in Γ.

Proposition 3.4 *If* Γ *is a fuzzy lattice ordered semigroup then,*
$\mu(a) - min(\mu(b), \mu(c)) = max(\mu(a-b), \mu(a-c)$, for all a,b,c in Γ.

CONCLUSION

So far in research algebraic structures were converted to lattice algebraic structure, in this paper we convert fuzzy ℓ - semigroup via fuzzy partial ordering. In the same way it will be interesting to convert other fuzzy lattice structure via fuzzy partial ordering.

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