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FUZZY PRIME L-FILTERS

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Abstract

In this paper, using the concept of fuzzy L-filter, definition of fuzzy prime L-filter in a lattice is defined. Some elementary properties, propositions, corollary and theorems of fuzzy prime L-filters are derived. Also some examples related to fuzzy prime L-filter are given.

Keywords: Fuzzy L-filter, level fuzzy L-filter, fuzzy prime L-filter.

1. INTRODUCTION

Nowadays in modern mathematics, the concept of fuzzy is an emerging topic. The concept of fuzzy sets was introduced in 1965 by L.A.Zadeh [11]. In that, the fuzzy group was introduced by Rosenfield [8]. Yuan and Wu [9] applied the concepts of fuzzy sets in lattice theory. The idea of fuzzy sublattice was introduced by Ajmal [1]. In paper [4], fuzzy L-filters and level fuzzy L-filters, theorems and examples are given. In this paper, the concept of fuzzy prime L-filter in lattices is introduced. The new definition of fuzzy prime L-filter is defined and examples are given. The properties of fuzzy prime L-filter are discussed. The union and intersection of two fuzzy prime L-filters are derived.

2. PRELIMINARIES

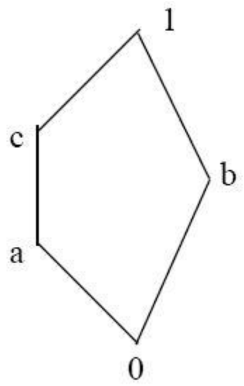
In this section, definition of fuzzy L-filter, level fuzzy L-filter and related examples are given.

2.1 Definition[1]

Let L be a lattice. Let μ be a fuzzy set in L. Then μ is called a fuzzy sublattice of L, if $\forall$ x, y $\in$ L,

(i) $\mu(x \vee y) \geqslant \min \{ \mu(x), \mu(y) \}$
(ii) $\mu(x \wedge y) \geqslant \min \{ \mu(x), \mu(y) \}$.

2.2 Example
Let L = {0, a, b, c, 1}. Let μ:L→ [0, 1] is a fuzzy subset in L defined by $\mu(0) = 0.6$, $\mu(a) = 0.5$, $\mu(b) = 0.4$, $\mu(c) = 0.7$, $\mu(1) = 0.8$. Then μ is a fuzzy sublattice of L.

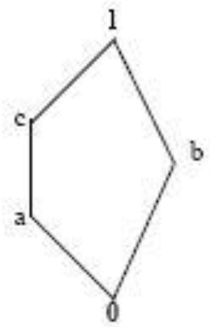


2.3 Definition[1]
Let μ be any fuzzy sublattice of a lattice and let $t \in [0, 1]$. The sublattice $\mu_t = \{x \in L \,/\, \mu(x) \geqslant t\}$ is called a level sublattice of μ.

2.4 Example
From example 2.2, let t = 0.6.Then $\mu_t = \{0, c, 1\}$. Then μ_t is a level sublattice of μ.

2.5 Definition[4]
A fuzzy subset μ: L→ [0,1] of L is called a fuzzy L-filter of L if $\forall$ x, y $\in$ L,
(i) $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(x \wedge y) \leq \min\{\mu(x), \mu(y)\}$.

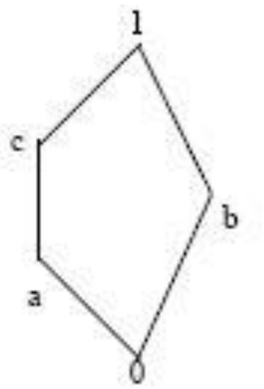
2.6 Example
Let L = {0, a, b, c, 1}. Let μ: L→ [0,1] is a fuzzy set in L defined by $\mu(0) = 0.3$, $\mu(a) = 0.3$, $\mu(b) = 0.3$, $\mu(c) = 0.3$, $\mu(1) = 0.7$. Then, μ is a fuzzy L-filter of L.



2.7 Definition [4]
Let μ_1 and μ_2 be any two fuzzy L-filters of a lattice L. μ_1 is said to be contained in μ_2 if $\mu_1(x) \leqslant \mu_2(x)$, $\forall$ x$\in$ L and is denoted by $\mu_1 \subseteq \mu_2$.

2.8 Definition [4]
Let μ be any fuzzy subset of a lattice L and let $t \in [0, 1]$. Then $\mu_t = \{ x \in L / \mu(x) \leq t\}$ is called level fuzzy L-filter of μ.

2.9 Example [4]
Let L={ 0, a, b, c, 1}. Let μ: L⟶ [0,1] is a fuzzy set in L defined by $\mu(0) = 0.3$, $\mu(a) = 0.3$, $\mu(b) = 0.3$, $\mu(c)= 0.3$, $\mu(1) =0.5$. Then μ is a fuzzy L-filter of L. In this example, let t = 0.3.



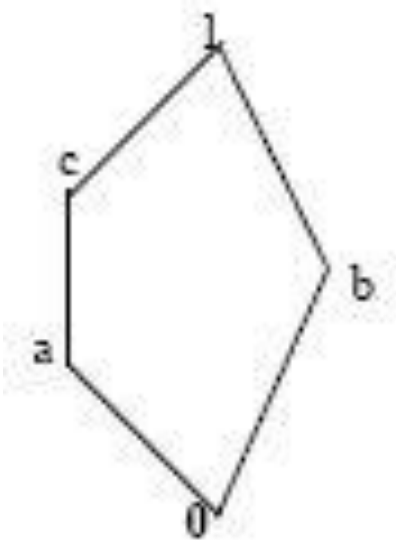
Then $\mu_t = \mu_{0.3} = \{0, a, b\}$.

2.10 Definition
Let μ be a fuzzy L-filter of a lattice L. The level fuzzy L-filters are defined by
$\mu_t = \{ x \in L / \mu(x) \leq t \}$
$\mu_s = \{ x \in L / \mu(x) \leq s \}$.
Clearly, $\mu_s \subseteq \mu_t$ whenever $t < s$.

3. FUZZY PRIME L-FILTERS
3.1 Definition
A fuzzy L-filter μ of a lattice L is said to be a fuzzy prime L-filter of L if
(i) μ is not a constant function and
(ii) for any two fuzzy L-filter σ and θ in L if $\sigma \vee \theta \subseteq \mu$, then either $\sigma \subseteq \mu$ or $\theta \subseteq \mu$.

3.2 Example
Let L = {0, a, b, 1} be a lattice and μ is a fuzzy L-filter of L. Then,
$\mu(0) = 0.6$, $\mu(a) = 0.4$, $\mu(b) = 0.4$, $\mu(1) = 0.4$.

Let σ and θ be any fuzzy L-filter of L. Then,
$\sigma(0) = 0.9$, $\sigma(a) = 0.3$, $\sigma(b) = 0.3$, $\sigma(1) = 0.3$ and
$\theta(0) = 0.5$, $\theta(a) = 0$, $\theta(b) = 0.4$, $\theta(1) = 0.4$.
Here, $\sigma \wedge \theta \subseteq \mu$, $\sigma \not\subset \mu$ but $\theta \subset \mu$.
Hence μ is a fuzzy prime L-filter of L.



3.3 Note
$\sigma \subseteq \mu$ means $\sigma(x) \leq \mu(x)$, for all $x \in L$.

3.4 Definition
A fuzzy L-filter μ of a lattice L is called fuzzy L-prime, if the ideal μ_t, where $t = \mu(0)$, is a prime L-filter of L.

3.5 Proposition
Let μ be any fuzzy L-filter of a lattice L such that each level fuzzy L-filter μ_t, $t \in \mathrm{Im}\mu$, is prime. If $\mu(x) < \mu(y)$ for some $x, y \in L$, then $\mu(x \vee y) = \mu(x)$.

3.6 Corollary
If μ is any fuzzy prime L-filter of a lattice L, then $\mu(x \vee y) = \min \{\mu(x), \mu(y)\}$, for all $x, y \in L$.

3.7 Theorem
Let μ be a fuzzy prime L-filter of a Lattice L. Then card $\mathrm{Im}\mu = 2$.

Proof
Since μ is non constant, card $\mathrm{Im}\mu \geq 2$.
Suppose that card $\mathrm{Im}\,\mu \geq 3$.
Let $\mu(1) = s$ and $k = \mathrm{Sup}\,\{ \mu(x) / x \in L \}$.
Then there exists $t, m \in \mathrm{Im}\mu$ such that $t < m < s$ and $t \leq k$.
Let σ and θ be two fuzzy subsets of L such that $\sigma(x) = ½ (t + m)$, for all $x \in L$ and

$$\theta(x) = \begin{cases} k, & \text{if } x \notin \mu_m = \{ x \in L / \mu(x) \geq m \} \\ s, & \text{if } x \in \mu_m . \end{cases}$$

Clearly, σ is a fuzzy L-filter of L.
To show that θ is a fuzzy L-filter of L.
Let x, y $\in$ L.

Case (i):
If x, y $\in \mu_m$, then $\theta(x) = s$, $\theta(y) = s$, $x\vee y\in\mu_m$ and $x \wedge y \in \mu_m$. Also,

$\theta(x\vee y) = s = \min \{ \theta(x), \theta(y) \}$
$\Rightarrow \theta(x\vee y) \geq \min \{ \theta(x), \theta(y) \}$
$\theta(x\wedge y) = s = \max \{ \theta(x), \theta(y) \}$.
$\Rightarrow \theta(x\wedge y) \geq \max \{ \theta(x), \theta(y) \}$.

Therefore θ is a fuzzy L-filter of L.

Case (ii):
If $x \in \mu_m$ and $y\notin \mu_m$, then $\theta(x) = s$, $\theta(y) = k$, $x\vee y \notin \mu_m$ and $x \wedge y \in \mu_m$. Also,

$\theta(x\vee y) = k = \min \{ \theta(x), \theta(y) \}$
$= \min \{ s, k \}$
$\Rightarrow \theta(x\vee y) \geq \min \{ \theta(x), \theta(y) \}$
$\theta(x\wedge y) = s = \max \{ \theta(x), \theta(y) \}$
$= \max \{ s, k \}$
$\Rightarrow \theta(x\wedge y) \geq \max \{ \theta(x), \theta(y) \}$.

Therefore θ is a fuzzy L-filter of L.

Case (iii):
If $x \notin \mu_m$ and $y \notin \mu_m$, then $\theta(x) = \theta(y) = k$, $x\vee y \notin \mu_m$ and $x\wedge y \notin \mu_m$. Also,

$\theta(x\vee y) = k = \min \{ \theta(x), \theta(y) \}$
$= \min \{ k, k \}$
$\Rightarrow \theta(x\vee y) \geq \min \{ \theta(x), \theta(y) \}$
$\theta(x\wedge y) = k = \max \{ \theta(x), \theta(y) \}$
$= \max \{ k, k \}$
$\Rightarrow \theta(x\wedge y) \geq \max \{ \theta(x), \theta(y) \}$.

Therefore θ is a fuzzy L-filter of L.

Claim:
$\sigma\vee\theta \subseteq \mu$.
Let x $\in$ L. Consider the following cases:

(i) Let x = 1. Then,

$$[\sigma\vee\theta](x) = \max_{x=y\vee z} \{\min (\sigma(y), \theta(z))\} \leq \tfrac{1}{2} (t + m) < s = \mu(1).$$

(ii) Let $x \neq 1$, $x \in \mu_m$. Then $\mu(x) \geq m$, and

$$[\sigma\vee\theta](x) = \max_{x=y\vee z} \{\min (\sigma(y), \theta(z))\}$$

$\leq \frac{1}{2}(t + m) < m$
$= \mu(x)$,
since $\min\{\sigma(y), \theta(z)\} \leq \sigma(y)$.

(iii) Let $x \neq 1$, $x \notin \mu_m$. Then for any $y, z \in L$ such that $x = y \vee z$, $y \notin \mu_m$ and $z \notin \mu_m$.

Thus $\theta(y) = k$ and $\theta(z) = k$.
Hence

$$[\sigma\wedge\theta](x) = \max_{x = y\vee z}\{\min(\sigma(y),\theta(z)\}$$
$$= \max\{\min(k, k)\}$$
$$= k \leq \mu(x).$$

Thus in any case, $[\sigma\vee\theta](x) \leq \mu(x)$.
Hence $\sigma\vee\theta \leq \mu$.
Now there exists $y\in L$ such that $\mu(y) = t$.
Then $\sigma(y) = \frac{1}{2}(t + m) > \mu(y)$.
$\Rightarrow \sigma(y) > \mu(y)$.
Hence $\sigma \subseteq \mu$.
Also, there exists $x\in L$ such that $\mu(x) = t$.
Then $x \in \mu_m$ and thus $\theta(x) = s > m = \mu(x)$.
$\Rightarrow \theta(x) > \mu(x)$
Hence $\theta \subseteq \mu$.
This shows that μ is not a fuzzy prime L-filter of L, which is a contradiction to the hypothesis.
Hence card Imμ = 2.

3.8 Theorem
Let μ be any fuzzy L-filter of a lattice, such that $1\in \text{Im}\mu$. Let θ be any fuzzy prime L-filter of L. Then $\mu\cap\theta$ is a fuzzy prime L-filter of the lattice $\mu_t = \{ x\in L / \mu(x) = 1 \}$.

Proof
To prove: μ is a fuzzy L-filter of L.
Let $x, y \in \mu_t$. Then $\mu(x) = 1$ and $\mu(y) = 1$.
Now,

(i) $\mu(x\vee y) \geq \min\{\mu(x), \mu(y)\}$
$= \min\{1,1\} = 1$.

(ii) $\mu(x\vee y) \geq \max\{\mu(x), \mu(y)\}$
$= \max\{1, 1\} = 1$.

Therefore μ is a fuzzy L-filter of L.
Also given θ is a fuzzy prime L-filter of L.

Claim:

$\mu \cap \theta$ is a fuzzy prime L-filter.

If θ is constant, say $\theta(x) = c$, for all $x \in L$.

Then for all $x \in \mu_t$,

$[\mu \cap \theta](x) = \min \{ \mu(x), \theta(x) \}$

$= \min \{ 1, c \} = c.$

Therefore $\mu \cap \theta$ is a fuzzy prime L-filter, since θ is a fuzzy prime L-filter of L.

Assume that θ is nonconstant.

Then there exists $\alpha \in [0, 1)$, such that

$$\theta(x) = \begin{cases} 1, & \text{if } x \in \theta_t \\ \alpha, & \text{if } x \in L - \theta_t \end{cases}$$

where $\theta_t = \{ x \in L \,/\, \theta(x) = 1 \}$.

⇨ θ_t is a prime L-filter.

⇨ $\theta_t \cap \mu_t$ is a prime L-filter of μ_t.

Next,

$$[\mu \cap \theta](x) = \begin{cases} 1, & \text{if } x \in \theta_t \cap \mu_t \\ \alpha, & \text{if } x \in \mu_t - (\theta_t \cap \mu_t) \end{cases}$$

Hence $\mu \cap \theta$ is a fuzzy prime L-filter of μ_t [since if μ is a prime fuzzy L-filter then CardImμ = 2].

3.9 Theorem

If $\{ \mu_i \,/\, i \in Z_+ \}$ is any collection of nonconstant fuzzy prime filter of a lattice L such that $\mu_1 \subseteq \mu_2 \subseteq \ldots \subseteq \mu_n \subseteq \ldots$, then the following statements are true:

(a). $\cup\, \mu_i$ is a fuzzy prime L-filter of L

(b). $\cap\, \mu_i$ is a fuzzy prime L-filter of L.

3.10 Theorem

Let L be a lattice and let μ be a fuzzy prime L-filter of L. Then $\mu(1) = 1$.

3.11 Theorem

Let L be a lattice and let μ be a fuzzy subset of L such that cardImμ = 2, $\mu(0) = 1$, and the set $\mu_1 = \{ x \in L: \mu(x) = \mu(1) \}$ is a prime L-filter of L. Then μ is a fuzzy prime L-filter of L.

CONCLUSION

The concept of fuzzy prime L-filters is established by giving examples, theorems and some properties. Using these, various results can be developed under the topic fuzzy prime L-filter.

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