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NOTES ON MULTI FUZZY RW-OPEN MAPS, MULTI FUZZY RW-CLOSED MAPS AND MULTI FUZZY RW-HOMEOMORPHISMS IN MULTI FUZZY TOPOLOGICAL SPACE

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Abstract

In this paper, we have studied some of the properties of multi fuzzy rw-open maps, multi fuzzy rw-closed maps and multi fuzzy rw-homeomorphism in multi fuzzy topological spaces and have proved some results on these.

Keywords: Multi fuzzy subset, multi fuzzy topological spaces, multi fuzzy rw-closed, multi fuzzy rw-open, multi fuzzy rw-continuous maps, multi fuzzy rw-irresolute maps, multi fuzzy rw-open maps, multi fuzzy rw-closed maps, multi fuzzy rw-homeomorphism.

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INTRODUCTION:

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [18] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C.L.Chang [5] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren [17], K.K.Azad [2], G.Balasubramanian and P.Sundaram [3, 4], S.R.Malghan and S.S.Benchalli [12, 13] and many others have contributed to the development of fuzzy topological spaces. We have introduced the concept of multi fuzzy rw-open maps, multi fuzzy rw-closed maps and multi fuzzy rw-homeomorphism in multi fuzzy topological spaces and have established some results.

1. PRELIMINARIES:

1.1 Definition[18]:
Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X → [0, 1].

1.2 Definition:
A **multi fuzzy subset** A of a set X is defined as an object of the form A = { ⟨ x, $A_1(x)$, $A_2(x)$, $A_3(x)$, …, $A_n(x)$ ⟩ / x∈X}, where $A_i : X \to [0, 1]$ for all i. It is denoted as A = ⟨ A_1, A_2, A_3, …, A_n ⟩.

1.3 Definition:
Let A and B be any two multi fuzzy subsets of a set X. We define the following relations and operations:
(i) A ⊆ B if and only if $A_i(x) \le B_i(x)$ for all i and for all x in X.
(ii) A = B if and only if $A_i(x) = B_i(x)$ for all i and for all x in X.
(iii) A^c = 1–A = ⟨ 1–A_1, 1–A_2, 1–A_3, …, 1–A_n ⟩.
(iv) A∩ B= { ⟨x, min{$A_1(x)$, $B_1(x)$ }, min{$A_2(x)$, $B_2(x)$}, …, min{$A_n(x)$, $B_n(x)$}⟩ / x∈X }.
(v) A∪B={ ⟨x, max {$A_1(x)$, $B_1(x)$ }, max{$A_2(x)$, $B_2(x)$}, …, max{$A_n(x)$, $B_n(x)$}⟩ / x∈X }.

1.4 Definition:
Let X be a set and ℑ be a family of multi fuzzy subsets of X. The family ℑ is called a multi fuzzy topology on X if and only if ℑ satisfies the following axioms
(i) $\overline{0}$, $\overline{1}$ ∈ℑ,
(ii) If { A_i ; i∈I } ⊆ ℑ, then $\bigcup_{i\in I} A_i \in \Im$,
(iii) If A_1, A_2, A_3,….. A_n∈ℑ, then $\bigcap_{i=1}^{i=n} A_i \in \Im$.

The pair (X, ℑ) is called a multi fuzzy topological space. The members of ℑ are called multi fuzzy open sets in X. A multi fuzzy set A in X is said to be multi fuzzy closed set in X if and only if A^c is a multi fuzzy open set in X.

1.5 Definition:
Let (X, ℑ) be a multi fuzzy topological space and A be a multi fuzzy set in X. Then ∩{ B : B^c ∈ℑ and B ⊇ A } is called multi fuzzy closure of A and is denoted by mfcl(A).

1.6 Definition:

Let (X, $\Im$) be a multi fuzzy topological space and A be a multi fuzzy set in X. Then $\cup\{B : B \in \Im$ and $B \subseteq A\}$ is called multi fuzzy interior of A and is denoted by mfint(A).

1.7 Definition:

Let (X, $\Im$) be a multi fuzzy topological space and A be multi fuzzy set in X. Then A is said to be

(i) multi fuzzy semiopen if and only if there exists a multi fuzzy open set V in X such that $V \subseteq A \subseteq$ mfcl(V).

(ii) multi fuzzy semiclosed if and only if there exists a multi fuzzy closed set V in X such that mfint(V) $\subseteq A \subseteq V$.

(iii) multi fuzzy regular open set of X if mfint(mfcl(A)) = A.

(iv) multi fuzzy regular closed set of X if mfcl(mfint(A)) = A.

(v) multi fuzzy regular semiopen set of X if there exists a multi fuzzy regular open set V in X such that$V \subseteq A \subseteq$ mfcl(V). We denote the class of multi fuzzy regular semiopen sets in multi fuzzy topological space X by MFRSO(X).

(vi) multi fuzzy generalized closed (mfg-closed) if mfcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is multi fuzzy open set and A is multi fuzzy generalized open if $\overline{1} - A$ is multi fuzzy generalized closed.

1.8 Definition:

An multi fuzzy set A of a multi fuzzy topological space (X, $\Im$) is called:

(i) multi fuzzy g-closed if mfcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is multi fuzzy open set in X. (ii) multi fuzzy g-open if its complement A^c is multi fuzzy g-closed set in X. (iii) multi fuzzy rg-closed if mfcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is multi fuzzy regular open set in X. (iv) multi fuzzy rg-open if its complement A^c is multi fuzzy rg-closed set in X. (v) multi fuzzy w-closed if mfcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is multi fuzzy semi open set in X. (vi) multi fuzzy w-open if its complement A^c is multi fuzzy w-closed set in X. (vii) multi fuzzy gpr-closed if pcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is multi fuzzy regular open set in X. (viii) multi fuzzy gpr-open if its complement A^c is multi fuzzy gpr-closed set in X.

1.9 Definition:

Let (X, $\Im$) be a multi fuzzy topological space. A multi fuzzy set A of X is called multi fuzzy regular w-clsoed(briefly, multi fuzzy rw-closed) if mfcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is multi fuzzy regular semiopen in multi fuzzy topological space X.

NOTE:

We denote the family of all multi fuzzy regular w-closed sets in multi fuzzy topological space X by MFRWC(X).

1.10 Definition:
A multi fuzzy set A of a multi fuzzy topological space X is called a multi fuzzy regular w-open (briefly, multi fuzzy rw-open) set if its complement A^C is a multi fuzzy rw-closed set in multi fuzzy topological space X.

NOTE:
We denote the family of all multi fuzzy rw-open sets in multi fuzzy topological space X by MFRWO(X).

1.11 Definition:
A mapping f : X⟶Y from a multi fuzzy topological space X to a multi fuzzy topological space Y is called

(i) multi fuzzy continuous if $f^{-1}(A)$ is multi fuzzy open in X for each multi fuzzy open set A in Y.
(ii) multi fuzzy generalized continuous (mfg-continuous) if $f^{-1}(A)$ is multi fuzzy generalized closed in X for each multi fuzzy closed set A in Y.
(iii) multi fuzzy semi continuous if $f^{-1}(A)$ is multi fuzzy semiopen in X for each multi fuzzy open set A in Y.
(iv) multi fuzzy almost continuous if $f^{-1}(A)$ is multi fuzzy open in X for each multi fuzzy regular open set A in Y.
(v) multi fuzzy irresolute if $f^{-1}(A)$ is multi fuzzy semiopen in X for each multi fuzzy semiopen set A in Y.
(vi) multi fuzzy gc-irresolute if $f^{-1}(A)$ is multi fuzzy generalized closed in X for each multi fuzzy generalized closed set A in Y.
(vii) multi fuzzy completely semi continuous if and only if $f^{-1}(A)$ is an multi fuzzy regular semiopen set of X for every multi fuzzy open set A in Y.
(viii) multi fuzzy w-continuous if and only if $f^{-1}(A)$ is an multi fuzzy w-closed set of X for every multi fuzzy closed A in Y.
(ix) multi fuzzy rg-continuous if $f^{-1}(A)$ is multi fuzzy rg-closed in X for each multi fuzzy closed set A in Y.
(x) multi fuzzy gpr-continuous if $f^{-1}(A)$ is multi fuzzy gpr-closed in X for each multi fuzzy closed set A in Y.
(xi) multi fuzzy almost-irresolute if $f^{-1}(A)$ is multi fuzzy semi open in X for each multi fuzzy regular semi open set A in Y.

1.12 Definition:
A mapping f : X⟶Y from a multi fuzzy topological space X to a multi fuzzy topological space Y is called

(i) multi fuzzy open mapping if f(A) is multi fuzzy open in Y for every multi fuzzy open set A in X.
(ii) multi fuzzy semiopen mapping if f(A) is multi fuzzy semiopen in Y for every multi fuzzy open set A in X.

1.13 Definition:
Let X and Y be multi fuzzy topological spaces. A map f : X⟶ Y is said to be multi fuzzy rw-continuous if the inverse image of every multi fuzzy open set in Y is multi fuzzy rw-open in X.

1.14 Definition:
Let X and Y be multi fuzzy topological spaces. A map f : X⟶Y is said to be a multi fuzzy rw-irresolute map if the inverse image of every multi fuzzy rw-open set in Y is a multi fuzzy rw-open set in X.

1.15 Definition:
Let (X, $\Im$) be a multi fuzzy topological space and A be a multi fuzzy set of X. Then multi fuzzy rw-interior and multi fuzzy rw-closure of A are defined as follows.
mfrwcl(A) = $\cap$\{ K : K is a multi fuzzy rw-closed set in X and A $\subseteq$ K \}.
mfrwint(A) = $\cup$ \{ G : G is a multi fuzzy rw-open set in X and G $\subseteq$ A \}.

Remark: It is clear that A $\subseteq$ mfrwcl(A) $\subseteq$ mfcl(A) for any multi fuzzy set A.

1.16 Theorem:
If A is a multi fuzzy regular open and multi fuzzy rg-closed in multi fuzzy topological space (X, $\Im$), then A is multi fuzzy rw-closed in X.

1.17 Definition:
Let (X, $\Im_1$) and (Y, $\Im_2$) be two multi fuzzy topological spaces. A map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called multi fuzzy rw-open if the image of every multi fuzzy open set in X is multi fuzzy rw-open in Y.

1.18 Definition:
Let (X, $\Im_1$) and (Y, $\Im_2$) be two multi fuzzy topological spaces. A map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called multi fuzzy rw-closed if the image of every multi fuzzy closed set in X is a multi fuzzy rw-closed set in Y.

1.19 Definition:
Let X and Y be multi fuzzy topological spaces. A bijection map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called multi fuzzy rw-homeomorphsim if f and f^{-1} are multi fuzzy rw-continuous.

NOTE:
The family of all multi fuzzy rw-homeomorphism from (X, $\Im$) onto itself is denoted by MFRW-H(X, $\Im$).

1.20 Definition:
A bijection map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called a multi fuzzy rwc-homemorphism if f and f^{-1} are multi fuzzy rw-irresolute. We say that spaces (X, $\Im_1$) and (Y, $\Im_2$) are multi fuzzy rwc-homeomorphism if there exsit a multi fuzzy rwc-homeomorphism from (X, $\Im_1$) onto (Y, $\Im_2$).

NOTE:
The family of all multi fuzzy rwc-homeomorphism from (X, $\Im$) onto itself is denoted by MFRWC-H(X, $\Im$).

2. SOME PROPERTIES:

2.1 Theorem:
Every multi fuzzy open map is a multi fuzzy rw-open map.

Proof:
Let f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) be a multi fuzzy open map andA be a multi fuzzy open set in multi fuzzy topological space X. Then f(A) is a multi fuzzy open set in multi fuzzy topological space Y. Since every multi fuzzy open set is multi fuzzy rw-open, f(A) is a multi fuzzy rw-open set in multi fuzzy topological space Y. Hence f is a multi fuzzy rw-open map.

2.2 Remark: The converse of the above theorem need not be true in general.

2.3 Example:
Let X = Y = { 1, 2, 3 } and the multi fuzzy sets A, B, C be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }, C = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 > }. Consider $\Im_1$ = { 0_X, 1_X, A, B, C } and $\Im_2$ = { 0_Y, 1_Y, A }. Then (X, $\Im_1$) and (Y, $\Im_2$) are multi fuzzy topological spaces. Let f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) be defined as f(1) = f(2) = 1 and f(3) = 3. Then this function is multi fuzzy rw-open but it is not multi fuzzy open, since the image of the multi fuzzy open set C in X is the multi fuzzy set C in Y which is not multi fuzzy open.

2.4 Theorem:
Let f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) and g : (Y, $\Im_2$) ⟶ (Z, $\Im_3$) be two multi fuzzy rw-open maps. Show that g•f need not be multi fuzzy rw-open.

Proof:
Consider the following example, let X = Y = Z = { 1, 2, 3 } and the multi fuzzy sets A, B, C, D be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = { <1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }, C = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }, D = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 > }. Consider $\Im_1$ = { 0_X, 1_X, A, D }

and $\Im_2$ = { 0_Y, 1_Y, A } and $\Im_3$ = { 0_Z, 1_Z, A, B, C }. Then (X, $\Im_1$), (Y, $\Im_2$) and (Z, $\Im_3$) are multi fuzzy topological spaces. Let f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) and g : (Y, $\Im_2$) ⟶ (Z, $\Im_3$) be the identity maps. Then f and g are multi fuzzy rw-open maps but their compositiong•f : (X, $\Im_1$) ⟶ (Z, $\Im_3$) is not multi fuzzy rw-open as D is multi fuzzy open in X but (g•f)(D) = D is not multi fuzzy rw-open in Z.

2.5 Theorem:

If f: (X, $\Im_1$)⟶ (Y, $\Im_2$) is multi fuzzy open map and g: (Y, $\Im_2$) ⟶ (Z, $\Im_3$) is multi fuzzy rw-open map, then their composition g•f : (X, $\Im_1$) ⟶ (Z, $\Im_3$) is multi fuzzy rw-open map.

Proof:

LetA be multi fuzzy open set in (X, $\Im_1$). Since f is multi fuzzy open map, f(A) is a multi fuzzy open set in (Y, $\Im_2$). Since g is a multi fuzzy rw-open map, g(f(A)) is multi fuzzy rw-open set in (Z, $\Im_3$). But g(f(A)) = (g•f)(A). Thus g•f is a multi fuzzy rw-open map.

2.6 Remark:

Every multi fuzzy w-open map is multi fuzzy rw-open but converse may not be true.

Proof:

Consider the example let X = { a, b}, Y = { x, y } and the multi fuzzy set A and B be defined as follows A = { < a, 0.7, 0.7, 0.7 >, < b, 0.8, 0.8, 0.8 > }, B = { < x, 0.7, 0.7, 0.7 >, < y, 0.6, 0.6, 0.6 > }. Then $\Im$ = { 0_X, 1_X, A } and σ = { 0_Y, 1_Y, B **}** be multi fuzzy topologies on X and Y respectively. Then the mapping *f* : (X, $\Im$) → (Y, σ) is defined by *f(a)* = x and *f(b)* = y is multi fuzzy rw-open but it is not multi fuzzy w-open.

2.7 Theorem:

A mapping *f* : (X, $\Im$)→ (Y, σ) is multi fuzzy rw-open if and only if for every multi fuzzy set A of X, *f (mfint(A))* ⊆ mfrwint(f(A)).

Proof:

Let *f* be a multi fuzzy rw-open mapping and A is a multi fuzzy open set in X. Now mfint(A) ⊆ A which implies that *f*(mfint(A) ⊆ *f(A).* Since *f* is a multi fuzzy rw-open mapping, *f(mfint(A))* is multi fuzzy rw-open set in Y such that *f(mfint(A))* ⊆ *f(A)* therefore *f(mfint(A))* ⊆ mfrwint *f(A).*

For the converse suppose thatA is a multi fuzzy open set of X. Then *f*(A) = *f* (mfint(A)) ⊆ mfrwint *f(A).* But mfwint(f(A)) ⊆ *f* (A). Consequently *f* (A) = mfwint(A) which implies that *f(A)* is a multi fuzzy rw-open set of Y and hence *f* is a multi fuzzy rw-open.

2.8 Theorem:
If $f : (X, \Im) \to (Y, \sigma)$ is a multi fuzzy rw-open map then
mfint ($f^{-1}(A)$) $\subseteq$ f^{-1}(mfrwint(A) for every multi fuzzy set A of Y.

Proof:
Let A be a multi fuzzy set of Y. Then mfint($f^{-1}(A)$) is a multi fuzzy open set in X. Since f is multi fuzzy rw-open *f (mfint (f* $^{-1}$(A))) is multi fuzzy rw-open in Y and hence *f(mfint* $f^{-1}(A)$) $\subseteq$ mfrwint(f($f^{-1}(A)$) $\subseteq$ mfrwint(f ($f^{-1}(A)$)) $\subseteq$ mfrwint(A). Thus mfint $f^{-1}(A) \subseteq f^{-1}$(mfrwint(A)).

2.9 Theorem:
A mapping $f : (X, \Im) \to (Y, \sigma)$ is multi fuzzy rw-open if and only if for each multi fuzzy set A of Y and for each multi fuzzy closed set U of X containing $f^{-1}(A)$ there is a multi fuzzy rw-closed V of Y such that $A \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof:
Suppose that f is a multi fuzzy rw-open map. Let A be the multi fuzzy closed set of Y and U be a multi fuzzy closed set of X such that $f^{-1}(A) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is multi fuzzy rw-closed set of Y such that $f^{-1}(V) \subseteq U$.
For the converse suppose that B is an multi fuzzy open set of X. Then $f^{-1}(f(B))^c \subseteq B^c$ and B^c is multi fuzzy closed set in X. By hypothesis there is a multi fuzzy rw-closed set V of Y such that $(f(B))^c \subseteq V$ and $f^{-1}(V) \subseteq B^c$. Therefore $B \subseteq (f^{-1}(V))^c$. Hence$V^c \subseteq f(B) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(B) = V^c$. Since V^c is multi fuzzy rw-open set of Y. Hence f (B) is multi fuzzy rw-open in Y and thus f is multi fuzzy rw-open map.

2.10 Theorem:
Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be multi fuzzy closed map. Then f is a multi fuzzy rw-closed map.

Proof:
Let A be multi fuzzy open set in (X, $\Im_1$). Since f is multi fuzzy closed map, f(A) is a multi fuzzy closed set in (Y, $\Im_2$). Since every multi fuzzy closed set is multi fuzzy rw-closed, f(A) is a multi fuzzy rw-closed set in (Y, $\Im_2$). Hence f is a multi fuzzy rw-closed map.

2.11 Remark: The converse of the above theorem need not be true in general.

2.12 Example:
Let X = Y= [0, 1]. The multi fuzzy sets A, B is defined as
A(x) = < 0.6, 0.6, 0.6 > if x = 1/3 and A(x) = < 1, 1, 1> otherwise, B(x) = < 0.8, 0.8, 0.8 > if x= 1/3 and B(x) = <1, 1, 1> otherwise. Consider $\Im_1 = \{0_X, 1_X, A\}$ and $\Im_2 = \{0_Y, 1_Y, B\}$. Then (X, $\Im_1$) and (Y, $\Im_2$) are multi fuzzy topological spaces. Let f: (X, $\Im_1$)$\longrightarrow$ (Y, $\Im_2$) be the identity map. Then f is a multi fuzzy rw-closed map but it is not

a multi fuzzy closed map, since the image of the multi fuzzy closed set A^C in X is not a multi fuzzy closed set in Y.

2.13 Theorem:
Show that the composition of two multi fuzzy rw-closed maps need not be a multi fuzzy rw-closed map.

Proof:
Consider the multi fuzzy topological spaces (X, $\Im_1$), (Y, $\Im_2$) and (Z, $\Im_3$) and mappings defined in example in Theorem 2.4. The maps f and g are multi fuzzy rw-closed but their composition is not multi fuzzy rw-closed, as A is a multi fuzzy closed set in X but (g•f)(A) = A is not multi fuzzy rw-closed in Z.

2.14 Theorem:
If f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) and g : (Y, $\Im_2$)⟶ (Z, $\Im_3$) be two maps. Then g•f : (X, $\Im_1$)⟶ (Z, $\Im_3$) is multi fuzzy rw-closed map if f is multi fuzzy closed and g is multi fuzzy rw-closed.

Proof:
Let A be a multi fuzzy closed set in (X, $\Im_1$). Since f is a multi fuzzy closed map, f(A) is a multi fuzzy closed set in (Y, $\Im_2$). Since g is a multi fuzzy rw-closed map, g(f(A)) is a multi fuzzy rw-closed set in (Z, $\Im_3$). But g(f(A)) = (g•f)(A). Thus g•f is multi fuzzy rw-closed map.

2.15 Theorem:
A map f : X⟶ Y is multi fuzzy rw-closed if for each multi fuzzy set D of Y and for each multi fuzzy open set E of X such thatE $\supseteq f^{-1}(D)$, there is a multi fuzzy rw-open set A of Y such that $D \subseteq A$ and $f^{-1}(A) \subseteq E$.

Proof:
Suppose that f is multi fuzzy rw-closed. Let D be a multi fuzzy subset of Y and E is a multi fuzzy open set of X such that $f^{-1}(D) \subseteq E$. Let A = 1_Y –f(1–E) is multi fuzzy rw-open set in multi fuzzy topological space Y. Note that $f^{-1}(D) \subseteq E$ which implies $D \subseteq A$ and $f^{-1}(A) \subseteq E$.
For the converse, suppose that E is a multi fuzzy closed set in X. Then $f^{-1}(1_Y - f(E)) \subseteq 1_X - E$ and $1_X - E$ is multi fuzzy open. By hypothesis, there is a multi fuzzy rw-open set A of Y such that $1_Y - f(E) \subseteq A$ and $f^{-1}(A) \subseteq 1_X - E$. Therefore $E \subseteq 1_X - f^{-1}(A)$. Hence $1_Y - A \subseteq f(E)$, $f(1_X - f^{-1}(A)) \subseteq 1_Y - A$ which implies $f(E) = 1_Y - A$. Since $1_Y - A$ is multi fuzzy rw-closed, f(E) is multi fuzzy rw-closed and thus f is multi fuzzy rw-closed.

2.16 Theorem:
Let f: (X, $\Im_1$) ⟶ (Y, $\Im_2$) be multi fuzzy irresolute and A be multi fuzzy regular semiopen in Y. Then $f^{-1}(A)$ is multi fuzzy regular semiopen in X.

Proof:

Let A be multi fuzzy regular semiopen in Y. To prove $f^{-1}(A)$ is multi fuzzy regular semiopen in X. That is to prove $f^{-1}(A)$ is both multi fuzzy semiopen and multi fuzzy semi-closed in X. Now A is multi fuzzy semiopen in Y. Since f is multi fuzzy irresolute, $f^{-1}(A)$ is multi fuzzy semiopen in X. Now A is multi fuzzy semi-closed in Y, as multi fuzzy regular semiopen set is multi fuzzy semi-closed. Then $1_Y - A$ is multi fuzzy semiopen in Y. Since f is multi fuzzy irresolute, $f^{-1}(1_Y - A)$ is multi fuzzy semiopen in X. But $f^{-1}(1_Y - A) = 1_X - f^{-1}(A)$ is multi fuzzy semiopen in X and so $f^{-1}(A)$ is multi fuzzy semi-closed in X. Thus $f^{-1}(A)$ is both multi fuzzy semiopen and multi fuzzy semi-closed in X and hence $f^{-1}(A)$ is multi fuzzy regular semiopen in X.

2.17 Theorem:

If a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is multi fuzzy irresolute and multi fuzzy rw-closed and A is multi fuzzy rw-closed set of X, then f(A) is a multi fuzzy rw-closed set in Y.

Proof:

Let A be a multi fuzzy closed set of X. Let $f(A) \subseteq E$, where E is multi fuzzy regular semiopen in Y. Since f is multi fuzzy irresolute, $f^{-1}(E)$ is a multi fuzzy regular semiopen in X, by Theorem 2.16 and $A \subseteq f^{-1}(E)$. Since A is a multi fuzzy rw-closed set in X, $mfcl(A) \subseteq f^{-1}(E)$. Since f is multi fuzzy rw-closed, f(mfcl(A)) is a multi fuzzy rw-closed set contained in the multi fuzzy regular semiopen set E, which implies $mfcl(f(mfcl(A))) \subseteq E$ and hence $mfcl(f(A)) \subseteq E$. Therefore f(A) is a multi fuzzy rw-closed set in Y.

2.18 Theorem:

If a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is multi fuzzy irresolute and multi fuzzy closed and A is a multi fuzzy rw-closed set in multi fuzzy topological space X, then f(A) is a multi fuzzy rw-clsoed set in multi fuzzy topological space Y.

Proof:

The proof follows from the Theorem 2.17 and the fact that every multi fuzzy closed map is a multi fuzzy rw-closed map.

2.19 Theorem:

Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be two mappings such that $g \bullet f : X \longrightarrow Z$ is a multi fuzzy rw-closed map, then

(i) if f is multi fuzzy continuous and surjective, then g is multi fuzzy rw-closed,

(ii) if g is multi fuzzy rw-irresolute and injective, then f is multi fuzzy rw-closed.

Proof:

(i) Let E be a multi fuzzy closed set in Y. Since f is multi fuzzy continuous, $f^{-1}(E)$ is a multi fuzzy closed set in X. Since $g \bullet f$ is a multi fuzzy rw-closed map, $(g \bullet f)(f^{-1}(E))$ is a multi fuzzy rw-closed set in Z. But $(g \bullet f)(f^{-1}(E)) = g(E)$, as f is surjective. Thus g is multi fuzzy rw-closed.

(ii) Let B be a multi fuzzy closed set of X. Then (g•f)(B) is a multi fuzzy rw-closed set in Z, since g•f is a multi fuzzy rw-closed map. Since g is multi fuzzy rw-irresolute, g^{-1}((g•f)(B)) is multi fuzzy rw-closed in Y. But g^{-1}((g•f)(B)) = f(B), as g is injective. Thus f is multi fuzzy rw-closed map.

2.20 Theorem:
If $f : (X, \Im) \to (Y, \sigma)$ is multi fuzzy almost irresolute and multi fuzzy rw-closed map and A is a multi fuzzy w-closed set of X, then *f* (A) is multi fuzzy rw-closed.

Proof:
Let *f(A)* $\subseteq$ O where O is a multi fuzzy regular semi open set of Y. Since *f* is multi fuzzy almost irresolute therefore f^{-1}(O) is a multi fuzzy semi open set of X such that A $\subseteq f^{-1}$(O). Since A is multi fuzzy w-closed of X which implies that mfcl□(A) $\subseteq f^{-1}$(O) and hence *f(mfcl(A))* $\subseteq$ O which implies that mfcl(f (mfcl(A)))□ $\subseteq$ O therefore mfcl(f(A)) $\subseteq$ O whenever *f(A)* $\subseteq$ O where O is a multi fuzzy regular semi open set of Y. Hence *f(A)* is a multi fuzzy rw-closed set of Y.

2.21 Theorem:
Every multi fuzzy homeomorphism is multi fuzzy rw- homeomorphism.

Proof:
Let a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a multi fuzzy homeomorphism. Then f and f^{-1} are multi fuzzy continuous. Since every multi fuzzy continuous map is multi fuzzy rw-continuous, f and f^{-1} are multi fuzzy rw-continuous. Therefore f is multi fuzzy rw- homeomorphism.

2.22 Remark: The converse of the above theorem need not be true.

2.23 Example:
Let X = Y = { 1, 2, 3 } and the multi fuzzy sets A, B, C be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > } and C = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 > }. Consider $\Im_1 = \{ 0_X, 1_X, A, C \}$ and $\Im_2 = \{ 0_Y, 1_Y, B \}$. Then $(X, \Im_1)$ and $(Y, \Im_2)$ are multi fuzzy topological spaces. Define a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ by f(1) = 1, f(2) = 3 and f(3) = 2. Here the function f is a multi fuzzy rw-homeomorphism but it is not a multi fuzzy homeomorphism, as the image of a multi fuzzy open set A in $(X, \Im_1)$ is A which is not a multi fuzzy open set in $(Y, \Im_2)$.

2.24 Theorem:
Let X and Y be multi fuzzy topological spaces and $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a bijective map. Then the following statements are equivalent.

(a) f^{-1} is multi fuzzy rw-continuous

(b) f is a multi fuzzy rw-open map

(c) f is a multi fuzzy rw-closed map.

Proof:
(a) ⟹(b). Let A be any multi fuzzy open set in X. Since f^{-1} is multi fuzzy rw-continuous, $(f^{-1})^{-1}(A) = f(A)$ is multi fuzzy rw-open in Y. Hence f is a multi fuzzy rw-open map.

(b) ⟹(c). Let A be any multi fuzzy closed set in X. Then 1_X–A is multi fuzzy rw-open in X. Since f is a multi fuzzy rw-open map, $f(1_X–A)$ is multi fuzzy rw-open in Y. But $f(1_X–A) = 1_Y – f(A)$, as f is a bijection map. Hence f(A) is multi fuzzy rw-closed in Y. Therefore f is multi fuzzy rw-closed.

(c)⟹(a). Let A be any multi fuzzy closed set in X. Then f(A) is a multi fuzzy rw-closed set in Y. But $(f^{-1})^{-1}(A) = f(A)$. Therefore f^{-1} is multi fuzzy rw-continuous.

2.25 Theorem:
Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a bijection and multi fuzzy rw-continuous map. Then the following statements are equivalent.
(a) f is a multi fuzzy rw-open map
(b) f is a multi fuzzy rw-homeomorphism
(c) f is a multi fuzzy rw-closed map.

Proof:
(a) ⟹(b). By hypothesis and assumption f is a multi fuzzy rw-homeomorphism.
(b) ⟹(c). Since f is a multi fuzzy rw-homeomorphism; it is multi fuzzy rw-open. So by the above Theorem 2.24, it is a multi fuzzy rw-closed map.
(c) ⟹(a). Let B be a multi fuzzy open set in X, so that1_X–B is a multi fuzzy closed set and f being multi fuzzy rw-clsoed, $f(1_X–B)$ is multi fuzzy rw-closed in Y. But $f(1_X–B) = 1_Y– f(B)$ thus f(B) is multi fuzzy rw-open in Y. Therefore f is a multi fuzzy rw-open map.

2.26 Theorem:
Every multi fuzzy rwc-homeomorphism is multi fuzzy rw-homeomorphism but not conversely.

Proof:
The proof follows from the fact that every multi fuzzy rw-irresolute map is multi fuzzy rw-continuous but not conversely.

2.27 Theorem:
Let $(X, \Im_1)$, $(Y, \Im_2)$ and $(Z, \Im_3)$ be multi fuzzy topological spaces and $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$, $g : (Y, \Im_2) \longrightarrow (Z, \Im_3)$ be multi fuzzy rwc-homeomorphisms. Then their composition $g{\bullet}f : (X, \Im_1) \longrightarrow (Z, \Im_3)$ is a multi fuzzy rwc-homeomorphism.

Proof:

LetA be a multi fuzzy rw-open set in (Z, $\Im_3$). Since g is a multi fuzzy rw-irrresolute, $g^{-1}(A)$ is a multi fuzzy rw-open set in (Y, $\Im_2$). Since f is a multi fuzzy rw-irresolute, $f^{-1}(g^{-1}(A))$ is a multi fuzzy rw-open set in (X, $\Im_1$). But $f^{-1}(g^{-1}(A)) = (g\bullet f)^{-1}(A)$. Therefore g•f is multi fuzzy rw-irresolute.

To prove that $(g\bullet f)^{-1}$ is multi fuzzy rw-irresolute. Let B be a multi fuzzy rw-open set in (X, $\Im_1$). Since f^{-1} is multi fuzzy rw-irresolute, $(f^{-1})^{-1}(B)$ is a multi fuzzy rw-open set in (Y, $\Im_2$). Also $(f^{-1})^{-1}(B) = f(B)$. Since g^{-1} is multi fuzzy rw-irresolute, $((g^{-1})^{-1})(f(B))$ is a multi fuzzy rw-open set in (Z, $\Im_3$). That is $((g^{-1})^{-1})(f(B)) = g(f(B)) = (g\bullet f)(B) = ((g\bullet f)^{-1})^{-1}(B)$. Therefore $(g\bullet f)^{-1}$ is multi fuzzy rw-irresolute. Thus g•f and $(g\bullet f)^{-1}$ are multi fuzzy rw-irresolute. Hence g•f is multi fuzzy rwc-homoeomorphism.

2.28 Theorem:

The set MFRWC-H(X, $\Im$) is a group under the composition of maps.

Proof:

Define a binary operation * : MFRWC-H(X, $\Im$) × MFRWC-H(X, $\Im$) ⟶ MFRWC-H(X, $\Im$) by f*g = g•f, for all f, g∈MFRWC-H(X, $\Im$) and • is the usual operation of composition of maps. Then by **theorem 2.20,** g•f ∈MFRWC-H(X, $\Im$). We know that the composition of maps is associative and the identity map I:(X, $\Im$) ⟶ (X, $\Im$) belonging to MFRWC-H(X, $\Im$) serves as the identity element. If f ∈MFRWC-H(X, $\Im$), then f^{-1}∈MFRWC-H(X, $\Im$) such that $f\bullet f^{-1} = f^{-1}\bullet f = I$ and so inverse exists for each element of MFRWC-H(X, $\Im$). Therefore (MFRWC-H(X, $\Im$), •) is a group under the operation of composition of maps.

2.29 Theorem:

Let f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) be a multi fuzzy rwc-homeomorphism. Then f induces an isomorphism from the group MFRWC-H(X, $\Im_1$) on to the group MFRWC-H(Y, $\Im_2$).

Proof:

Using the map f, we define a map Ψ_f : MFRWC-H(X, $\Im_1$)⟶MFRWC-H(Y, $\Im_2$) by $\Psi_f(h) = f\bullet h\bullet f^{-1}$, for every h ∈MFRWC-H(X, $\Im_1$). Then Ψ_f is a bijection. Further, for all h_1h_2∈MFRWC-H(X, $\Im_1$), $\Psi_f(h_1\bullet h_2) = f\bullet(h_1\bullet h_2)\bullet f^{-1} = (f\bullet h_1\bullet f^{-1})\bullet (f\bullet h_2\bullet f^{-1}) = \Psi_f(h_1)\bullet\Psi_f(h_2)$. Therefore Ψ_f is a homeomorphism and so it is an isomorphism induced by f.

CONCLUSION

In the study of the structure of multi fuzzy topological spaces, we notice that multi fuzzy rw-closed and multi fuzzy rw-open sets with special properties always play an important role. In this paper, we define multi fuzzy rw-open maps and multi fuzzy rw-closed maps in multi fuzzy topological spaces and investigate the relationship among these multi fuzzy rw-open maps and multi fuzzy rw-closed maps in multi fuzzy topological spaces. Some characterization theorems of multi fuzzy rw-open maps and multi fuzzy rw-closed maps in multi fuzzy topological spaces are obtained. We hope that the research along this direction can be continued, and in fact, this work would serve as a foundation for further study of the theory of topology, it will be necessary to carry out more theoretical research to establish a general framework for the practical application. This multi fuzzy set is a tool to analyze solve multi-experts system problems.

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