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IVF Almost Generalized Semi-Precontinuous Mappings

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Abstract

In this paper we introduce interval valued fuzzy almost generalized semi-precontinuous mappings. We investigate some of its properties. Also we provide the relation between interval valued fuzzy almost generalized semi-pre continuous mappings and other interval valued fuzzy continuous mappings.

Keywords: IVF-set, IVF-topological space, IVF-point, IVF-generalized semi-preclosed set, IVF-continuous mappings.

AMS Subject Classification(2010): 54A40, 08A72.

1 INTRODUCTION

The concept of fuzzy subset was introduced and studied by L. A. Zadeh [13] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] introduced semi-preclosed sets and Dontchev [4] introduced generalized semi-preclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by Saraf and Khanna [12]. Tapas Kumar Mondal and S. K. Samantha [9] introduced the topology of interval valued fuzzy sets. Jeyabalan. R and Arjunan. K, [6] introduced interval valued fuzzy generalized semi-preclosed sets. In this paper, we introduce that IVF-almost generalized semi-precontinuous mappings and some properties are investigated.

2 PRELIMINARIES

Definition 2.1 [9] Let X be a non empty set. A mapping $\overline{A}: X \to D[0,1]$ is called an interval valued fuzzy set (briefly $IVFS$) on X , where $D[0,1]$ denotes the family of all closed subintervals of [0,1] and, for all $x \in X$, where $A^-(x)$ and $A^+(x)$ are fuzzy sets of X such that $A^-(x) \le A^+(x)$, for all $x \in X$. Thus $\overline{A}(x)$ is an interval (a closed subset of [0,1]) and not a number fom the interval [0,1] as in the case of fuzzy set.

Notation 2.2 D^X denotes the set of all interval valued fuzzy subsets of a non empty set X .

Definition 2.3 [9] Let X be a non empty set. Let $x_0 \in X$ and $\alpha \in D[0,1]$ be fixed such that $\alpha \neq [0,0]$. Then the interval valued fuzzy subset $p_{x_0}^{\alpha}$ is called an interval valued fuzzy point defined by,

$$p_{x_0}^{\alpha} = \begin{cases} \alpha \ if \ x = x_0 \\ [0,0] \ if \ x \neq x_0 \end{cases}.$$

Definition 2.4 [9] Let $\overline{A}$ and $\overline{B}$ be any two $IVFS$ of X , that is $\overline{A} = \{< x,[A^-(x), A^+(x)] >: x \in X\},\ \overline{B} = \{< x,[B^-(x), B^+(x)] >: x \in X\}$. We define the following relations and operations:

(i) $\overline{A} \subseteq \overline{B}$ if and only if $A^-(x) \le B^-(x)$ and $A^+(x) \le B^+(x)$, for all $x \in X$.

(ii) $\overline{A} = \overline{B}$ if and only if $A^-(x) = B^-(x)$, and $A^+(x) = B^+(x)$, for all $x \in X$.

(iii) $(\overline{A})^c = \overline{1} - \overline{A} = \{< x,[1 - A^+(x), 1 - A^-(x)] >: x \in X\}$.

(iv) $\overline{A} \cap \overline{B} = \{< x,[\min[A^-(x), B^-(x)], \min[A^+(x), B^+(x)]] >: x \in X\}$.

(v) $\overline{A} \cup \overline{B} = \{< x,[\max[A^-(x), B^-(x)], \max[A^+(x), B^+(x)]] >: x \in X\}$.

We denote by $\overline{0}_X$ and $\overline{1}_X$ for the interval valued fuzzy sets $\{< x,[0,0] >,\ for\ all\ x \in X\}$ and $\{< x,[1,1] >,\ for\ all\ x \in X\}$ respectively.

Definition 2.5 [9] Let X be a set and $\Im$ be a family of interval vlued fuzzy sets $(IVFSs)$ of X . The family $\Im$ is called an interval valued fuzzy topology $(IVFT)$ on X if and only if $\Im$ satisfies the following axioms:

(i) $\overline{0}_X, \overline{1}_X \in \Im$,

(ii) If $\{\overline{A}_i : i \in I\} \subseteq \Im$, then $\bigcup_{i \in I} \overline{A}_i \in \Im$,

(iii) If $\overline{A}_1, \overline{A}_2, \overline{A}_3, \ldots, \overline{A}_n \in \Im$, then $\bigcap_{i=1}^{n} \overline{A}_i \in \Im$.

The pair $(X, \Im)$ is called an interval valued fuzzy topological space $(IVFTS)$.

The members of $\Im$ are called interval valued fuzzy open sets $(IVFOS)$ in X.

An interval valued fuzzy set $\overline{A}$ in X is said to be interval valued fuzzy closed set $(IVFCS)$ in X if and only if $(\overline{A})^c$ is an $IVFOS$ in X.

Definition 2.6 [9] Let $(X,\Im)$ be an $IVFTS$ and $\overline{A} = \left\{< x,[A^-(x), A^+(x)] >: x \in X\right\}$ be an $IVFS$ in X. Then the interval valued fuzzy interior and interval valued fuzzy closure of $\overline{A}$ denoted by $ivfint(\overline{A})$ and $ivfcl(\overline{A})$ are defined by

$$ivfint(\overline{A}) = \bigcup \left\{\overline{G} : \overline{G} \textit{ is an IVFOS in } X \textit{ and } \overline{G} \subseteq \overline{A}\right\},$$

$$ivfcl(\overline{A}) = \bigcap \left\{\overline{K} : \overline{K} \textit{ is an IVFCS in } X \textit{ and } \overline{A} \subseteq \overline{K}\right\}.$$

For any $IVFS$ $\overline{A}$ in $(X,\Im)$, we have $ivfcl(\overline{A}^c) = (ivfint\ (\overline{A}))^c$ and $ivfint(\overline{A}^c) = (ivfcl(\overline{A}))^c$.

Definition 2.7 *An* $IVFS$ $\overline{A} = \left\{< x,[A^-(x), A^+(x)] >: x \in X\right\}$ *in an* $IVFTS$ $(X,\Im)$ *is said to be an*

(i) IVF regular closed set ($IVFRCS$) if $\overline{A} = ivfcl(ivfint(\overline{A}))$;

(ii) IVF semi-closed set ($IVFSCS$) if $ivfint\ (ivfcl(\overline{A})) \subseteq \overline{A}$;

(iii) IVF preclosed set ($IVFPCS$) if $ivfcl(ivfint\ (\overline{A})) \subseteq \overline{A}$;

(iv) IVF α closed set ($IVF\alpha CS$) if $ivfcl(ivfint\ (ivfcl\ (\overline{A}))) \subseteq \overline{A}$;

(v) IVF β closed set ($IVF\beta CS$) if $ivfint(ivfcl\ (ivfint(\overline{A}))) \subseteq \overline{A}$.

Definition 2.8 *An* $IVFS$ $\overline{A} = \left\{< x,[A^-(x), A^+(x)] >: x \in X\right\}$ *in an* $IVFTS$ $(X,\Im)$ *is said to be an*

(i) interval valued fuzzy generalized closed set ($IVFGCS$) if $ivfcl$ $(\overline{A}) \subseteq \overline{U}$, whenever $\overline{A} \subseteq \overline{U}$ and $\overline{U}$ in an $IVFOS$;

(ii) interval valued fuzzy generalized regular closed set ($IVFGRCS$) if $ivfcl\ (\overline{A}) \subseteq \overline{U}$, whenever $\overline{A} \subseteq \overline{U}$ and $\overline{U}$ is an $IVFROS$.

Definition 2.9 *An* $IVFS$ $\overline{A} = \left\{< x,[A^-(x), A^+(x)] >: x \in X\right\}$ *is an* $IVFTS$ $(X,\Im)$ *is said to be an*

(i) interval valued fuzzy semi-preclosed set ($IVFSPCS$) if there exist on

IVFPCS $\overline{B}$, such that $ivfint\overline{B} \subseteq \overline{A} \subseteq \overline{B}$;

(ii) interval valued fuzzy semi-preopen set (*IVFSPOS*) if there exist on *IVFPOS* $\overline{B}$, such that $\overline{B} \subseteq \overline{A} \subseteq ivfcl(\overline{B})$.

Definition 2.10 *Let* $\overline{A}$ *be an IVFS is an IVFTS* $(X, \Im)$. *Then the interval valued fuzzy semi-preinterior of* $\overline{A}$ $(ivfspint(\overline{A}))$ *and the interval valued fuzzy semi-preclosure of* $\overline{A}(ivfspcl(\overline{A}))$ *are defined by*

$$ivfspint(\overline{A}) = \bigcup\{\overline{G} : \overline{G} \text{ is an IVFSPOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},$$

$$ivfspcl(\overline{A}) = \bigcap\{\overline{K} : \overline{K} \text{ is an IVFSPCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.$$

For any *IVFS* $\overline{A}$ in $(X, \Im)$, we have $ivfspcl(\overline{A}^c) = (ivfspint(\overline{A}))^c$ and $ivfspint(\overline{A}^c) = (ivfspcl(\overline{A}))^c$.

Definition 2.11 *[6] An IVFS* $\overline{A}$ *in IVFTS* $(X, \Im)$ *is said to be an interval valued fuzzy generalized semi-preclosed set (IVFGSPCS) if* $ivfspcl(\overline{A}) \subseteq \overline{U}$, *whenever* $\overline{A} \subseteq \overline{U}$ *and* $\overline{U} \in \Im$.

Definition 2.12 *[6] The complement* $\overline{A}^c$ *of an IVFGSPCS* $\overline{A}$ *in an IVFTS* $(X, \Im)$ *is called an interval valued fuzzy generalized semi-preopen set (IVFGSPOS) in* X.

Definition 2.13 *Let* $\overline{A}$ *be an IVFS is an IVFTS* $(X, \Im)$. *Then the interval valued fuzzy generalized semi-preinterior of* $\overline{A}$ $(ivfgspint(\overline{A}))$ *and the interval valued fuzzy generalized semi-preclosure of* $\overline{A}(ivfgspcl(\overline{A}))$ *are defined by*

$$ivfgspint(\overline{A}) = \bigcup\{\overline{G} : \overline{G} \text{ is an IVFGSPOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},$$

$$ivfgspcl(\overline{A}) = \bigcap\{\overline{K} : \overline{K} \text{ is an IVFGSPCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.$$

For any *IVFS* $\overline{A}$ in $(X, \Im)$, we have $ivfgspcl(\overline{A}^c) = (ivfgspint(\overline{A}))^c$ and $ivfgspint(\overline{A}^c) = (ivfgspcl(\overline{A}))^c$.

Definition 2.14 *An IVFTS* $(X, \Im)$ *is called an interval valued fuzzy semi-pre* $T_{1/2}$ *space* $(IVFSPT_{1/2})$ *if every IVFGSPCS is an IVFSPCS in* X.

Definition 2.15 *[9] An IVFS* $\overline{A}$ *of a IVFTS of* $(X, \Im)$ *is said to be an interval valued fuzzy neighbourhood(IVFN) of an IVFP* $p_{x_0}^{\alpha}$ *if there exists an IVFOS* $\overline{B}$ *in* X *such that* $p_{x_0}^{\alpha} \in \overline{B} \subseteq \overline{A}$.

Definition 2.16 *Let* $(X,\Im)$ *and* (Y,σ) *be IVFTSs. Then a map* $g:X\to Y$ *is called an*

- (i) interval valued fuzzy continuous (*IVF* continuous mapping) if $g^{-1}(\overline{B})$ is *IVFOS* in X for all $\overline{B}$ in σ.
- (ii) interval valued fuzzy semi-continuous mapping (*IVFS*-continuous mapping) if $g^{-1}(\overline{B})$ is *IVFSOS* in X for all $\overline{B}$ in σ.
- (iii) interval valued fuzzy α-continuous mapping (*IVF*α-continuous mapping) if $g^{-1}(\overline{B})$ is *IVF*α*OS* in X for all $\overline{B}$ in σ.
- (iv) interval valued fuzzy pre-continuous mapping (*IVFP*-continuous mapping) if $g^{-1}(\overline{B})$ is *IVFPOS* in X for all $\overline{B}$ in σ.
- (v) interval valued fuzzy β-continuous mapping (*IVF*β-continuous mapping) if $g^{-1}(\overline{B})$ is *IVF*β*OS* in X for all $\overline{B}$ in σ.
- (vi) interval valued fuzzy semi-precontinuous mapping (*IVFSP*-continuous mapping) if $g^{-1}(\overline{B})$ is *IVFSPOS* in X for all $\overline{B}$ in σ.

Definition 2.17 *An IVFS* $\overline{A}$ *is said to be interval valued fuzzy dense (IVFD) in another IVFS* $\overline{B}$ *in an IVFT* $(X,\Im)$, *if* $ivfcl(\overline{A})=\overline{B}$.

Definition 2.18 *Let* $(X,\Im)$ *and* (Y,σ) *be IVFTSs. Then a map* $g:X\to Y$ *is called interval valued fuzzy generalized continuous (IVFG continuous) mapping if* $g^{-1}(\overline{B})$ *is IVFGCS in* X *for all* $\overline{B}$ *in* σ^c.

Definition 2.19 *A mapping* $g:(X,\Im)\to(Y,\sigma)$ *is called an interval valued fuzzy almost generalized semi-precontinuous (IVFaGSP continuous) mapping if* $g^{-1}(\overline{V})$ *is an IVFGSPCS in* X *for every IVFRCS* $\overline{V}$ *in* Y.

Example 2.20 *Let* $X=\{a,b\}$, $Y=\{u,v\}$ *and*

$$\overline{K}_1=\{<a,[0.1,0.2]>,<b,[0.3,0.4]>\},$$
$$\overline{L}_1=\{<u,[0.3,0.4]>,<v,[0.4,0.6]>\}.$$

Then $\Im=\{\overline{0}_X,\overline{K}_1,\overline{1}_X\}$ and $\sigma=\{\overline{0}_Y,\overline{L}_1,\overline{1}_Y\}$ are *IVFT* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping.

3 MAIN RESULTS

Theorem 3.1 *Every IVF continuous mapping is an IVFaGSP continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVF* continuous mapping. Let $\overline{V}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an *IVFCS*, $\overline{V}$ is an *IVFCS* in Y. Then

$g^{-1}(\overline{V})$ is an *IVFCS* in X, by hypothesis. Since every *IVFCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.2 *The converse of the above theorem* 3.1 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.1,0.2] >, < b,[0.3,0.4] >\},$$
$$\overline{L}_1 = \{< u,[0.8,0.9] >, < v,[0.6,0.7] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an *IVF* continuous mapping. Since $\overline{L}_1^c = \{< u,[0.1,0.2] >, < v,[0.3,0.4] >\}$ is an *IVFCS* in Y but $g^{-1}(\overline{L}_1^c) = \{< a,[0.1,0.2] >, < b,[0.3,0.4] >\}$ is not an *IVFCS* in X, because $ivfcl(g^{-1}(\overline{L}_1^{\,c})) = \overline{1}_X \neq g^{-1}(\overline{L}_1^c)$.

Theorem 3.3 *Every IVFG continuous mapping is an IVFaGSP continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVF* continuous mapping. Let $\overline{V}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an *IVFCS*, $\overline{V}$ is an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an *IVFGCS* in X. Since every *IVFGCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.4 *The converse of the above theorem* 3.3 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.4,0.5] >, < b,[0.6,0.7] >\},$$
$$\overline{L}_1 = \{< a,[0.6,0.7] >, < b,[0.7,0.8] >\},$$
$$\overline{M}_1 = \{< u,[0.4,0.8] >, < v,[0.7,0.9] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{L}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{M}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an *IVFG* continuous mapping, since $\overline{M}_1^c$ is *IVFCS* in Y but $g^{-1}(\overline{M}_1^c)$ is not an *IVFGCS* in X, because $g^{-1}(\overline{M}_1^c)\subseteq\overline{K}_1$ and $g^{-1}(\overline{M}_1^c)\subseteq\overline{L}_1$ but $ivfcl(g^{-1}(\overline{M}_1^c)) = \overline{1}_X \not\subset \overline{K}_1$ and $ivfcl(g^{-1}(\overline{M}_1^c)) = \overline{1}_Y \not\subset \overline{L}_1$.

Theorem 3.5 *Every IVFS continuous mapping is an IVFaGSP continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVF* continuous mapping. Let $\overline{V}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an *IVFCS*, $\overline{V}$ is an *IVFCS* in Y. Then

$g^{-1}(\overline{V})$ is an *IVFSCS* in X. Since every *IVFSCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.6 *The converse of the above theorem* 3.5 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{<a,[0.1,0.2]>, <b,[0.3,0.4]>\},$$
$$\overline{L}_1 = \{<u,[0.3,0.4]>, <v,[0.4,0.6]>\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an *IVFS* continuous mapping. Since $\overline{L}_1^c = \{<u,[0.6,0.7]>, <v,[0.4,0.6]>\}$ is an *IVFCS* in Y but $g^{-1}(\overline{L}_1^c) = \{<a,[0.6,0.7]>, <b,[0.4,0.6]>\}$ is not an *IVFSCS* in X, because $ivfint(ivfcl(g^{-1}(\overline{L}_1^{\,c}))) = ivfint(\overline{1}_X) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$.

Theorem 3.7 *Every IVFP continuous mapping is an IVFaGSP continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVF* continuous mapping. Let $\overline{V}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an *IVFCS*, $\overline{V}$ is an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an *IVFPCS* in X. Since every *IVFPCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.8 *The converse of the above theorem* 3.7 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{<a,[0.3,0.4]>, <b,[0.4,0.7]>\},$$
$$\overline{L}_1 = \{<u,[0.1,0.2]>, <v,[0.3,0.4]>\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an *IVFP* continuous mapping. Since $\overline{L}_1^c = \{<u,[0.8,0.9]>, <v,[0.6,0.7]>\}$ is an *IVFCS* in Y and $g^{-1}(\overline{L}_1^c) = \{<a,[0.8,0.9]>, <b,[0.6,0.7]>\}$ is not an *IVFPCS* in X, because $ivfcl(ivfint(g^{-1}(\overline{L}_1^{\,c}))) = ivfcl(\overline{K}_1) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$.

Theorem 3.9 *Every IVFSP continuous mapping is an IVFaGSP continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVFSP* continuous mapping. Let $\overline{V}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an *IVFSPCS*, $\overline{V}$ is an *IVFSPCS* in Y.

Then $g^{-1}(\overline{V})$ is an *IVFSPCS* in X. Since every *IVFSPCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.10 *The converse of the above theorem* 3.9 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.3,0.4] >, < b,[0.4,0.6] >\},$$
$$\overline{L}_1 = \{< u,[0.1,0.2] >, < v,[0.3,0.4] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an *IVFSP* continuous mapping. Since $\overline{L}_1^c = \{< u,[0.8,0.9] >, < v,[0.6,0.7] >\}$ is an *IVFCS* in Y but $g^{-1}(\overline{L}_1^c) = \{< a,[0.8,0.9] >, < b,[0.6,0.7] >\}$ is not an *IVFSPCS* in X, because there exist no *IVFPCS* $\overline{B}$ in X such that $ivfint(g^{-1}(\overline{L}_1^c)) \subseteq \overline{B} \subseteq g^{-1}(\overline{L}_1^c)$.

Theorem 3.11 *Every* $IVF\beta$ *continuous mapping is an* *IVFaGSP* *continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVFSP* continuous mapping. Let $\overline{V}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an $IVF\beta CS$, $\overline{V}$ is an $IVF\beta CS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVF\beta CS$ in X. Since every $IVF\beta CS$ is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.12 *The converse of the above theorem* 3.11 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.5,0.7] >, < b,[0.3,0.4] >\},$$
$$\overline{L}_1 = \{< u,[0.3,0.4] >, < v,[0.4,0.6] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an $IVF\beta$ continuous mapping. Since $\overline{L}_1^c = \{< u,[0.6,0.7] >, < v,[0.4,0.6] >\}$ is an *IVFCS* in Y but $g^{-1}(\overline{L}_1^c) = \{< a,[0.6,0.7] >, < b,[0.4,0.6] >\}$ is not an $IVF\beta CS$ in X, because $ivfint(ivfcl(ivfint(g^{-1}(\overline{L}_1^c)))) = ivfint(ivfcl(\overline{K}_1)) = ivfint(\overline{1}_X) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$.

Theorem 3.13 *Every* $IVF\alpha$ *continuous mapping is an* *IVFaGSP* *continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an *IVFSP* continuous mapping. Let $\overline{V}$ be an

IVFRCS in Y. Since every *IVFRCS* is an $IVF\alpha CS$, $\overline{V}$ is an $IVF\alpha CS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVF\alpha CS$ in X. Since every $IVF\alpha CS$ is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.14 *The converse of the above theorem* 3.13 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.3,0.4] >,< b,[0.4,0.6] >\},$$
$$\overline{L}_1 = \{< u,[0.1,0.2] >,< v,[0.3,0.4] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFT* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFaGSP* continuous mapping but not an $IVF\alpha$ continuous mapping. Since $\overline{L}_1^c = \{< u,[0.8,0.9] >,< v,[0.6,0.7] >\}$ is an *IVFCS* in Y but $g^{-1}(\overline{L}_1^c) = \{< a,[0.8,0.9] >,< b,[0.6,0.7] >\}$ is not an $IVF\alpha CS$ in X, because $ivfcl(ivfint(ivfcl(g^{-1}(\overline{L}_1^c)))) = ivfcl(ivfint(\overline{1}_X)) = ivfcl(\overline{1}_X) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$.

Theorem 3.15 *Let* $g:(X,\Im)\to(Y,\sigma)$ *be a mapping where* $g^{-1}(\overline{V})$ *is an IVFRCS in* X, *for every IVFCS* $\overline{V}$ *in* Y. *Then* g *is an IVFaGSP continuous mapping*.

Proof. Let $\overline{A}$ be an *IVFRCS* in Y. Since every *IVFRCS* is an *IVFCS*, $\overline{V}$ is an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an *IVFRCS* in X. Since every *IVFRCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping. Then $g^{-1}(\overline{V})$ is an *IVFRCS* in X. Since every *IVFRCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFaGSP* continuous mapping. W

Remark 3.16 *The converse of the above theorem* 3.15 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.3,0.4] >,< b,[0.4,0.7] >\},$$
$$\overline{L}_1 = \{< u,[0.1,0.2] >,< v,[0.3,0.4] >\}.$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFT* on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an *IVFGSP* continuous mapping but not a mapping as defined in theorem 3.15, since $\overline{L}_1^c = \{< u,[0.8,0.9] >,< v,[0.6,0.7] >\}$ is an *IVFCS* in Y and $g^{-1}(\overline{L}_1^c) = \{< a,[0.8,0.9] >,< b,[0.6,0.7] >\}$ is not an *IVFRCS* in X, because $ivfcl(ivfint(g^{-1}(\overline{L}_1^c))) = ivfcl(\overline{K}_1)) = \overline{1}_X \neq g^{-1}(\overline{L}_1^c)$.

Theorem 3.17 *Every IVFGSP continuous mapping is an IVFaGSP - continuous mapping.*

Proof. Let $g: X \to Y$ be an *IVFGSP*-continuous mapping. Let $\overline{A}$ be an *IVFRCS* in Y. Then $\overline{A}$ is an *IVFCS* in Y. By hypothesis $g^{-1}(\overline{A})$ is an *IVFGSPS* in X. Hence g is an *IVFaGSP* continuous mapping.

Remark 3.18 *The converse of the above theorem* 3.17 *need not be true from the following example: Let* $X = \{a,b\}$, $Y = \{u,v\}$ *and*

$$\overline{K}_1 = \{< a,[0.4,0.5] >,< b,[0.6,0.7] >\},$$
$$\overline{L}_1 = \{< a,[0.6,0.7] >,< b,[0.7,0.8] >\},$$
$$\overline{M}_1 = \{< u,[0.4,0.8] >,< v,[0.7,0.9] >\},$$
$$\overline{N}_1 = \{< u,[0.3,0.5] >,< v,[0.5,0.7] >\},$$

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{L}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{M}_1, \overline{N}_1, \overline{1}_Y\}$ are *IVFT* on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an *IVFaGSP* continuous mapping but not an *IVFGSP* continuous mapping, since $\overline{M}_1^c = \{< u,[0.2,0.6] >,< v,[0.1,0.3] >\}$ is *IVFCS* in Y but $g^{-1}(\overline{M}_1^c)$ is not an *IVFGSPCS* in Y, but $g^{-1}(\overline{M}_1^c)$ is not an *IVFGSPCS* in Y, $g^{-1}(\overline{M}_1^c) = \{< a,[0.2,0.6] >,< b,[0.1,0.3] >\} \subseteq \overline{K}_1$ but $ivfspcl(g^{-1}(\overline{M}_1^c)) = \overline{1}_X \not\subset \overline{K}_1$.

Theorem 3.19 *Let* $p_{x_0}^{\alpha}$ *be an IVFP in* X. *A mapping* $g: X \to Y$ *is an IVFaGSP continuous mapping, then for every IVFO* $\overline{A}$ *in* Y *with* $g(p_{x_0}^{\alpha}) \in \overline{A}$, *there exists an IVFOS* $\overline{B}$ *in* X *with* $p_{x_0}^{\alpha} \in \overline{B}$ *such that* $g^{-1}(\overline{A})$ *is IVFD in* $\overline{B}$.

Proof. Let $\overline{A}$ be an *IVFROS* in Y. Then $\overline{A}$ is an *IVFOS* in Y. Let $g(p_{x_0}^{\alpha}) \in \overline{A}$, then there exists an *IVFOS* $\overline{B}$ in X such that $p_{x_0}^{\alpha} \in \overline{B}$ and $ivfcl(g^{-1}(\overline{A})) = \overline{B}$. Since $\overline{B}$ is an *IVFOS*, $ivfcl(g^{-1}(\overline{A}))$ is also an *IVFOS* in X. Therefore $ivfint(ivfcl(g^{-1}(\overline{A}))) = ivfcl(g^{-1}(\overline{A}))$.
Now $g^{-1}(\overline{A}) \subseteq ivfcl(g^{-1}(\overline{A})) = ivfint(ivfcl(g^{-1}(\overline{A}))) \subseteq ivfcl(ivfint(ivfcl(g^{-1}(\overline{A}))))$. This implies $g^{-1}(\overline{A})$ is an *IVFβOS* in X and hence an *IVFGSPOS* in X. Thus g is an *IVFaGSP* continuous mappings. W

Theorem 3.20 *Let* $f: X \to Y$ *be a mapping where* X *is an* $IVFSPT_{1/2}$ *space. Then the following are equivalent:*

(i) g is an *IVFaGSP* continuous mapping.

(ii) $ivfspcl(g^{-1}(\overline{A})) \subseteq g^{-1}(ivfcl(\overline{A}))$ for every $IVFSPOS$ $\overline{A}$ in Y,
(iii) $ivfspcl(g^{-1}(\overline{A})) \subseteq g^{-1}(ivfcl(\overline{A}))$ for every $IVFSOS$ $\overline{A}$ in Y,
(iv) $g^{-1}(\overline{A}) \subseteq ivfspint(g^{-1}(ivfint(ivfcl(\overline{A}))))$ for every $IVFPOS$ $\overline{A}$ in Y

Proof. $(i) \Leftrightarrow (ii)$ Let $\overline{A}$ be an $IVFSPOS$ in Y. Then by definition 2.9, there exists an $IVFPOS$ $\overline{B}$ such that $\overline{B} \subseteq \overline{A} \subseteq ivfcl(\overline{B})$ and $\overline{B} \subseteq ivfint(ivfcl(\overline{B}))$. Now $ivfcl(ivfint(ivfcl(\overline{A}))) \supseteq ivfcl(ivfint(ivfcl(\overline{B}))) \supseteq ivfcl(\overline{B}) \supseteq \overline{A}$. Hence $\overline{A} \subseteq ivfcl(ivfint(ivfcl(\overline{A})))$. Therefore $ivfcl(\overline{A}) \subseteq ivfcl(ivfint(ivfcl(\overline{A})))$. But $ivfcl(ivfint(ivfcl(\overline{A}))) \subseteq ivfcl(\overline{A})$. Hence $ivfcl(ivfint(ivfcl(\overline{A}))) = ivfcl(\overline{A})$. This implies $ivfcl(\overline{A})$ is an $IVFRCS$ in $(X, \Im)$. By hypothesis $g^{-1}(ivfcl(\overline{A}))$ is an $IVFGSPCS$ in X and hence $g^{-1}(ivfcl(\overline{A}))$ is an $IVFSPCS$ in X, since X is an $IVFSPT_{1/2}$ space. This implies $ivfspcl(g^{-1}(ivfcl(\overline{A}))) = g^{-1}(ivfcl(\overline{A}))$. Now $ivfspcl(g^{-1}(\overline{A})) \subseteq ivfspcl(g^{-1}(ivfcl(\overline{A}))) = g^{-1}(ivfcl(\overline{A}))$. Thus $ivfspcl(g^{-1}(\overline{A})) \subseteq g^{-1}(ivfcl(\overline{A}))$.
$(ii) \Leftrightarrow (iii)$ Since every $IVFSOS$ is an $IVFSPOS$, proof is similar in $(i) \Rightarrow (ii)$.
$(iii) \Leftrightarrow (i)$ Let $\overline{A}$ be an $IVFRCS$ in Y. Then $\overline{A} = ivfcl(ivfint(\overline{A}))$. Therefore $\overline{A}$ is an $IVFSOS$ in Y.
By hypothesis, $ivfspcl(g^{-1}(\overline{A})) \subseteq g^{-1}(ivfcl(\bar{(}A))) = g^{-1}(\overline{A}) \subseteq ivfspcl(g^{-1}(\overline{A}))$.
Hence $g^{-1}(\overline{A})$ is an $IVFSPCS$ and hence is an $IVFGSPCS$ in X. Thus g is an $IVFaGSP$ continuous mapping.
$(i) \Leftrightarrow (iv)$ Let $\overline{A}$ be an $IVFPOS$ in Y. Then $\overline{A} \subseteq ivfint(ivfcl(\overline{A}))$. Since $ivfint(ivfcl(\overline{A}))$ is an $IVFROS$ in Y, by hypothesis , $g^{-1}(ivfint(ivfcl(\overline{A})))$ is an $IVFGSPOS$ in X. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(ivfint(ivfcl(\overline{A})))$ is an $IVFSPOS$ in X.
Therefore $g^{-1}(\overline{A}) \subseteq g^{-1}(ivfint(ivfcl(\overline{A}))) = ivfspint(g^{-1}(ivfint(ivfcl(\overline{A}))))$.
$(iv) \Leftrightarrow (i)$ Let $\overline{A}$ be an $IVFROS$ in Y. Then $\overline{A}$ is an $IVFPOS$ in X. By hypothesis, $g^{-1}(\overline{A}) \subseteq ivfspint(g^{-1}(ivfint(ivfcl(\overline{A}))))$ $= ivfspint(g^{-1}(\overline{A}))$ $\subseteq g^{-1}(\overline{A})$. This implies $g^{-1}(\overline{A})$ is an $IVFSPOS$ in X and hence is an $IVFGSPOS$ in X. Therefore g is an $IVFaGSP$ continuous mapping. ▯

Theorem 3.21 *Let $g: X \to Y$ be a mapping. If g is an $IVFGSP$ continuous mapping, then $ivfgspcl(g^{-1}(\overline{A}) \subseteq g^{-1}(ivfcl(barA))$ for every $IVFSPOS$ $\overline{A}$ in Y.*

Proof. let $\overline{A}$ be an $IVFSPOS$ in Y. Then $ivfcl(\overline{A})$ is an $IVFRCS$ in Y.By hypothesis $g^{-1}(ivfcl(\overline{A}))$ is an $IVFGSPCS$ in X. Then $ivfgspcl(g^{-1}(ivfcl(\overline{A}))) = g^{-1}(ivfcl(\overline{A}))$. Now

$ivfspcl(g^{-1}(\overline{A})) \subseteq ivfgspcl(g^{-1}(ivfcl(\overline{A}))) = g^{-1}(ivfcl(\overline{A}))$.That is $ivfgspcl(g^{-1}(\overline{A})) \subseteq g^{-1}(ivfcl(\overline{A}))$.

Theorem 3.22 *Let* $g: X \to Y$ *be a mapping where* X *is an* $IVFSPT_{1/2}$ *space. If* g *is an* $IVFaGSP$ *continuous mapping, then* $ivfint(ivfcl(ivfint(g^{-1}(\overline{B})))) \subseteq g^{-1}(ivfspcl(\overline{B}))$ *for every* $\overline{B} \in IVFRC(Y)$. **Proof.** Let $\overline{B} \subseteq Y$ be an $IVFRCS$. By hypothesis, $g^{-1}(\overline{B})$ is an $IVFGSPCS$ in X. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(\overline{B})$ is an $IVFSPCS$ in X. Therefore $ivfspcl(g^{-1}(\overline{B})) = g^{-1}(\overline{B})$. Now $ivfint(ivfcl(ivfint(g^{-1}(\overline{B})))) \subseteq g^{-1}(\overline{B}) \bigcup ivfint(ivfcl(ivfint(g^{-1}(\overline{B})))) \subseteq ivfspcl(g^{-1}(\overline{B})) = g^{-1}(\overline{B}) = g^{-1}(ivfspcl(\overline{B}))$. Hence $ivfint(ivfcl(ivfint(g^{-1}(\overline{B})))) \subseteq g^{-1}(ivfspcl(\overline{B}))$.

Theorem 3.23 *Let* $g: X \to Y$ *be a mapping where* X *is an* $IVFSPT_{1/2}$ *space. If* g *is an* $IVFaGSP$ *continuous mapping, then* $g^{-1}(ivfspint(\overline{B})) \subseteq ivfcl(ivfint(ivfcl(g^{-1}(\overline{B}))))$ *for every* $\overline{B} \in IVFRO(Y)$.
Proof. This theorem can be easily proved by taking complement in theorem 3.22.

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