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A STUDY ON FUZZY NORMAL HX IDEAL OF A HX RING

R. Muthuraj[1] **and N.Ramila Gandhi**[2]

[1] *PG and Research Department of Mathematics, H.H.The Rajah's College, Pudukkottai – 622 001, Tamilnadu, India.*
E-mail : rmr1973@yahoo.co.in
[2]*Department of Mathematics, PSNA College of Engg. and Technology, Dindigul-624 622, T amilnadu , India.*
E-mail : satrami@yahoo.com

Abstract

In this paper, we introduce the concept of fuzzy normal HX ideals of a HX ring and discussed its properties by establishing the relationship among them. We also discuss fuzzy normal HX ideal under homomorphism and anti homomorphism of a HX ring.

Keywords: Fuzzy HX ideal, fuzzy normal HX ideal, homomorphism, anti homomorphism, image and pre-image of fuzzy subsets, upper level subset of fuzzy normal HX ideal.

AMS Subject Classification (2000): 20N25, 03E72, 03F055 , 06F35, 03G25.

1. INTRODUCTION

In 1965,Zadeh [15] introduced the concept of fuzzy sets and studied their properties.He defined fuzzy subset of a non-empty set as a collection of objects with grade of membership in a continuum, with each object being assigned a value between 0 and 1 by a membership function. Fuzzy set theory was guided by the assumption that classical sets were not natural, appropriate or useful notions in describing the real life problems, because every object encountered in this real physical world carries some degree of fuzziness. Further the concept of grade of membership is not a probabilistic concept. An algebraic structure will have an underlying set, binary operations, unary operations, and constants, that have some of the properties like commutativity, associativity, identity elements, inverse elements, and distributivity. Different kinds of structures will have different operations and properties. Rings will have the three operations of addition, subtraction, and multiplication, but don't need division. Most of rings will have commutative multiplication. In 1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [3] introduced the concept of HX group. In 1982 Wang-jin Liu [4] introduced the concept of fuzzy ring and fuzzy ideal. In 1988 Professor Li Hong Xing [5] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1,2] gave the structures of HX ring on a class of ring. T.K.Mukherjee and M.K.Sen [7] fuzzified certain results on rings.

R. Muthuraj et.al [8] introduced the concept of homomorphism and anti homomorphism of fuzzy HX ideals of a HX ring. In this paper we define a new algebraic structure of a fuzzy normal HX ideal and study some of their properties. We define the concept of fuzzy subsets of a fuzzy normal HX ideals of a HX ring and discuss some of their related properties also we introduce the concept of an image and pre-image of a fuzzy subsets and discuss some of its properties with fuzzy normal HX ideal under homomorphism and anti homomorphism.

2. PRELIMINARIES

In this section, we define the notion of fuzzy normal HX ideals of a HX ring and discussed some of its related results. Throughout this paper, $R = (R ,+, \cdot)$ is a Ring, e is the additive identity element of R and xy, we mean $x \cdot y$.

2.1 Definition [9]

Let R be a ring. Let μ be a fuzzy set defined on R . Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^{μ} of $\Re$ is called a fuzzy HX ring on $\Re$ or a fuzzy ring induced by μ if the following conditions are satisfied. For all A , B $\in \Re$,

i. $\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

ii. $\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

where $\lambda^{\mu}(A) = \max\{\mu(x) \ / \ \text{for all } x \in A \subseteq R\}$.

2.2 Definition

Let R be a ring. Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy HX right ideal λ^{μ} of $\Re$ is called a fuzzy normal HX right ideal on $\Re$ or a fuzzy normal right ideal induced by μ if the following conditions are satisfied.

For all A,B $\in \Re$, $\lambda^{\mu}(AB) = \lambda^{\mu}(BA)$, where $\lambda^{\mu}(A) = \max\{\mu(x) \ / \ \text{for all } x \in A \subseteq R\}$.

2.3 Definition

Let R be a ring. Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy HX left ideal λ^μ of $\Re$ is called a fuzzy normal HX left ideal on $\Re$ or a fuzzy normal left ideal induced by μ if the following conditions are satisfied.

For all A,B $\in \Re$, $\lambda^\mu(AB) = \lambda^\mu(BA)$, where $\lambda^\mu(A) = \max\{ \mu(x) /$ for all $x \in A \subseteq R \}$.

2.4 Definition

Let R be a ring. Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\Re$ is called a fuzzy normal HX ideal on $\Re$ or a fuzzy normal ideal induced by μ if it is both fuzzy normal HX right ideal and fuzzy normal HX left ideal on $\Re$. That is, For all A, B $\in \Re$,

i. $\lambda^\mu(A - B) \geq \min\{ \lambda^\mu(A), \lambda^\mu(B) \}$,

ii. $\lambda^\mu(AB) \geq \max\{ \lambda^\mu(A), \lambda^\mu(B) \}$

iii. $\lambda^\mu(AB) = \lambda^\mu(BA)$, where $\lambda^\mu(A) = \max\{ \mu(x) /$ for all $x \in A \subseteq R \}$.

2.5 Theorem

If μ is a fuzzy normal right ideal of a ring R then the fuzzy subset λ^μ is a fuzzy normal HX right ideal of a HX ring $\Re$.

Proof

Let μ be a fuzzy normal right ideal of R.

By Theorem 3.2 [10], λ^μ is a fuzzy HX right ideal of a HX ring $\Re$.

Now, For all A,B $\in \Re$,

$$\begin{aligned} \lambda^\mu(AB) &= \max\{ \mu(xy) / \text{ for all } x \in A \subseteq R, y \in B \subseteq R \} \\ &= \max\{ \mu(yx) / \text{ for all } x \in A \subseteq R, y \in B \subseteq R \} \\ &= \lambda^\mu(BA) \end{aligned}$$

Therefore, $\lambda^\mu(AB) = \lambda^\mu(BA)$.

Hence, λ^μ is a fuzzy normal HX right ideal on $\Re$.

2.6 Theorem

If μ is a fuzzy normal left ideal of a ring R then the fuzzy subset λ^μ is a fuzzy normal HX left ideal of a HX ring $\Re$.

Proof

Let μ be a fuzzy normal left ideal of R.

By Theorem 4.2 [10], λ^μ is a fuzzy HX left ideal of a HX ring $\Re$.

Now, For all $A,B \in \Re$,

$$\lambda^{\mu}(AB) = \max\{\mu(xy) / \text{ for all } x\in A \subseteq R,\ y\in B \subseteq R\}$$
$$= \max\{\mu(yx) / \text{ for all } x\in A \subseteq R,\ y\in B \subseteq R\}$$
$$= \lambda^{\mu}(BA)$$

Therefore, $\lambda^{\mu}(AB) = \lambda^{\mu}(BA)$.

Hence, λ^{μ} is a fuzzy normal HX left ideal on $\Re$.

2.7 Theorem

If μ is a fuzzy normal ideal of a ring R then the fuzzy subset λ^{μ} is a fuzzy normal HX ideal of a HX ring $\Re$.

Proof

It is clear.

2.8 Theorem

Let μ and η be any two fuzzy sets on R. Let λ^{μ} and γ^{η} be any two fuzzy normal HX right ideals of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy normal HX right ideal of a HX ring $\Re$.

Proof

Let λ^{μ} and γ^{η} be any two fuzzy normal HX right ideals of a HX ring $\Re$.

By Theorem 3.3[10], $\lambda^{\mu} \cap \gamma^{\eta}$ is a fuzzy HX right ideal of a HX ring $\Re$.

Let $A,B \in \Re$

$$(\lambda^{\mu}\cap\gamma^{\eta})(AB) = \min\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\}$$
$$= \min\{\lambda^{\mu}(BA), \gamma^{\eta}(BA)\}$$
$$= (\lambda^{\mu}\cap\gamma^{\eta})(BA).$$
$$(\lambda^{\mu}\cap\gamma^{\eta})(AB) = (\lambda^{\mu}\cap\gamma^{\eta})(BA).$$

Hence, $\lambda^{\mu} \cap \gamma^{\eta}$ is a fuzzy normal HX right ideal of a HX ring $\Re$.

2.9 Theorem

Let μ and η be any two fuzzy sets on R. Let λ^{μ} and γ^{η} be any two fuzzy normal HX left ideals of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy normal HX left ideal of a HX ring $\Re$.

Proof

Let λ^{μ} and γ^{η} be any two fuzzy normal HX left ideals of a HX ring $\mathfrak{R}$.

By Theorem 4.3 [10] , $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy HX left ideal of a HX ring $\mathfrak{R}$.

Let A,B $\in \mathfrak{R}$

$$\begin{aligned} (\lambda^{\mu}\cap\gamma^{\eta})(AB) &= \min\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\} \\ &= \min\{\lambda^{\mu}(BA), \gamma^{\eta}(BA)\} \\ &= (\lambda^{\mu}\cap\gamma^{\eta})(BA). \end{aligned}$$

Hence, $\lambda^{\mu} \cap \gamma^{\eta}$ is a fuzzy normal HX left ideal of a HX ring $\mathfrak{R}$.

2.10 Theorem

Let μ and η be any two sets on R. Let λ^{μ} and γ^{η} be any two fuzzy normal HX ideals of a HX ring $\mathfrak{R}$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy normal HX ideal of a HX ring $\mathfrak{R}$.

Proof

It is clear.

2.11 Remark

i. The intersection of family of fuzzy normal HX ideals of a HX ring $\mathfrak{R}$ is also fuzzy normal HX ideal of $\mathfrak{R}$.
ii. Let R be a ring. Let μ and η be fuzzy normal ideals of R and $\mu \cap \eta$ is also a fuzzy normal ideal of R then $\varphi^{\mu \cap \eta}$ is a fuzzy normal HX ideal of $\mathfrak{R}$ induced by $\mu \cap \eta$ of R.

2.12 Theorem

If λ^{μ} , γ^{η} , $\varphi^{\mu \cap \eta}$ are fuzzy normal HX ideals of a HX ring $\mathfrak{R}$ induced by the fuzzy sets μ , η , $\mu \cap \eta$ of R then $\varphi^{\mu \cap \eta} = \lambda^{\mu} \cap \gamma^{\eta}$.

Proof

It is clear.

2.13 Theorem

Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy normal HX ring and γ^{η} be a fuzzy normal HX right (left) ideal of a HX ring $\mathfrak{R}$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy normal HX right (left) ideal of a HX ring $\mathfrak{R}$.

Proof

It is clear.

2.14 Theorem

Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy normal HX ring and γ^{η} be a fuzzy normal HX ideal of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy normal HX ideal of a HX ring $\Re$.

Proof

It is clear.

2.15 Theorem

Let μ and η be any two fuzzy sets of R. Let $\Re \subset 2^{R} - \{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy normal HX right ideals of $\Re$ then, their union $(\lambda^{\mu} \cup \gamma^{\eta})$ is also a fuzzy normal HX right ideal of $\Re$.

Proof

Let λ^{μ} and γ^{η} be any two fuzzy normal HX right ideals of $\Re$.

By Theorem 3.4[10], $(\lambda^{\mu} \cup \gamma^{\eta})$ is a fuzzy HX right ideal of $\Re$.

Let A,B $\in \Re$,

$$
\begin{aligned}
(\lambda^{\mu} \cup \gamma^{\eta})(AB) &= \max\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\} \\
&= \max\{\lambda^{\mu}(BA), \gamma^{\eta}(BA)\} \\
&= (\lambda^{\mu} \cup \gamma^{\eta})(BA). \\
(\lambda^{\mu} \cup \gamma^{\eta})(AB) &= (\lambda^{\mu} \cup \gamma^{\eta})(BA).
\end{aligned}
$$

Hence, $\lambda^{\mu} \cup \gamma^{\eta}$ is a fuzzy normal HX right ideal of a HX ring $\Re$.

2.16 Theorem

Let μ and η be any two fuzzy sets of R. Let $\Re \subset 2^{R} - \{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy normal HX left ideals of $\Re$ then, their union $(\lambda^{\mu} \cup \gamma^{\eta})$ is also a fuzzy normal HX left ideal of $\Re$.

Proof

Let λ^{μ} and γ^{η} be any two fuzzy normal HX left ideals of $\Re$.

By Theorem 4.4[10], $\lambda^{\mu} \cup \gamma^{\eta}$ is a fuzzy HX left ideal of $\Re$.

Let A,B $\in \Re$,

$$(\lambda^{\mu} \cup \gamma^{\eta})(AB) = \max\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\}$$
$$= \max\{\lambda^{\mu}(BA), \gamma^{\eta}(BA)\}$$
$$= (\lambda^{\mu} \cup \gamma^{\eta})(BA).$$
$$(\lambda^{\mu} \cup \gamma^{\eta})(AB) = (\lambda^{\mu} \cup \gamma^{\eta})(BA).$$

Hence, $\lambda^{\mu} \cup \gamma^{\eta}$ is a fuzzy normal HX left ideal of a HX ring $\Re$.

2.17 Theorem

Let μ and η be any two fuzzy sets of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy normal HX ideals of $\Re$ then, their union $(\lambda^{\mu} \cup \gamma^{\eta})$ is also a fuzzy normal HX ideal of $\Re$.

Proof

It is clear.

2.18 Remark

i. Union of family of fuzzy normal HX ideals of a HX ring $\Re$ is also fuzzy normal HX ideal of $\Re$.
ii. Let R be a ring. Let μ and η be fuzzy normal ideals of R then $\varphi^{\mu \cup \eta}$ is a fuzzy normal HX ideal of $\Re$ induced by $\mu \cup \eta$ of R.

2.19 Theorem

Let R be a ring. Let μ and η be fuzzy sets of R. If λ^{μ} , γ^{η} , $\varphi^{\mu \cup \eta}$ are fuzzy normal HX ideals of a HX ring $\Re$ induced by μ , η , $\mu \cup \eta$ of R then $\varphi^{\mu \cup \eta} = \lambda^{\mu} \cup \gamma^{\eta}$.

Proof

It is clear.

2.20 Theorem

Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy normal HX ring and γ^{η} be a fuzzy normal HX right (left) ideal of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cup \gamma^{\eta}$ is also a fuzzy normal HX right (left) ideal of a HX ring $\Re$.

Proof

It is clear.

2.21 Theorem

Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy normal HX ring and γ^{η} be a fuzzy normal HX ideal of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cup \gamma^{\eta}$ is also a fuzzy normal HX ideal of a HX ring $\Re$.

Proof

It is clear.

3. CARTESIAN PRODUCT OF FUZZY NORMAL HX IDEAL OF A HX RING

3.1 Theorem

Let μ and η be any two fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R1} - \{\phi\}$ and $\Re_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings. If λ^{μ} and γ^{η} are any two fuzzy normal HX right ideals of $\Re_1$ and $\Re_2$ respectively then, $\lambda^{\mu} \times \gamma^{\eta}$ is also a fuzzy normal HX right ideal of a HX ring $\Re_1 \times \Re_2$.

Proof

Let λ^{μ} and γ^{η} are any two fuzzy normal HX right ideals of $\Re_1$ and $\Re_2$ respectively.

By Theorem 3.5[10], $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX right ideal of a HX ring $\Re_1 \times \Re_2$.

Let (A, B) , (C,D) $\in \Re_1 \times \Re_2$,

$$
\begin{aligned}
(\lambda^{\mu} \times \gamma^{\eta})((A, B)\cdot(C ,D)) &= (\lambda^{\mu} \times \gamma^{\eta})(AC, BD) \\
&= \min\{\lambda^{\mu}(AC), \gamma^{\eta}(BD)\} \\
&= \min\{\lambda^{\mu}(CA), \gamma^{\eta}(DB)\} \\
&= (\lambda^{\mu} \times \gamma^{\eta})(CA, DB) \\
&= (\lambda^{\mu} \times \gamma^{\eta})((C, D)\cdot(A, B)) \\
(\lambda^{\mu} \times \gamma^{\eta})((A, B)\cdot(C ,D)) &= (\lambda^{\mu} \times \gamma^{\eta})((C, D)\cdot(A, B)).
\end{aligned}
$$

Hence, $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy normal HX right ideal of the HX ring $\Re_1 \times \Re_2$.

3.2 Theorem

Let μ and η be any two fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R1} - \{\phi\}$ and $\Re_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings. If λ^{μ} and γ^{η} are any two fuzzy normal HX left ideals of $\Re_1$ and $\Re_2$ respectively then, $\lambda^{\mu} \times \gamma^{\eta}$ is also a fuzzy normal HX left ideal of a HX ring $\Re_1 \times \Re_2$.

Proof

Let λ^{μ} and γ^{η} are any two fuzzy normal HX left ideals of $\Re_1$ and $\Re_2$ respectively.

By Theorem 4.5[10], $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX left ideal of a HX ring $\Re_1 \times \Re_2$.

Let $(A, B), (C,D) \in \Re_1 \times \Re_2$,

$$\begin{aligned}
(\lambda^{\mu} \times \gamma^{\eta})((A, B)\cdot(C ,D)) &= (\lambda^{\mu} \times \gamma^{\eta})(AC, BD) \\
&= \min\{\lambda^{\mu}(AC), \gamma^{\eta}(BD)\} \\
&= \min\{\lambda^{\mu}(CA), \gamma^{\eta}(DB)\} \\
&= (\lambda^{\mu} \times \gamma^{\eta})(CA, DB) \\
&= (\lambda^{\mu} \times \gamma^{\eta})((C, D)\cdot(A, B)) \\
(\lambda^{\mu} \times \gamma^{\eta})((A, B)\cdot(C ,D)) &= (\lambda^{\mu} \times \gamma^{\eta})((C, D)\cdot(A, B)).
\end{aligned}$$

Hence, $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy normal HX left ideal of the HX ring $\Re_1 \times \Re_2$.

3.3 Theorem

Let μ and η be any two fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R1} - \{\phi\}$ and $\Re_2 \subset 2^{R2} - \{\phi\}$ be any two HX rings. If λ^{μ} and γ^{η} are any two fuzzy normal HX ideals of $\Re_1$ and $\Re_2$ respectively then, $\lambda^{\mu} \times \gamma^{\eta}$ is also a fuzzy normal HX ideal of a HX ring $\Re_1 \times \Re_2$.

Proof

It is clear.

4. HOMOMORPHISM AND ANTI HOMOMORPHISM OF A FUZZY NORMAL HX IDEAL

In this section, we introduce the concept of an image, pre-image of fuzzy subset of a HX ring and discussed the properties of homomorphic and anti homomorphic images and pre-images of fuzzy normal HX ideal of a HX ring $\Re$.

4.1 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively.Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let μ be a fuzzy subset of R_1. Let λ^{μ} be a fuzzy normal HX right ideal of $\Re_1$ then $f(\lambda^{\mu})$ is a fuzzy normal HX right ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ^{μ} is a fuzzy normal HX right ideal of $\Re_1$.

By theorem 3.7 [8], f (λ^{μ}) is a fuzzy HX right ideal of $\Re_2$.

There exist $X, Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$,

$$\begin{aligned}
(f(\lambda^{\mu}))(f(X)\,f(Y)) &= (f(\lambda^{\mu}))(f(XY)), \\
&= \lambda^{\mu}(XY) \\
&= \lambda^{\mu}(YX) \\
&= (f(\lambda^{\mu}))(f(YX)) \\
&= (f(\lambda^{\mu}))(f(Y)f(X)) \\
(f(\lambda^{\mu}))(f(X)f(Y)) &= (f(\lambda^{\mu}))(f(Y)f(X)).
\end{aligned}$$

Hence, f (λ^{μ}) is a fuzzy normal HX right ideal of $\Re_2$.

4.2 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let μ be a fuzzy subset of R_1. Let λ^{μ} be a fuzzy normal HX left ideal of $\Re_1$ then f (λ^{μ}) is a fuzzy normal HX left ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ^{μ} is a fuzzy normal HX left ideal of $\Re_1$.

By theorem 3.7 [8], f (λ^{μ}) is a fuzzy normal HX left ideal of $\Re_2$.

There exist $X, Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$,

$$\begin{aligned}
f(\lambda^{\mu}))(f(X)\,f(Y)) &= (f(\lambda^{\mu}))(f(XY)), \\
&= \lambda^{\mu}(XY) \\
&= \lambda^{\mu}(YX) \\
&= (f(\lambda^{\mu}))(f(YX)) \\
&= (f(\lambda^{\mu}))(f(Y)f(X)) \\
(f(\lambda^{\mu}))(f(X)f(Y)) &= (f(\lambda^{\mu}))(f(Y)f(X)).
\end{aligned}$$

Hence, f (λ^{μ}) is a fuzzy normal HX left ideal of $\Re_2$.

4.3 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively.

Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let μ be a fuzzy subset of R_1. Let λ^μ be a fuzzy normal HX ideal of $\Re_1$ then $f(\lambda^\mu)$ is a fuzzy normal HX ideal of $\Re_2$, if λ^μ has a supremum property and λ^μ is f-invariant.

Proof

It is clear.

4.4 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings. Let α be a fuzzy subset of R_2. Let η^α be a fuzzy normal HX right ideal of $\Re_2$ then $f^{-1}(\eta^\alpha)$ is a fuzzy normal HX right ideal of $\Re_1$.

Proof

Let α be a fuzzy subset of R_2 and η^α be a fuzzy normal HX right ideal of $\Re_2$.

By Theorem 3.9[8], $f^{-1}(\eta^\alpha)$ is a fuzzy HX right ideal of $\Re_1$.

For any $X,Y \in \Re_1$, $f(X), f(Y) \in \Re_2$,

$$\begin{aligned}
(f^{-1}(\eta^\alpha))(XY) &= \eta^\alpha(f(XY)) \\
&= \eta^\alpha(f(X)\, f(Y)) \\
&= \eta^\alpha(f(Y)\, f(X)) \\
&= \eta^\alpha(f(YX)) \\
&= (f^{-1}(\eta^\alpha))(YX) \\
(f^{-1}(\eta^\alpha))(XY) &= (f^{-1}(\eta^\alpha))(YX)
\end{aligned}$$

Hence, $f^{-1}(\eta^\alpha)$ is a fuzzy normal HX right ideal of $\Re_1$.

4.5 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings. Let α be a fuzzy subset of R_2. Let η^α be a fuzzy normal HX left ideal of $\Re_2$ then $f^{-1}(\eta^\alpha)$ is a fuzzy normal HX left ideal of $\Re_1$.

Proof

Let α be a fuzzy subset of R_2 and η^α be a fuzzy normal HX left ideal of $\Re_2$.

By Theorem 3.9[8], $f^{-1}(\eta^\alpha)$ is a fuzzy HX left ideal of $\Re_1$.

For any $X,Y \in \Re_1$, $f(X), f(Y) \in \Re_2$,

$$(f^{-1}(\eta^\alpha))(XY) = \eta^\alpha(f(XY))$$

$$= \eta^{\alpha}(f(X)\, f(Y))$$
$$= \eta^{\alpha}(f(Y)\, f(X))$$
$$= \eta^{\alpha}(f(YX))$$
$$= (f^{-1}(\eta^{\alpha}))\,(YX)$$
$$(f^{-1}(\eta^{\alpha}))\,(XY) = (f^{-1}(\eta^{\alpha}))\,(YX)$$

Hence, $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX left ideal of $\Re_1$.

4.6 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism on HX rings. Let α be a fuzzy subset of R_2. Let η^{α} be a fuzzy normal HX ideal of $\Re_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX ideal of $\Re_1$.

Proof

It is clear.

4.7 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let μ be a fuzzy subset of R_1. Let λ^{μ} be a fuzzy normal HX right ideal of $\Re_1$, then $f(\lambda^{\mu})$ is a fuzzy normal HX left ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ^{μ} is a fuzzy normal HX right ideal of $\Re_1$.

By Theorem 3.8[8], $f(\lambda^{\mu})$ is a fuzzy HX left ideal of $\Re_2$.

There exist $X, Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$.

$$f(\lambda^{\mu}))\,(f(X)\, f(Y)) = (f(\lambda^{\mu}))\,(f(YX)),$$
$$= \lambda^{\mu}(YX)$$
$$= \lambda^{\mu}(XY)$$
$$= (f(\lambda^{\mu}))\,(f(XY))$$
$$= (f(\lambda^{\mu}))\,(f(Y)f(X))$$
$$(f(\lambda^{\mu}))\,(f(X)f(Y)) = (f(\lambda^{\mu}))\,(f(Y)f(X)).$$

Hence, $f(\lambda^{\mu})$ is a fuzzy normal HX left ideal of $\Re_2$.

4.8 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let μ be a fuzzy subset of R_1 . Let λ^{μ} be a fuzzy normal HX left ideal of $\Re_1$, then $f(\lambda^{\mu})$ is a fuzzy normal HX right ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

Let μ be a fuzzy subset of R_1 and λ^{μ} is a fuzzy normal HX right ideal of $\Re_1$.

By Theorem 3.8[8], $f(\lambda^{\mu})$ is a fuzzy HX right ideal of $\Re_2$

There exist $X, Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$

$$
\begin{aligned}
f(\lambda^{\mu}))(f(X)\,f(Y)) &= (f(\lambda^{\mu}))(f(YX)), \\
&= \lambda^{\mu}(YX) \\
&= \lambda^{\mu}(XY) \\
&= (f(\lambda^{\mu}))(f(XY)) \\
&= (f(\lambda^{\mu}))(f(Y)f(X)) \\
(f(\lambda^{\mu}))(f(X)f(Y)) &= (f(\lambda^{\mu}))(f(Y)f(X)).
\end{aligned}
$$

Hence, $f(\lambda^{\mu})$ is a fuzzy normal HX right ideal of $\Re_2$.

4.9 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let μ be a fuzzy subset of R_1 . Let λ^{μ} be a fuzzy normal HX ideal of $\Re_1$, then $f(\lambda^{\mu})$ is a fuzzy normal HX ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof

It is clear.

4.10 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be an anti homomorphism on HX rings. Let α be a fuzzy subset of R_2. Let η^{α} be a fuzzy normal HX right ideal of $\Re_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX left ideal of $\Re_1$.

Proof

Let α be a fuzzy subset of R_2 and η^{α} be a fuzzy normal HX right ideal of $\Re_2$.

By Theorem 3.10[8], $f^{-1}(\eta^{\alpha})$ is a fuzzy HX left ideal of $\mathfrak{R}_1$.

For any $X,Y \in \mathfrak{R}_1$, then $f(X), f(Y) \in \mathfrak{R}_2$,

$$\begin{aligned}
(f^{-1}(\eta^{\alpha}))(XY) &= \eta^{\alpha}(f(XY)) \\
&= \eta^{\alpha}(f(Y)\,f(X)) \\
&= \eta^{\alpha}(f(X)\,f(Y)) \\
&= \eta^{\alpha}(f(YX)) \\
&= (f^{-1}(\eta^{\alpha}))(YX) \\
(f^{-1}(\eta^{\alpha}))(XY) &= (f^{-1}(\eta^{\alpha}))(YX)
\end{aligned}$$

Hence, $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX left ideal of $\mathfrak{R}_1$.

4.11 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be an anti homomorphism on HX rings. Let α be a fuzzy subset of R_2. Let η^{α} be a fuzzy normal HX left ideal of $\mathfrak{R}_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX right ideal of $\mathfrak{R}_1$.

Proof

Let α be a fuzzy subset of R_2 and η^{α} be a fuzzy normal HX right ideal of $\mathfrak{R}_2$.

By Theorem 3.10[8], $f^{-1}(\eta^{\alpha})$ is a fuzzy HX right ideal of $\mathfrak{R}_1$.

For any $X,Y \in \mathfrak{R}_1$, then $f(X), f(Y) \in \mathfrak{R}_2$

$$\begin{aligned}
(f^{-1}(\eta^{\alpha}))(XY) &= \eta^{\alpha}(f(XY)) \\
&= \eta^{\alpha}(f(Y)\,f(X)) \\
&= \eta^{\alpha}(f(X)\,f(Y)) \\
&= \eta^{\alpha}(f(YX)) \\
&= (f^{-1}(\eta^{\alpha}))(YX) \\
(f^{-1}(\eta^{\alpha}))(XY) &= (f^{-1}(\eta^{\alpha}))(YX)
\end{aligned}$$

Hence, $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX right ideal of $\mathfrak{R}_1$.

4.12 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be an anti homomorphism on HX rings. Let α be a fuzzy subset of R_2. Let η^{α} be a fuzzy normal HX ideal of $\mathfrak{R}_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy normal HX ideal of $\mathfrak{R}_1$.

Proof

It is clear.

REFERENCES

[1] Bing-xueYao and Yubin-Zhong, 2007, The construction of power ring, Fuzzy information and Engineering (ICFIE),ASC 40,pp.181-187.
[2] Bing-xueYao and Yubin-Zhong, 2009, Upgrade of algebraic structure of ring, Fuzzy information and Engineering , 2:219-228.
[3] Li Hong Xing, HX group, 1987,BUSEFAL,33(4), 31-37.
[4] Liu. W.J., Fuzzy invariant subgroups and fuzzy ideals, Fuzzy sets and systems,8: 133-139.
[5] Li Hong Xing, HX ring, 1988, BUSEFAL , 34(1) 3-8.
[6] Mashinchi.M and Zahedi. M.M, On fuzzy ideals of a ring, J.Sci.I.R.Iran,1(3), 208-210(1990).
[7] Mukherjee.T.K & Sen.M.K., On fuzzy ideals in rings,Fuzzy sets and systems,21, 99-104,1987.
[8] Muthuraj.R, Ramila Gandhi.N, Homomorphism and anti homomorphism of fuzzy HX ideals of a HX ring, Discovery, Volume 21, Number 65, July 1, 2014, pp 20-24.
[9] Muthuraj.R, Ramila Gandhi.N, pseudo fuzzy cosets of a HX ring, SS international journal of Pure and applied mathematics. Vol.1 Issue.1 2015, pp 22-30.
[10] [10] Muthuraj.R, Ramila Gandhi.N, Properties of fuzzy HX ideal of a HX ring, International journal of Mathematics And its Applications. Vol.4, Issue.2-B(2016),pp 41-50.
[11] Rosenfeld. A., Fuzzy groups,J.Math.Anal.,35(1971),512-517.
[12] Shaoquan Sun,Fuzzy rings with operators, Department of Mathematics,pp.111-115.
[13] Yao Bing- xue, Homomorphism and Isomorphism of Regular Power Ring, Chinese Quarterly Journal Of Mathematics, Vol . 15,No. 2, June 2000.
[14] Wang Qing –hua ,Fuzzy rings and fuzzy subrings ,pp.1-6.
[15] Zadeh.L.A., Fuzzy sets,Information and control,8,338-353.