

## Pathos Lict Jump Graph of a Tree

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### Abstract

Let  $G$  be a nonempty graph. jump graph  $J(G)$  of  $G$  is the graph whose vertex set is the set of edges of  $G$  in which two vertices of  $J(G)$  are adjacent if and only if the edges of  $G$  are not adjacent in  $G$ . In this paper we define a new type of graph called pathos lict jump graph of tree. We characterize the planarity and outer planarity and traversability of pathos lict jump graph of a tree.

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### Introduction

The concept of pathos of a graph  $G$  was introduced by Harary[6], as a collection of minimum number of line disjoint open paths whose union is  $G$ . The path number of a graph  $G$  is the number of paths in pathos. All graphs considered here are finite, undirected and simple. We refer[6] for unexplained terminology and notation. If  $G$  is a planar graph, the inner vertex number  $i(G)$  of  $G$  is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of  $G$  in the plane. A graph  $G$  is said to be minimally non-outerplanar if  $i(G) = 1$ . Let  $G = (V(G), E(G))$  be a graph, and  $V(G)$  and  $E(G)$  are called the order and size of  $G$ , respectively.  $C(G)$  denote the crossing number of a graph. The line graph  $L(G)$  of  $G$  has the edges of  $G$  as its vertices and two vertices of  $L(G)$  are adjacent if and only if they are adjacent in  $G$ . We call the complement of line graph  $L(G)$  is the jump graph  $J(G)$  of  $G$  followed by [2]. That is, the jump graph  $J(G)$  is the graph defined on  $E(G)$ , in which two vertices are adjacent if and only if they are not adjacent in  $G$ . Since both  $L(G)$  and

$J(G)$  are defined on the edge set of  $G$ , hence isolated vertices of  $G$  play no role in both graphs. We assume that the graph  $G$  under consideration is nonempty and has no isolated vertices in that follows. Cut vertex jump graph was introduced by Venkanagouda M Goudar and Nagendrappa [2]. The lict graph  $n(G)$  of a graph  $G$  is introduced by M H Muddebihal and V R Kulli [3]. Also by Venkanagouda M Goudar, Nagendrappa and M H Muddebihal introduced the new concept called lict jump graph of a graph [1]. Using these concepts, we introduced the new type of graph called pathos lict jump graph  $J_{pn}(T)$  of a tree  $T$ . The vertices of pathos lict jump graph  $J_{pn}(T)$  has the edges, cut vertices and path of pathos of  $T$ . Two vertices of  $J_{pn}(T)$  are adjacent if and only if edges are not adjacent in  $T$  or edges are adjacent in  $T$ , the edges are incident to the cut vertex of  $T$  and edges lies on a path. Since the system of pathos for a tree is not unique, the corresponding pathos lict jump graph of a tree is not unique. All graphs considered here are finite, undirected and simple. We refer [6] for unexplained terminology and notation.

The line degree of a line  $uv$  in a tree  $T$ . The pathos length is the number of edges which lie on a particular path  $P_i$  of pathos of a tree  $T$ . A pendent pathos is a path  $P_i$  of pathos having unit length which corresponds to a pendent edge in  $T$ . A pathos vertex is a vertex in  $J_P(T)$  corresponding to a path of pathos of  $T$ . If  $G$  is planar, the inner point number  $i(G)$  of a graph  $G$  is the minimum number of points not belonging to the boundary of the exterior region in any embedding of  $G$  in the plane. A graph  $G$  is said to be minimally nonouterplanar if  $i(G) = 1$

### Preliminaries

We need the following results for our further results

**Theorem 1:** [6] If  $G$  is a  $(p, q)$  graph whose vertices have degree  $d_i$  then  $L(G)$  has  $q$  vertices and  $q_L = -q + \frac{1}{2} \sum d_i^2$  edges.

**Theorem 2:** [7] The line graph  $L(G)$  of a graph is planar if and only if  $G$  is planar,  $\Delta(G) \leq 4$  and if  $\deg v = 4$  for a vertex  $v$  of  $G$ , then  $v$  is a cut vertex.

**Theorem 3:** [6] A graph is planar if and only if it has no sub graph homeomorphic to  $K_5$  or  $K_{3,3}$ .

**Theorem 4:** [6] A graph is outerplanar if and only if it has no sub graph homeomorphic to  $K_4$  or  $K_{2,3}$ .

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**Theorem 5:** For any  $(p, q)$  graph  $T$ , whose vertices have degree  $d_i$ , cut vertices  $c$  have degree  $c_j$ , then  $J_{pn}(T)$  has  $q+c+k$  vertices and  $\sum \deg c_j + \binom{q}{2} + q$  edges, where  $k$  is the path number.

**Proof:** By definition of pathos lict jump graph, the number of vertices is the union of number of edges, the number of cut vertices and number of path of pathos of T. Hence  $J_{pn}(T)$  has  $q + c+k$  vertices where k is a path number

Further, the jump graph is the complement of line graph and also line graph have  $q_L = -q + \frac{1}{2} \sum d_i^2$  edges. Clearly the union of jump graph and the line graph is a complete graph. Hence the number of edges in the jump graph is the number of edges in complete graph minus number of edges in line graph. This implies that the number of edges in jump graph  $= \frac{q(q-1)}{2} - \left(-q + \frac{1}{2} \sum d_i^2\right)$ . Also the number of edges in the lict jump graph is sum of the number of edges incident with cut vertices  $\deg c_j$  and the number of edges in line graph. Hence the number of edges in pathos lict jump graph is the sum of number of edges in jump graph, the of edges in lict graph and the edges lies on the pathos which is the number of edges of T. Thus

$$E|J_{pn}(G)| = \frac{q(q-1)}{2} - \left(-q + \frac{1}{2} \sum d_i^2\right) + \sum \deg c_j - q + \frac{1}{2} \sum d_i^2 + q = \sum \deg c_j + \binom{q}{2} + q$$

edges

**Theorem 6:** For any nontrivial graph G, the pathos lict jump graph  $J_{pn}(G)$  is planar if and only if i).  $q \leq 4$ , (ii). there is an unique cutvertex  $c_j$  such that  $\deg c_j \leq 3$ .

**Proof:** Suppose  $J_{pn}(G)$  is planar. Assume that  $q > 4$ . By the definition of  $J_{pn}(G)$ , it forms  $K_5$  as a subgraph, which is non planar, a contradiction.

Also if there is a cutvertex  $c_j$  such that  $\deg c_j > 3$ . By definition of lict graph, lict graph of  $K_{1,4}$  is  $K_5$  which is non planar. Since pathos lict jump graph contains lict graph as a subgraph, hence  $J_{pn}(G)$  is non planar, which is a contradiction.

Conversely suppose G satisfies both conditions, we have the following cases:

**Case (i):** Suppose G is a graph which contains the edges such that  $q \leq 4$ . By the definition of lict jump graph, all four vertices forms a graph which is  $K_4$ . By the definition of pathos lict jump graph, each pathos vertex is adjacent to at most three vertices of  $J_{pn}(G)$ , which is planar.

Further, suppose G is a graph which contains a cutvertex  $c_j$  such that  $\deg c_j \leq 3$ . By the lict jump graph, it forms  $K_4$  as a subgraph. By the definition of pathos lict jump graph, the pathos vertex does not loose any planarity. Hence  $J_{pn}(G)$  is planar.

**Theorem 7:** For any nontrivial graph G, the pathos lict jump graph  $J_{pn}(G)$  is outerplanar if and only if G is a path  $P_n$ ,  $n \leq 3$ .

**Proof:** Suppose pathos lict jump graph  $J_{pn}(G)$  is outerplanar. Assume that  $G$  is a path such that  $P_n, n > 3$ . clearly, each edge is adjacent to at most two edges and incident to at most two cut vertices and hence by the definition of lict jump graph, each vertex  $e_j$  of  $J_n(G)$  is adjacent to all remaining vertices  $e_j$  then it has  $K_4$  has sub graph which is non outer planar, by the definition of pathos lict jump graph, which is nonouterplanar, a contradiction.

Conversely, Suppose  $G$  is a path  $P_n, n \leq 3$ . Clearly each edge is adjacent to at most one adjacent and at most one non adjacent edge. By the definition of  $J_{pn}(G)$ , it has  $K_3$  has a sub graph which is outer planar.

**Theorem 8:** For any nontrivial graph  $G$ , the pathos lict jump graph  $J_{pn}(G)$  is minimally non outerplanar if and only if  $G$  is a path  $P_4$ , or  $G$  is  $K_{1,3}$ .

**Proof:** Suppose  $J_{pn}(G)$  is minimally non outer planar. We have the following cases

**Case (i):** Assume that  $G$  is  $K_{1,n}, n \geq 4$ . By the definition of lict jump graph,  $J_n(K_{1,n}) = K_{n+1}$  which is not minimally non outer planar, hence by the definition of pathos lict jump graph,  $J_{pn}(K_{1,n})$  has  $K_{n+1}$  as a subgraph, which minimally non outer planar.

**Case (ii):** Assume that  $G$  is a path. In  $P_n, n \geq 5$ , each edge is adjacent to at most two edges and incident to at most two cut vertices and hence by the definition of lict jump graph, each vertex  $e_j$  of  $J_n(G)$  is adjacent to all remaining vertices  $e_j$  then it has  $K_5$  has sub graph which is minimally non outer planar, hence definition of pathos lict jump graph,  $J_{pn}(K_{1,n})$  has  $K_{n+1}$  as a subgraph, which minimally non outer planar, a contradiction.

Conversely, suppose  $G$  is  $K_{1,3}$ . By definition of lict graph lict graph of  $K_{1,3}$  is  $K_4$ . Since there is no nonadjacent edge in  $K_{1,3}$  and hence lict graph of  $G$  is the lict jump graph of  $G$  and is  $K_{1,3}$  which has inner vertex number one. Since the pathos vertex does not affects the planarity, hence  $J_{pn}(K_{1,3})$  is minimally non outer planar.

Also if  $G$  is a path  $P_4$ , clearly each edge is adjacent to at most one adjacent and at most one non adjacent edge. By the definition of  $J_n(G)$ , it has  $K_3$  has a sub graph and a pathos vertex is adjacent to all vertices of  $J_{pn}(G)$ , which is minimally non outer planar.

**Theorem 9:** For any nontrivial graph  $G$ , the pathos lict jump graph  $J_{pn}(G)$  without pendent pathos is hamiltonian if and only if  $G$  is a tree other than  $K_{1,3} + x$ .

**Proof:** Suppose pathos lict jump graph  $J_{pn}(G)$  is hamiltonian. If  $J_{pn}(G)$  contains a pendent pathos, it becomes a pendent vertex in  $J_{pn}(G)$  which is non hamiltonian.

Assume that  $G$  is a graph  $K_{1,3} + x$ . By the definition of lict graph, each block is complete and every cut vertex lies on exactly two blocks of  $n(T)$ . By definition of  $J_n(G)$ , each nonadjacent edge of  $G$  is adjacent in  $J_n(G)$ . Clearly it forms a block contains  $e_1e_2e_3e_4c_1e_5e_6e_7c_2, \dots, e_i e_{i+1} e_1$ . By the definition of pathos lict jump graph, the each pathos vertex is adjacent to two edges, clearly which is non hamiltonian, a contradiction.

Conversely, suppose  $G$  is a graph other than  $K_{1,3+x}$ . We have the follow cases

**Case (i):** Let  $G$  be a path  $P_n$ . Let  $V[n(P_n)] = (e_1, e_2, \dots, e_n) (c_1, c_2, \dots, c_{n-2})$  where  $(c_1, c_2, c_{n-2})$  are cut vertices of  $P_n$ . Since each block is a triangle and each block consists vertices as  $B_1 = e_1, c_1, e_2, B_2 = e_2, c_2, e_3, \dots, B_m = e_{n-1}, c_{n-2}, e_n$ . In  $J_n(G)$ , the nonadjacent edges of  $G$  are adjacent. The pathos vertex is adjacent to all vertices of  $J_{pn}(G)$ , clearly it forms a cycle as  $e_1c_1e_2c_2, \dots, e_{n-1}c_{n-2}e_n p_1 e_1$  containing all the vertices of  $J_{pn}(G)$ . Hence  $J_{pn}(G)$  is Hamiltonian

**Case (ii):** Let  $G$  be a tree  $T$ . We have the following sub cases:

**Sub Case (i):** Suppose  $T$  is a star with  $n$  vertices. Then  $L(G) = K_{1, n}, n(T) = L(T) + K_1$ , which contains a Hamiltonian cycle  $e_1e_2, \dots, e_nK_1e_1$ . Hence  $n(T)$  is Hamiltonian. By definition of  $J_{pn}(G)$ , the nonadjacent edges of  $G$  are adjacent in  $J_{pn}(G)$  and the pathos vertices does not affects the hamiltonian. Hence  $J_{pn}(G)$  is Hamiltonian.

**Sub Case (ii):** If  $T$  has more than one cut vertex without  $K_{1,3+x}$ , then in  $n(T)$ , each block is complete and every cut vertex lies on exactly two blocks of  $n(T)$ . By definition of  $J_{pn}(G)$ , each nonadjacent edge of  $G$  is adjacent in  $J_{pn}(G)$ . Clearly it forms a block contains  $e_1e_2e_3 p_1e_4c_1e_5e_6 p_1 e_7c_2, \dots, e_i e_{i+1} p_i e_1$ , which is Hamiltonian.

**Theorem 10:** For any nontrivial graph  $G$ , the pathos lict jump graph  $J_{pn}(G)$  is always non eulerian.

**Proof:** Suppose  $G$  is Eulerian. We have the following cases

**Case (i):** Assume that  $G$  is  $K_{1, n}$  for  $n$  is odd. By definition of lict graph, lict graph of  $K_{1, n}$  is  $K_{n+1}$ . Since there is no nonadjacent edge in  $G$  and hence lict graph is the lict jump graph of  $G$ , which is  $K_{n+1}$ . Clearly  $n+1$  is even. So every vertex of  $J_n(G)$  is odd degree. By the definition of pathos lict jump graph, one pathos vertex  $p_1$  is adjacent to one vertex and the remaining pathos vertices are adjacent to two vertices, which is non Eulerian, a contradiction. Also if  $G$  is  $K_{1, n}$   $n$  is even. By the definition of lict jump graph,  $J_n(G)$  is a complete graph with  $(n+1)$  vertices which is odd. By the definition of pathos lict jump graph, each pathos vertex is adjacent to two vertices of  $J_{pn}(G)$ . The degree of the vertex  $c_i$ , the cutvertex of  $G$  is odd, hence it is noneulerian.

**Case (ii):** Assume that  $G$  is a tree other than  $K_{1, n}$ . Clearly  $J_{pn}(G)$  has  $K_4 - x$  as a subgraph, which is non eulerian.

**Theorem 11:** For any graph  $G$ , the pathos lict jump graph  $J_{pn}(G)$  is a block if and only if  $G$  has no pendent pathos..

**Proof:** Suppose pathos lict jump graph  $J_{pn}(G)$  is a block. Assume that  $G$  has a pendent pathos. By the definition of pathos lict jump graph, it contains a pendent edge, which is a separable graph, a contradiction.

Conversely, suppose  $G$  is a graph without pendent paths. We have the following cases:

**Case (i):** Let  $G$  be a path  $P_n$ . Clearly each edge is adjacent to at least one adjacent or at least one nonadjacent edge. By the definition of  $J_{pn}(G)$ , it forms a graph without cut vertex. Hence  $J_{pn}(G)$  is a block.

**Case (ii):** Suppose  $G$  is a tree. Clearly each edge is adjacent to at least one adjacent or one nonadjacent edge. By the definition of  $J_{pn}(G)$ , it forms a block.

**Case (iii):** Suppose  $G$  is a block. By definition of  $J_n(G)$ , all the vertices of  $J_{pn}(G)$  are adjacent. Hence  $J_{pn}(G)$  is a complete graph, which is a block. Hence for any graph  $G$ ,  $J_{pn}(G)$  is always a block.

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