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A Dynamic Optimization Model for Sustainable Tourism

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Abstract

In this study, the optimal control theory is applied to find the solution to the economic and environmental problems in the development of sustainable tourism. Tourism specialization may be a successful tool to achieve fast economic growth, on the other side, it may be detrimental for natural resources. The aim of this study is to find the balance between economic benefits or the number of visitors and environmental costs to reach a sustainable development in the tourist destination of Tanah Lot, Bali Indonesia. Our analysis is based on the Pontryagin's maximum principle used to find the best possible number of tourist visiting the destination. We shows that if the tourist number is optimally determined then the long-run sustainable growth will be possible. We found that the optimal solution to this problem is $(X^{*}; Q^{*}) = (150{,}150;\ 3{,}003{,}019)$ which represents a condition where the sustainability is attainable.

Keywords: Dynamics optimisation, optimal control theory, sustainable tourism

1. INTRODUCTION

In the last two decades, articles discussing sustainable tourism from a mathematical perspective have increased significantly. In fact, the mathematical concepts that are applied are really sophisticated, such as the application of fuzzy theory, see for example in [6], [7], and [12]. These three articles offer a more comprehensive method

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of risk and preference analysis in tourism development. Besides, the optimal control theory is also applied in tourism, see for examples in [2] and [20]. The optimal control theory is applied to calculate the number of tourists who may visit a destination per unit time. These articles also offer a form of discussion of sustainable tourism using a more formal and mathematical approach. These include the models in [17] and [18] in which both articles discuss sustainable tourism in a dynamic optimization framework. The virtue of these articles is in the application of a more formal (or quantitative) mathematical model in presenting the dynamics of sustainable tourism.

The simplest thing to understand from the mathematical models presented in [17] or [5] might be the simplest in terms of ease of understanding when the results of these models are fitted with definitions of sustainability. For example, the definition of sustainability, as quoted in [3]: "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (World Commission on Environment and Development, 1987). As explained in [17], this definition assumes a relatively irrefutable social objective and is capable of permeating current environmental policies. That is, the definition of sustainability is very much in line with the mathematical models mentioned above.

Basically, the two models in [16] and [17] or [5] referred to above, refer to the bioeconomic model in [10] which is adapted for the purposes of tourism management. This bioeconomic model develops a dynamic model that describes the interrelated behavior between economic and environmental changes associated with tourism for a certain time span. In particular, this dynamic optimization model is developed for a particular tourism area and then optimizes the sustainable benefits.

When we talk about sustainable tourism, we are really talking about the concept of sustainable development. Sustainable development is an activity that seeks to make the lowest possible impact on the local environment and culture. At the same time, these activities can increase community income, create jobs, and protect the local ecosystem. The concept of sustainable development is an initiative from the formation of the concept of sustainable tourism or ecotourism [13].

The concept of sustainable tourism development basically aims to find a balance between social, economic and environmental goals. In another words, how to reduce the negative impact of tourism development, so that the benefits obtained from tourism can be benefited forever. Sustainable tourism constructs tourism as a triangular relationship between people around the destination and the surrounding environment, tourists and the tourism industry [13].

Among the three components, the tourism industry has the strongest influence. In this context, sustainable tourism aims to find a long-term balance between the three partners in the triangle. In addition, this concept aims to minimize environmental and cultural damage, as well as optimize visitor satisfaction, and maximize long-term economic growth for the region. It is a way to strike a balance between the growth potential of tourism and the need for environmental conservation [17].

When we talk about tourism, it is not something strange if we also mention the quality of the environment where the tourist destination is located. Likewise, when we talk about environmental quality, of course it is very closely related to natural resources. Natural resources referred to in this discussion are resources that can be renewed and are natural in the sense that these resources will eventually experience natural death or human disturbance, such as production forests or fisheries. So what is the relationship between tourism and natural resources? [8]

Before discussing further, it is necessary to know that renewable natural resources have the nature of being left without human disturbance to reach a maximum point where the growth rates will be balanced with destruction rates [16]. This maximum point is also known as the carrying capacity of natural resources for sustainability [10]. If the natural resources are used for natural tourism purposes, then the natural carrying capacity can be interpreted as the number of tourists allowed to visit at each unit of time for environmental preservation.

2. TOURIST DESTINATION DYNAMIC

Basically tourism depends on various attributes of natural resources or environment. In this discussion the model simplification assumes that the condition or quality of tourist destination can be measured accurately. Suppose that X represents the dynamic of environmental quality of the tourist destination for a certain period of time and suppose also that Q represents the number of tourist visiting the destination. Q can also be considered as the production of the tourist resource. The relationship between X and Q for a certain period t can be expressed as

$$\frac{\partial X(t)}{\partial t} = \dot{X} = \alpha\, Q(t) \tag{1}$$

In this case, it is assumed that every tourist will consume or degrade the quality of the environment during their visit. It is also assumed that the environment of the tourist destination will decompose through biodegrade process with the speed of β $(0 < \beta < 1)$, hence

$$\frac{\partial X(t)}{\partial t} = \dot{X} = \alpha\, Q(t) - \beta X(t) \tag{2}$$

For simplicity it is also assumed that all the resources which are the main assets of the tourist destination can be renewed, while non-renewable resources are not included in the model. This simplification allows us to create a mathematical model based on a balance between the number of visitors and the quality of the environment [11]. Another assumption for simplification is that the environmental quality of the renewable resource can be gradually improved by itself in proportion to the initial state of the renewable resource.

The model does not yet account for benefits of tourism. To introduce this aspect of the model, the model must incorporate the objectives of the tourism industry. Assume that the goal of the tourism industry is economic benefit. This mathematically can be expressed as

$$\Pi(t) = F(Q(t)) - G(X(t)) \tag{3}$$

That is, the economic benefit of tourism industry is the different between the function of visitors and the function of quality environment. From year to year, the goal of the industry is to maintain and, if possible, increase net benefit. This objective translates to a long-term goal of maximizing the sum of discounted profits over time. By taking into account for the time value of money, tourism industry maximizes the sum of discounted benefit. Formally, the model assumes that benefit are discounted when summing over time to account for the time value of money. This gives

$$\Pi(t) = \int_0^\infty [F(Q(t)) - G(X(t))]\, e^{-rt}\, dt \tag{4}$$

where F and G are utility functions.

3. OPTIMAL DYNAMIC SOLUTION

The The control variable in this problem is the number of visitors. The resulting number of visitors, in turn, influences environmental quality. Government or tourism industry seeks to manipulate the number of tourists entering the tourist area in order to preserve environmental quality. The model currently allows only a single control variable representing the total number of visitors. Formally, the problem can be expressed as the dynamic equations:

$$\max_Q \Pi(t) = \int_0^\infty [F(Q(t)) - G(X(t))]\, e^{-\rho t}\, dt \tag{5}$$

with respect to

$$\begin{aligned} &\dot{X} = \alpha\, Q(t) - \beta X(t) \\ &X(t) \geq 0, Q(t) \geq 0 \\ &X(0) \text{ given} \end{aligned} \tag{6}$$

The optimal solution for the above problem is found by setting into a version of Hamiltonian [1], that is

$$\mathcal{H} = \{F\big(Q(t)\big) - G\big(X(t)\big)\} + \lambda(t)[\alpha\, Q(t) - \beta X(t)]. \tag{7}$$

with necessary conditions

$$\frac{\partial \mathcal{H}}{\partial Q(t)} = F_Q(\cdot) + \alpha\lambda(t) = 0 \tag{8}$$

$$-\dot{\lambda}(t) + \rho\lambda(t) = \frac{\partial \mathcal{H}}{\partial X(t)} = G_X(\cdot) + \beta\lambda(t) \tag{9}$$

First derivative of (8) giving

$$\dot{\lambda}(t) = -F_{QQ}(\cdot)\dot{Q}(\cdot)/\alpha$$

and then combining (6) (7) and (8) giving

$$\dot{Q}(\cdot) = \frac{(\beta + \rho)F_Q(\cdot) - \alpha G_X(\cdot)}{F_{QQ}(\cdot)} \tag{10}$$

The subscripts above represent partial derivatives with respect to X and Q, λ is the co-state variable and subscripts denote partial derivatives with respect to the variable, ρ represents the discount rate, t the time period and e is the exponential operator. The steady-state solution is found by solving (10) when $\dot{Q} = 0$, giving

$$\alpha G_X(\cdot) = (\beta + \rho)F_Q(\cdot) \tag{11}$$

and solving (6) when $\dot{X} = 0$, giving

$$Q(t) = \beta X(t)/\alpha \tag{12}$$

The gradient of the curve can be found by carrying out total differential on (11) giving

$$\alpha G_{XX}(\cdot)dX = (\beta + \rho)F_{QQ}(\cdot)dQ \tag{13}$$

Hence, the gradient for isocline curve $\dot{Q} = 0$, giving

$$\frac{dQ}{dX} = \frac{\alpha G_{XX}(\cdot)}{(\beta + \rho)F_{QQ}(\cdot)}$$

Equilibrium solutions are characterized by zero changes over time in both environmental quality (X) and the number of tourist visitors (Q), that is solutions where both $\dot{X} = 0$ and $\dot{Q} = 0$ based on equations (6) and (10).

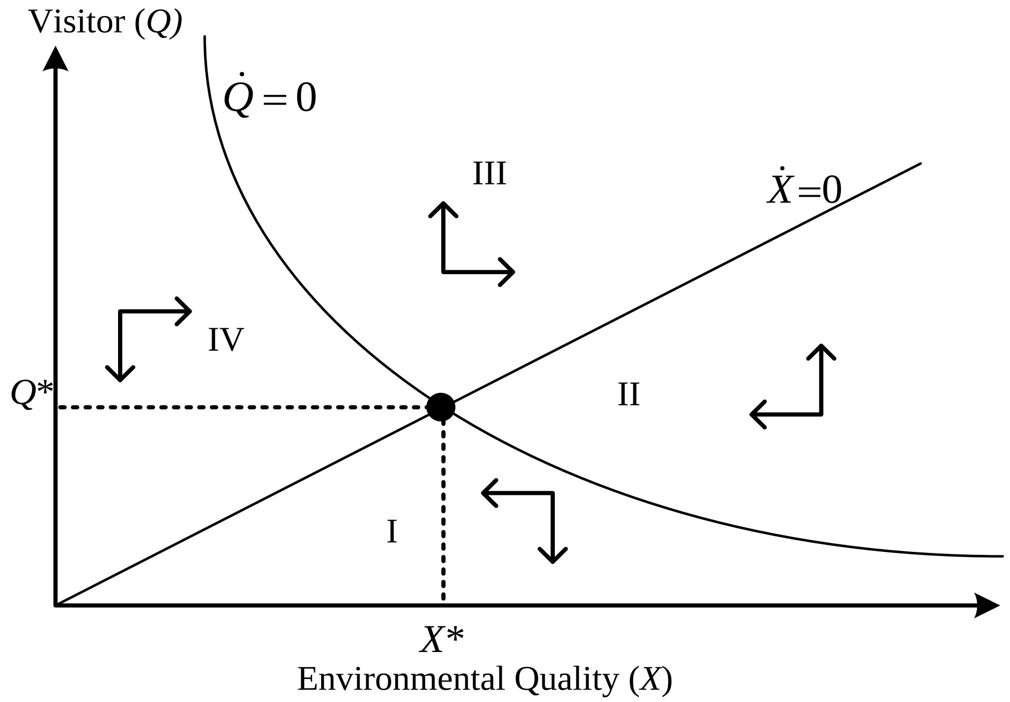

Figure 1. Phase diagram for equilibrium solution of sustainable environmental

Figure 1 shows a phase diagram illustrating the equilibrium solutions for a sustainable environment. The diagram consist of four areas called isosectors. For instance, Isosector II is located on the right of curve $\dot{X} = 0$ and above curve $\dot{Q} = 0$. When we draw an imaginary line vertically in this area, every point situated in this area shows that it is less Q needed to reach $\dot{X} = 0$. As a consequence, when the number of visitor is increased over time then the quality of the environment are minimized. In other words, maximizing benefits means having to increase the number of visitors over time. Because the number of visitors is a control in this system, the visitor curve is very important in environmental conservation. The points where the visitor curve ($\dot{Q}$) crosses the environmental sustainability curve $\dot{X} = 0$ as described above represent a steady-state, sustainable solutions for tourism.

4. CASE STUDY

Tanah Lot, Bali, is a tourist attraction that is most visited by foreign tourists. It is located in Tabanan, about 20 kilometres North West of Denpasar, the temple sits on a large offshore rock which has been shaped continuously over the years by the ocean tide.

Types of tourists who visit are divided into foreign and domestic tourists. The number of tourist visits in the last 5 years has fluctuated. With an average growth of only 0.01% with an average visit of 3,332,720 tourists. Table 2 below shows the number of tourists visited Tanah Lot during period of 2014-2018

Table 1. The number of tourists visited Tanah Lot during period of 2014-2020

Year	2014	2015	2016	2017	2018
Number	3,125,000	3,179,617	3,525,335	3,497,825	3,335,822

Source: Tanah Lot Tourism Management Agency (2019)

Fitting the data with the square root function gives $f(Q) = 260200Q^{0.5} + 2896400$ and assume that the growth of the environmental quality follow a function of $g(X) = -0.005X^2$, so the challenge facing the government is to find the point of sustainability the maximize the benefit over time. We also assume that the speed of biodegradation in the area is 1% ($\beta = 0.1$), the speed of visitor producing pollution 2% ($\alpha = 0.005$), and the discounted rate $\rho = 0$ This problem can be expressed as

$$\max_{Q} \int_0^\infty e^{-\rho t}(260200Q(t)^{0.5} + 2896400 \ - 0.001X(t)^2)dt \qquad (14)$$

with respect to

$$\dot{X} = 0.02Q(t) - 0.01X(t)$$
$$X(t) \geq 0, Q(t) \geq 0.$$

Using equations (11) and (12), we have a system of nonlinear equation with 2 variables.

$$-0.000015\left(\frac{13010.00}{\sqrt{Q}} - 0.00005X\right)Q^{1.5} = 0$$
$$0.005Q - 0.1X = 0$$

Solving the system gives the equilibrium solution of $(X^*, Q^*) = (1.5\ \times 10^5, 3.0 \times 10^6)$. Point (X^*, Q^*) is also a sustainable solution since it is the intersection between curves $\dot{Q} = 0$ and $\dot{X} = 0$.

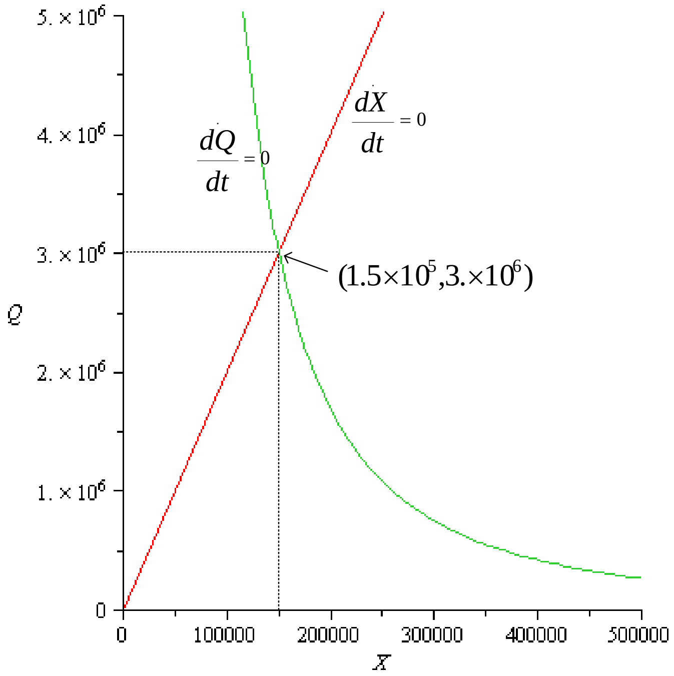

Figure 2. Optimal equilibrium solution for environmental sustainability

5 CONCLUSION

Although the model presented above is in a very simple form, it offers a relevant idea in the development of sustainable tourism. Application to empirical studies requires assumptions that must be met, such as the biodegradation in the area and the speed of visitor producing pollution. The advantage of this model is that it is still general form and of course it can be applied to various conditions and it is very flexible to modify. One thing to note about this model is that in order to achieve the dual goal that is maintaining visitor numbers and at the same time maintaining environmental quality, the optimal dynamic solution is likely to vary from destination to destination, except for the goal function of destinations are identical.

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