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Dominator Coloring Number of Regular Graphs

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Abstract

Graph coloring and domination in graphs have plenty of applications in computer, communication, biological and social network. The dominator coloring of the graph is obtained by merging the concept of graph node coloring and domination. It is defined as a proper coloring of nodes in which each node of the graph dominates all nodes of at least one color class. The smallest number of colors used for dominator coloring of nodes of a graph is called the dominator coloring number of the graph. In this paper the dominator coloring number of some regular graphs are obtained.

1. INTRODUCTION

A network or graph consists of a collection of nodes and edges. Graph theory techniques enable the analysis of complex structures in networks; hence have plenty of applications in domains such as biological network, computer network, communication network, social network, etc.

A subset D_S of the node set of graph G which is such that each node $V - D_S$ is adjacent to at least one member of D_S is called the dominating set [1]. The dominating set has application in computer and communication networks such as cell-phone network, mobile ad-hoc network, wireless sensor network etc. for routing and broadcasting the information between the nodes to the mobile devices. In biological network, the importance of dominating sets could be seen in the study of the human PPI network which holds details of the protein that operate and enable the biological processes within the cell [2, 3, 4].

A proper coloring of a graph G is a function $f: V \rightarrow Z_+$such that for $v, w \in V,\ f(v) \neq f(w)$ whenever v and w are adjacent nodes in G. Among all proper colorings of nodes of G, the smallest number of colors used in a node coloring is called the chromatic number, denoted by $\chi(G)$. Graph coloring is an optimization problem which has many applications in circuit testing, assignment of mobile frequency, map coloring, allocation of register etc. Graph coloring techniques have applications in biological networks; in particular to PPI networks to help improve the quality of initial protein complexes. Thus these findings support to improve the existing protein complex detection methods [5].

As domination and coloring have application in interconnection networks, the dominator coloring or χ_d coloring of a graph which is the amalgamation of node coloring and domination shall also be used in the study of behavior and properties of interconnection networks; in particular to mobile ad-hoc networks, vehicular adhoc network, air traffic network etc. The χ_d coloring of a graph is defined as a proper coloring of nodes in a graph such that every node of G dominates at least a color class. The χ_d coloring number denoted by $\chi_d(G)$ is smallest number of colors used in the χ_d coloring of G [7].

The notion of dominator coloring or χ_d coloring was introduced by Hedetniemi et al[6]. Gera et.al [7-9] also studied this concept and has found the dominator coloring of various graphs and its association with chromatic number and domination number. The dominator coloring number in P_4- free graphs was given by Chellali and Maffray [10]. Merouane [11, 12] gave some bounds for planar and star-free graphs, exact values for split graphs and a characterization of trees. Arumugam et.al [13, 14] showed various bounds on dominator coloring, investigated about the algorithmic aspects of dominator coloring and gave a bound on Mycielskian of graph. The dominator coloring or χ_d coloring of Prism graph, Quadrilateral Snake, Triangle Snake, Barbell graph, Degree splitting graph, m-splitting graph and m-shadow graph of path graph, Circular arc overlap graphs, closed Sun graph, closed Helm graph, generalized Flower Snark, Triangular belt, Alternate Triangular belt, Hajós graph, Trampoline graph, cyclic snake graphs , central graph, middle graph, total graph and line graphs of various graphs were discussed in various papers by various authors [15-27].

A graph in which every node has the same valency or degree is known as a regular graph. The study of regular graphs has applications in coding theory and computer science. Regular graphs are hard to be attacked since almost all nodes look the same i.e., they all have the same number of neighbors.

In this paper the dominator coloring number of some regular graphs (constructed using an algorithm) are obtained.

2. DOMINATOR COLORING NUMBER OF REGULAR GRAPHS

As a matter of fact, instead of trying to generate all k-regular graphs, we considered the k- regular graph constructed using the following algorithm [28]

Algorithm 2.1:

Condition: $0 < k < n$.

First all the nodes are placed in a circle.

If n is odd:

For i in range $(1, k)$:

If i is odd:

Make edge for each node x steps away (i)

If n is even:

If k is even:

Count Even numbers $= \frac{n}{2}$

For i in range (Count Even numbers):

Make edge for each node x steps away $(i + 1)$

If k is odd:

Count odd numbers $= \frac{n-1}{2} + 1$

For i in range (Count odd numbers):

Make edge for each node x steps away $\left(\frac{n}{2}\right) - i$

i.e., a regular graph is constructed as explained below

Case 1: n is odd and k must be even.

All the nodes are placed in a circle. Then the nodes are joined x positions away where x is all odd numbers between 1 to k.

Example:

$n = 9$	$k = 2$	Produce edge for each node x steps away 1
$n = 9$	$k = 4$	Produce edge for each node x steps away 1 Produce edge for each node x steps away 3
$n = 9$	$k = 6$	Produce edge for each node x steps away 1 Produce edge for each node x steps away 3 Produce edge for each node x steps away 5
$n = 9$	$k = 8$	Produce edge for each node x steps away 1 Produce edge for each node x steps away 3 Produce edge for each node x steps away 5 Produce edge for each node x steps away 7

Case 2: n is even and k is even

All the nodes are placed in a circle. Then the nodes are joined x positions away where x is a range of numbers from 1 to $\frac{n}{2}$ and the range of these numbers is limited by the number of even numbers from 1 to k including k.

Example:

$n = 10$	$k = 2$	Produce edge for each node x steps away 1
$n = 10$	$k = 4$	Produce edge for each node x steps away 1 Produce edge for each node x steps away 2
$n = 10$	$k = 6$	Produce edge for each node x steps away 1 Produce edge for each node x steps away 2 Produce edge for each node x steps away 3
$n = 10$	$k = 8$	Produce edge for each node x steps away 1 Produce edge for each node x steps away 2 Produce edge for each node x steps away 3 Produce edge for each node x steps away 4

Case 3: n is even and k is odd

All the nodes are placed in a circle. Then the nodes are joined x positions away where x is a range of numbers from $\frac{n}{2}$ to 1 and the range of these numbers is limited by the number of odd numbers from 1 to k including k.

Example:

$n = 10$	$k = 1$	Produce edge for each node x steps away 5
$n = 10$	$k = 3$	Produce edge for each node x steps away 5 Produce edge for each node x steps away 4
$n = 10$	$k = 5$	Produce edge for each node x steps away 5 Produce edge for each node x steps away 4 Produce edge for each node x steps away 3
$n = 10$	$k = 7$	Produce edge for each node x steps away 5 Produce edge for each node x steps away 4 Produce edge for each node x steps away 3 Produce edge for each node x steps away 2
$n = 10$	$k = 9$	Produce edge for each node x steps away 5 Produce edge for each node x steps away 4 Produce edge for each node x steps away 3 Produce edge for each node x steps away 2 Produce edge for each node x steps away 1

Proposition 2.2: If C_n is a cycle on n nodes where $n \geq 3$ then its dominator coloring number is given by [8] $\chi_d(C_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & if\ n = 4 \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n = 5 \\ \left\lceil \frac{n}{3} \right\rceil + 2 & otherwise \end{cases}$

Theorem 2.3: If G is a connected 2 regular graph on n nodes where $n \geq 3$ then its dominator coloring number is given by $\chi_d(G) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & if\ n = 4 \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n = 5. \\ \left\lceil \frac{n}{3} \right\rceil + 2 & otherwise \end{cases}$

Proof:

Given G is a connected 2 regular graph on n nodes. Since G is connected and degree of every node is exactly 2, there exists an Eulerian cycle. Thus all connected 2 regular graphs with n nodes are isomorphic to n cycle. Hence by proposition 2.2 we have $\chi_d(G) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & if\ n = 4 \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n = 5. \\ \left\lceil \frac{n}{3} \right\rceil + 2 & otherwise \end{cases}$

Theorem 2.4: If G is a disconnected 2 regular graph on n nodes with k components then its dominator coloring number $\chi_d(G)$ satisfies the relation $\max(\chi_d(G_i)) + k - 1 \leq \chi_d(G) \leq \sum_{i=1}^{k} \chi_d(G_i)$.

Proof:

Let G be a disconnected 2 regular graph on n nodes with k components $G_1, G_2, \ldots G_k$. Let $\chi_d(G_i)$ be the dominator coloring number of each of the k components $G_1, G_2, \ldots G_k$ of G. Since each component G_i of G is connected and 2 regular, it must contain some cycle $C_k, k \geq 3$. In other words a disconnected 2 regular graph G on n nodes is isomorphic to disjoint union of cycles. Then

$$\chi_d(G) \leq \sum_{i=1}^{k} \chi_d(G_i) \qquad (1)$$

Let G_i be the component of G with maximum dominator coloring number $\chi_d(G_i)$. For every $j \neq i$, each G_j needs atleast a new color to dominate the entire color class of G_j. Hence the $(k-1)$ components of G need atleast $(k-1)$ colors to dominate its entire color class. Therefore

$$\chi_d(G) \geq \max(\chi_d(G_i)) + k - 1 \qquad (2)$$

Combining equations (1) and (2), we have the result $\max(\chi_d(G_i)) + k - 1 \leq \chi_d(G) \leq \sum_{i=1}^{k} \chi_d(G_i)$.

Theorem 2.5: For any $n \in \mathbb{Z}_+$, n is even, $n \geq 4$, if G is a $n-2$ regular graph on n nodes, then its dominator coloring number is $\chi_d = \frac{n}{2}$

Proof:

Let $\mathrm{V(G)} = \{v_i / 1 \leq i \leq n\}$ be the node set of the regular graph. The edges of the $n-2$ regular graph are constructed using algorithm 2.1.

For dominator coloring, the nodes are assigned colors as follows:

The nodes $v_i$and $v_{i+\frac{n}{2}}$ when $1 \leq i \leq \frac{n}{2}$, are given color i.

Then the nodes $v_i$and $v_{i+\frac{n}{2}}$ when $1 \leq i \leq \frac{n}{2} - 1$, dominate color class $i+1$. The nodes v_n and $v_{\frac{n}{2}}$ dominate color class 1.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of $n-2$ regular graph on n(even) nodes is given by $\chi_d = \frac{n}{2}$.

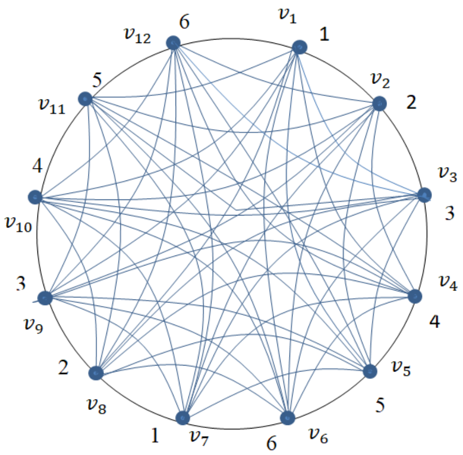

Figure 1: Dominator coloring number of 10-regular graph on 12 vertices is $\chi_d(G) = 6$

(Here $n = 12, n - 2 = 10$ and $\frac{n}{2} = 6$)

Theorem 2.6: For any $n \in \mathbb{Z}_+$, n is even, $n \geq 4$, if G is a $\frac{n}{2}$ regular graph on n nodes then its dominator coloring number is $\chi_d = \begin{cases} \frac{n}{2} & when\, n = 6\, (or)\ n(mod\ 4) \equiv 0 \\ 4 & when\ n \neq 6\ and\ n(mod\ 4) \equiv 2 \end{cases}$

Proof:

Let $V(G) = \{v_i / 1 \leq i \leq n\}$ be the node set of the regular graph. The edges of the $\frac{n}{2}$ regular graph are constructed using algorithm 2.1.

For dominator coloring, the nodes are assigned colors as follows:

Let $q = \left\lceil \frac{n}{4} \right\rceil$.

Case 1: When $n = 6$

The nodes v_i, v_{i+q} and v_{i+2q} when $1 \leq i \leq q$, are given colors $\left\lceil \frac{i}{q} \right\rceil, \left\lceil \frac{i}{q} \right\rceil + 1$ and $\left\lceil \frac{i}{q} \right\rceil + 2$ respectively.

Then the nodes v_1, v_n dominate color class 2, the nodes v_2, v_3 dominate color class 3 and the nodes v_4, v_5 dominate color class 1 respectively.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and $\chi_d = \frac{n}{2}$ when $n = 6$

Case 2: When $n(mod\ 4) \equiv 0$

The nodes v_i and $v_{i+\frac{n}{2}}$ when $1 \leq i \leq \frac{n}{2}$, are given color i.

The nodes v_i and $v_{i+\frac{n}{2}}$ when $1 \leq i \leq \frac{n}{4}$, dominate color class $i + \frac{n}{4}$. And the nodes v_i and $v_{i+\frac{n}{2}}$ when $\frac{n}{4} + 1 \leq i \leq \frac{n}{2}$, dominate color class $i - \frac{n}{4}$.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and $\chi_d = \frac{n}{2}$ when $n(mod\ 4) \equiv 0$.

Case 3: When $n(mod\ 4) \equiv 2\ and\ n \neq 6$

The nodes v_i, v_{i+q} and v_{i+2q} when $1 \leq i \leq q$, are given color $\left\lceil \frac{i}{q} \right\rceil$, $\left\lceil \frac{i}{q} \right\rceil + 1$ and $\left\lceil \frac{i}{q} \right\rceil + 2$ respectively. The nodes v_{n+1-i} when $1 \leq i \leq n - 3q$, are given color 4.

The node v_1 dominates color class 2. The nodes v_{i+1} and v_{i+q+1} when $1 \leq i \leq q$, dominate color class $\left\lceil \frac{i}{q} \right\rceil + 2$ and $\left\lceil \frac{i}{q} \right\rceil + 3$ respectively. The nodes v_{n+1-i} when $1 \leq i \leq q - 1$, dominate color class 2. And the nodes v_{i+2q+1} when $1 \leq i \leq q - 2$, dominate color class 1.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and $\chi_d = 4$ when $n(mod\ 4) \equiv 2$ and $n \neq 6$.

Thus the dominator coloring number of $\frac{n}{2}$ regular graph on n(even) nodes and $n \geq 4$ is given by

$$\chi_d = \begin{cases} \frac{n}{2} & if\ n = 6\ (or)\ n(mod\ 4) \equiv 0 \\ 4 & if\ n(mod\ 4) \equiv 2\ and\ n \neq 6 \end{cases}.$$

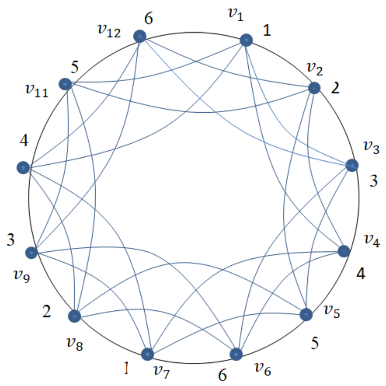

Figure 2(a)

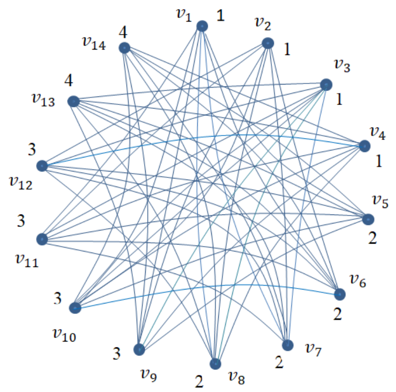

Figure 2(b)

Figure 2(a): Dominator coloring number of 6-regular graph on 12 vertices is $\chi_d(G) = 6$

$(Here\ n = 12\ and\ n(mod\ 4) \equiv 0\ )$

Figure 2(b): Dominator coloring number of 7-regular graph on 14 vertices is $\chi_d(G) = 4$

$(Here\ n = 14\ and\ \ n(mod\ 4) \equiv 2\ )$

Theorem 2.7: For any $n \in \mathbb{Z}_+, n$ is odd and $n \geq 5$, if G is a $n-3$ regular graph on n nodes, then its dominator coloring number is $\chi_d = \left\lceil \frac{n}{2} \right\rceil$.

Proof:

Let $V(G) = \{v_i / 1 \leq i \leq n\}$ be the node set of the regular graph. The edges of the $n-3$ regular graph are constructed using algorithm 2.1.

For dominator coloring, the nodes are assigned colors as follows:

Let $q = \left\lfloor \frac{n}{4} \right\rfloor$.

Case 1: When $n > 5\ and\ n(mod\ 4) \equiv 3$

The nodes v_{4i-3}, v_{4i-1} when $1 \leq i \leq q$, are given color $2i - 1$ respectively. The nodes v_{4i-2}, v_{4i} when $1 \leq i \leq q$, are given color $2i$ respectively. The nodes v_{n-2}, v_n are given color $\left\lceil \frac{n}{2} \right\rceil - 1$ and the node v_{n-1} is given color $\left\lceil \frac{n}{2} \right\rceil$.

The nodes v_{4i-3}, v_{4i-1} when $1 \leq i \leq q$, dominate color class $2i$. The nodes v_{4i-2}, v_{4i} when $1 \leq i \leq q$, dominate color class $2i-1$ respectively. The nodes v_{n-2}, v_{n-1}, v_n dominate color class $\left\lceil \frac{n}{2} \right\rceil$.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and $\chi_d = \left\lceil \frac{n}{2} \right\rceil$.

Case 2: When $n > 5\ and\ n(mod\ 4) \equiv 1$

The nodes v_{4i-3}, v_{4i-1} when $1 \leq i \leq q$, are given color $2i - 1$. The nodes v_{4i-2}, v_{4i} when $1 \leq i \leq q$, are given color $2i$ respectively. The node v_n is given color $\left\lceil \frac{n}{2} \right\rceil$.

The nodes v_{4i-3}, v_{4i-1} when $1 \leq i \leq q$, dominate color class $2i$ and the nodes v_{4i-2}, v_{4i} when $1 \leq i \leq q$, dominate color class $2i - 1$ respectively. The node v_n dominates color class $\left\lceil \frac{n}{2} \right\rceil$.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and $\chi_d = \left\lceil \frac{n}{2} \right\rceil$.

Case 3: When $n = 5$

The nodes v_{4i-3}, v_{4i-1} when $1 \leq i \leq q$, are given color $2i - 1$. The nodes v_{4i-2}, v_{4i} when $1 \leq i \leq q$, are given color $2i$ respectively. The node v_n is given color $\left\lceil \frac{n}{2} \right\rceil$.

The nodes v_{i+1} when $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$, dominate color class i. And the nodes v_1 and v_n dominate color class $\left\lceil \frac{n}{2} \right\rceil$.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and and $\chi_d = \left\lceil \frac{n}{2} \right\rceil$.

Thus the dominator coloring number of $n - 3$ regular graph on n(odd) nodes and $n \geq 5$, is given by $\chi_d = \left\lceil \frac{n}{2} \right\rceil$.

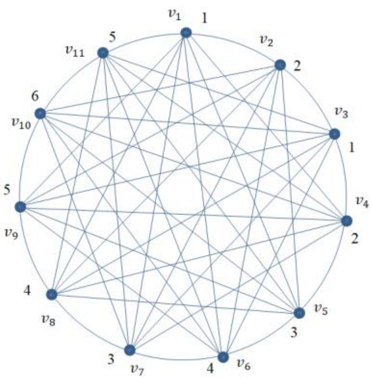

Figure 3(a)

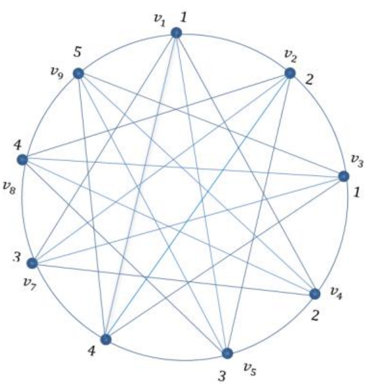

Figure 3(b)

Figure 3(a): Dominator coloring number of 8-regular graph on 11 vertices is $\chi_d(G) = 6$

$(Here\ n = 11, n(mod\ 4) \equiv 3, n - 3 = 8\ and\ \chi_d(G) = \left\lceil \frac{n}{2} \right\rceil = 6)$

Figure 3(b): Dominator coloring number of 6-regular graph on 9 vertices is $\chi_d(G) = 5$

$(Here\ n = 9, n(mod\ 4) \equiv 1, n - 3 = 6\ and\ \chi_d(G) = \left\lceil \frac{n}{2} \right\rceil = 5)$

Theorem 2.8: For any $n \in \mathbb{Z}_+$, $n > 5$ and $n(mod\ 4) \equiv 1$, if G is a $\left\lfloor \frac{n}{2} \right\rfloor$ regular graph on n nodes, then its dominator coloring number is $\chi_d = 4$.

Proof:

Let $V(G) = \{v_i / 1 \le i \le n\}$ be the node set of the regular graph. The edges of the $\left\lfloor \frac{n}{2} \right\rfloor$regular graph are constructed using algorithm 2.1.

For dominator coloring, the nodes are assigned colors as follows:

Let $q = \left\lfloor \frac{n}{2} \right\rfloor$. The nodes v_i when $1 \le i \le q$, are painted with color 1 when i is odd and color 2 when i is even. The node $v_{\left\lceil \frac{n}{2} \right\rceil}$ is given color 1. The nodes $v_{i+\left\lceil \frac{n}{2} \right\rceil}$ when $1 \le i \le q$, are painted with color 3 when i is odd and color 4 when i is even.

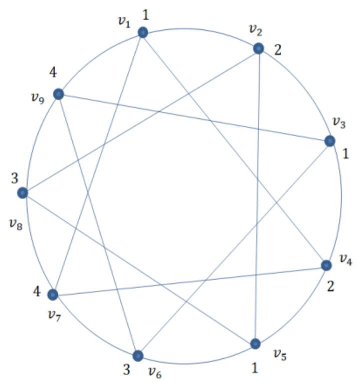

Figure 4: Dominator coloring number of 4 - regular graph on 9 vertices is $\chi_d(G) = 4$.

$$\left(Here\ n = 9,\ \ n(mod\ 4) \equiv 1\ and\ \left\lfloor \frac{n}{2} \right\rfloor = 4 \right)$$

The nodes v_i when $1 \le i \le q$, dominate color class 2 when i is odd and color class 1 when i is even. The node $v_{\left\lceil \frac{n}{2} \right\rceil}$ dominates color class 2. The nodes $v_{i+\left\lceil \frac{n}{2} \right\rceil}$ when $1 \le i \le q$, dominate color class 4 when i is odd and color 3 when i is even.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of $\left\lfloor \frac{n}{2} \right\rfloor$ regular graph on n(odd) nodes, $n > 5$ and $n(mod\ 4) \equiv 1$ is given by $\chi_d = 4$.

Theorem 2.9: For any $n \in \mathbb{Z}_+$, $n \ge 5$ and $n(mod\ 4) \equiv 3$, if G is a $\left\lceil \frac{n}{2} \right\rceil$ regular graph on n nodes, then its dominator coloring number is $\chi_d = 4$.

Proof:

Let $V(G) = \{v_i / 1 \le i \le n\}$ be the node set of the regular graph. The edges of the

$\left\lceil\frac{n}{2}\right\rceil$ regular graph are constructed using algorithm 2.1.

For dominator coloring, the nodes are assigned colors as follows:

Let $q = \left\lceil\frac{n}{2}\right\rceil$. The nodes v_i when $1 \leq i \leq q$, are painted with color 1 when i is odd and color 2 when i is even. The nodes v_{i+q} when $1 \leq i \leq q-1$, are painted with color 3 when i is odd and color 4 when i is even.

The nodes v_i when $1 \leq i \leq q$, dominate color class 2 when i is odd and dominate color class 1 when i is even. The nodes v_{i+q} when $1 \leq i \leq q-1$, dominate color class 4 when i is odd and dominate color class 3 when i is even.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of $\left\lceil\frac{n}{2}\right\rceil$ regular graph on n(odd) nodes, $n \geq 5$ and $n(mod\ 4) \equiv 3$ is given by $\chi_d = 4$.

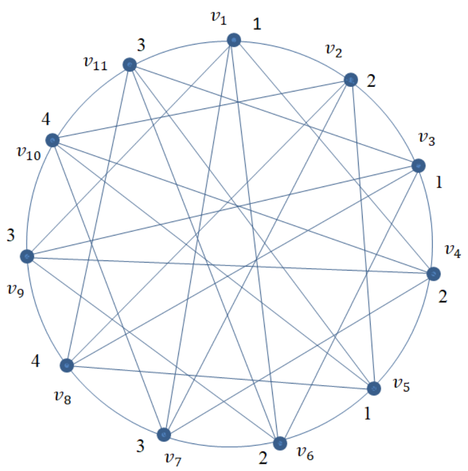

Figure 5: Dominator coloring number of 6 - regular graph on 11vertices is $\chi_d(G) = 4$.

$$\left(Here\ n = 11,\ n(mod\ 4) \equiv 3\ and\ \left\lceil\frac{n}{2}\right\rceil = 6\right)$$

Theorem 2.10: For any $n \in \mathbb{Z}_+$, $n \geq 5$ and $n(mod\ 4) \equiv 1$, if G is a $\left\lceil\frac{n}{2}\right\rceil + 1$ regular graph on n nodes, then its dominator coloring number is $\chi_d = 5$.

Proof:

Let $\mathrm{V(G)} = \{v_i / 1 \leq i \leq n\}$ be the node set of the regular graph. The edges of the $\left\lceil\frac{n}{2}\right\rceil + 1$ regular graph are constructed using algorithm 2.1.

For dominator coloring, the nodes are assigned colors as follows:

Let $q = \left\lfloor \frac{n}{4} \right\rfloor$. The nodes v_i when $1 \leq i \leq 2q$, are painted with color 1 when i is odd and color 2 when i is even and the nodes $v_{i+\left\lfloor \frac{n}{2} \right\rfloor}$ when $1 \leq i \leq 2q$, are painted with color 3 when i is odd and color 4 when i is even. The node v_n is given color 5.

The nodes v_i when $1 \leq i \leq 2q$, dominate color class 2 when i is odd and dominate color class 1 when i is even. The nodes $v_{i+\left\lfloor \frac{n}{2} \right\rfloor}$ when $1 \leq i \leq 2q$, dominate color class 4 when i is odd and color class 3 when i is even. The node v_n dominates color class 5.

Every adjacent node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of $\left\lceil \frac{n}{2} \right\rceil + 1$ regular graph on n(odd) nodes, $n > 3$ and $n(mod\ 4) \equiv 1$ is given by $\chi_d = 5$.

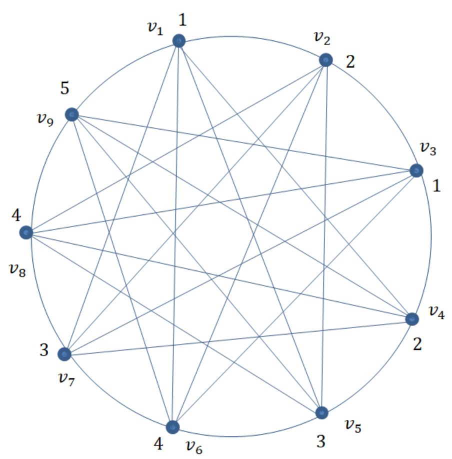

Figure 6: Dominator coloring number of 6 - regular graph on 9 vertices is $\chi_d(G) = 5$.

$$\left(Here\ n = 9,\ \ n(mod\ 4) \equiv 1\ and\ \left\lceil \frac{n}{2} \right\rceil + 1 = 6 \right)$$

3. CONCLUSION

In the field of graph theory, node coloring and domination have plenty of application in computer and communication network such as cell-phone network, mobile ad-hoc network, vehicular adhoc network, wireless sensor network etc. for routing and broadcasting the information between the nodes to the mobile devices. So the amalgamation of domination and coloring of graphs called the dominator coloring shall also be used in the study of interconnection networks; in particular to mobile ad-hoc network and vehicular adhoc network. In this paper, the dominator coloring number of some regular graphs are obtained.

REFERENCES

[1] T.W. Haynes, S.T. Hedetniemi, Peter Slater, Fundamentals of Domination in graphs, Marcel Dekker, New York, (1998).

[2] Jose C. Nacher, Tatsuya Akutsu, Minimum dominating set-based methods for analyzing biological networks, Science Direct, 102(1), (2016), 57–63.

[3] Haiying Wang, Huiru Zheng , Fiona Browne, Chaoyang Wang, Minimum dominating sets in cell cycle specific protein interaction networks, IEEE XPlore, (2015).

[4] T. Milenkovic, V. Memisevic, A. Bonato, and Przˇulj, Dominating biological networks, PLoS One, 6(8), (2011), e23016.

[5] S. Khor, *Application of graph coloring to biological networks,* IET Systems Biology, 4(3), **(**2010), 185- 192.

[6] Hedetniemi, S.M., Hedetniemi, S.T., Mcrae, A.A., Blour, J.R.S.: Dominator coloring of graphs. (2006, preprint)

[7] R. M. Gera, S Horton, C. Rasmussen, Dominator Colorings and Safe Clique Partitions, Congressus Numerantium 181, (2006), 19-32.

[8] R. M. Gera, On dominator coloring in graphs, Graph Theory Notes N.Y. LII (2007), 52, 25–30.

[9] R. M. Gera, On the dominator colorings in Bipartite graphs, IEEE Computer Society (2007), 1, 947-952.

[10] Mustapha Chellali, Frederric Maffray, Dominator Colorings in some classes of Graphs, Graphs and Combinatorics, (2012), 28(1), 97–107

[11] Merouane, Houcine Boumediene, et al., Dominated Colorings of Graphs, Graphs and Combinatorics, (2014), 31(3) 713-727

[12] Boumediene Merouane, Houcine & Chellali, Mustapha, An algorithm for the dominator chromatic number of a tree, Journal of Combinatorial Optimization, (2013), 30(1), 27-33.

[13] Arumugam S, Chandrasekar K Raja, et al., Algorithmic aspects of dominator colorings in graphs, *Lecture Notes in Comput.Sci.* 7056 (2011) 19–30

[14] Arumugam S, Jay Bagga and K Raja Chandrasekar, On dominator colorings in graphs, Proc. Indian Acad. Sci. (Math. Sci.), (2012), 122(4), 561–571.

[15] T. Manjula and R. Rajeswari, Dominator Coloring of Prism graph, Applied Mathematical Sciences, (2015), 9(38), 1889–1894.

[16] T. Manjula and R. Rajeswari, Dominator Coloring of Quadrilateral snake, Triangle Snake graph and Barbell graph, IEEE XPLORE, (2016), 115 - 119.

[17] Samir K. Vaidya and Minal S. Shukla, Dominator Coloring of Some Degree Splitting Graphs, International Journal of Mathematics And Scientific Computing , (2015), 5(2),

[18] T. Manjula and R. Rajeswari, Dominator Chromatic number of m-splitting graph and m-shadow graph of path graph, Int. J. Biomedical Engineering and Technology, (2018), 27(1/2), 100-113.

[19] A Sudhakaraiah V Ragavalakshmi, Dominator Chromatic Number of Circular-Arc Overlap Graphs, Mathematical Theory and Modelling, (2012), 2(9), 1-6.

[20] T. Manjula and R. Rajeswari, Dominator Coloring of certain graphs, International Journal of Engineering and Advanced Technology, (2018), 8(2s), 262 – 268.

[21] T. Manjula and R. Rajeswari, Dominator Coloring number of some class of graphs, International Journal of Engineering and Advanced Technology, (2018), 8(2s), 269 – 274.

[22] Manjula T and Rajeswari R, Dominator coloring of Hajós and Trampoline graph, Journal of Physics: Conference Series (2021), 1770(1), 012080.

[23] Manjula T and Rajeswari R, An analytical modeling on χ_d coloring of certain graphs for applications of air traffic and air scheduling management (2021), DOI: 10.1108/AEAT-04-2021-0104.

[24] Manjula T and Rajeswari R, Dominator coloring of Central and Middle graph of Closed Helm graph, Test Engineering Mathematics, (2020), 82, 6228-6232.

[25] Manjula T and Rajeswari R, Dominator chromatic number of certain graph, Journal of Combinatorial Mathematics and Combinatorial Computing, (2020), 112, 43-52.

[26] K. Kavitha, N. G. David, Dominator Coloring of Central Graphs, International Journal of Computer Applications, (2012), 51(12), 11-14.

[27] K. Kavitha, N. G. David, Dominator Chromatic number of Middle and Total graphs, International Journal of Computer Applications, (2012), 49(20), 42-46.

[28] https://math.stackexchange.com/a/1911863 (2016)