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An Approximate Solution for Fuzzy Volterra Integro-differential Equations using Variational Iteration Technique

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Abstract

In this research work, we utilize the concept of Fuzzy Volterra Integro-Differential equations and also the Variational iteration Method. The Variational Iteration method(VIM) has been implemented to examine the critical types of illustrates. The integro-differential equation is of the general form :

$$p^j(x) = f(x) + C \int_a^b k(x,r)p(r)dr$$

where a and b be the limits in the integration, C be the constant and $k(x,r)$ be the kernel with two variables x and r. The approach obtained by the VIM are nearly convergent to the exact solution. The final result obtained from the variational iteration Method are very convenient, accurate and really effective. Based on the fuzzy concept with the VIM, we represent an approximate exact solution in the form of convergent series. The advantage of this overture is an algorithmic structure of the VIM are also has been set. To demonstrate the method dormant, we have proposed problems which have been compared with the exact solutions. A Numerical example has been clarified by this concept.

Keywords: Fuzzy differential equations, Volterra Integro-Differential Equations, Variational Iteration Method

AMS subject classifications: 03E72, 34A07, 94D05

1. INTRODUCTION

In this paper work, we introduced the Variational iteration Method for calculating the iterations of SFVIDEs. In 1964 July, the concept of fuzzy set was introduced.

The proposal of the grade for membership is the image that evolved the support of fuzzy set theory[20]. The fuzzy sets, linguistic variables and possibility distribution are the first three fundamental concepts in the fuzzy logic for all subareas. The fourth conception is very efficient because it is the foundation for maximum industrial applications of fuzzy logic. The word fuzzy logic is nearly new in two disimilar perception. In a limited sensation, fuzzy logic designate to a logical system that helps the hypothesis of classical two-valued logic under uncertainty condition. In a wide sense, the fuzzy logic mean to all technologies and theories which take on the fuzzy set, which are with unsharp boundaries.

The fuzzy system[17][16] is an efficient part of fuzzy calculus which is applying to the forecasts for future events which are unknown. In plenty of real life problems [10], the fuzzy set theory in mathematical models has also been applied as a powerful tool for processing vague, modeling under uncertainty conditions and subjective information. The fuzzy set concept is mainly about the decision-making process. Nowadays, the fuzzy set is applied for advanced information technology process in industrial process. Numerical solutions are established by Friedman and Kandel [4].

Various types of differential equations, integral equations and integro differential equations has been applied in the molding of various physical systems. An Integro-Differential are the equations in which the function with unknown under an integral sign appears and an equation consisting of both integrals and differentiation appears respectively. The vision of theoretical point on the fuzzy differential equations are very necessary topic because of these necessity condition many of the authors are excited in the fuzzy differential equations (FDEs) work. For example, Kavela introduced FDEs for Cauchy problem [12] in 1986. The main intention for the study of the system of Fuzzy Volterra Integro-differential equations is to develop equations which helps to expand the methodology of formulation and to discover the solutions for the problems that are tangled. The study of the system of fuzzy Volterra Integro-differential equations is bearable for the analysis method by traditional techniques.

The solution for the system of Fuzzy Volterra Integro-differential equations is one of the main aims in copious field of engineering and also other applied sciences[8][2]. Lately, from the finest of our mastery many of the researches solve the approximate solutions for the system of Fuzzy Volterra Integro-differential equations(SFVIDEs). Newly, the variational Iteration Method(VIM) application can be adopted in linear and nonlinear problems which has been staunch by the scientist. J.H. He, who is the scientist approached the Variational Iteration method which helps to obtain our results nearly to exact solutions. The main interest for this article is to existing extra implementation of the familiar Variational iteration Method (VIM) which furnish approximate solutions for SFVIDEs. In this analysis paper, we subject the system of fuzzy Volterra Integro-Differential equations with Variational Iteration method(VIM) and proposed a numerical scheme to obtain successive approximation uncertainity aspect as follows:

$x_1'(s) = f_1(s) + \int_a^s k_1(s,r)g_1(r, x_1(r), x_2(r), ..., x_n(r))dr, s \geq a,$

$x_2'(s) = f_2(s) + \int_a^s k_2(s,r)g_2(r, x_1(r), x_2(r), ..., x_n(r))dr, s \geq a,$.

....

$$x_n'(s) = f_n(s) + \int_a^s k_n(s,r)g_n(\eta, x_1(r), x_2(r), ..., x_n(r))dr, s \geq a \quad (1)$$

which has the following fuzzy initial conditions as:

$$[\underline{x_1}(a), \overline{x_1}(a)] = \beta_1, [\underline{x_2}(a), \overline{x_2}(a)] = \beta_2, ..., [\underline{x_n}(a), \overline{x_n}(a)] = \beta_n \quad (2)$$

Here the kernel functions of the above equations be $k_1(s,r), k_2(s,r), ..., k_n(s,r)$, the continuous fuzzy valued and linear or non linear continuous real valued functions in the equations are $f_1, f_2, ..., f_n$ and $g_1, g_2, ..., g_n$ respectively. Thus $x_1, x_2, .., x_n$ and $\beta_1, \beta_2, ..., \beta_n$ are the unknown fuzzy valued functions and fuzzy numbers respectively. The nonlinear system of fuzzy Volterra Integro-Differential equations (FVIDE) may naturally measure the efficacy of this VIM for finding the approximate solutions.

This research paper has been arranged in the following basis. In section 2, there are some basic definitions and necessary concepts about fuzzy concepts and theorem about fuzzy logic has also been introduced. In section 3, the FVIDE is applied to the proposed method for finding the nearly equal results. In section 4, the VIM has been clarified. In VIM, we examine the correction functional and the restricted variation for finding the successive approximations for the system of equations and also we utilize the concept of VIM with FVIDE. For this concept we have the system of equations with fuzzy initial conditions and we apply our proposed VIM to get our exact solutions and we have a theorem which is based on the clear vision of this concept. Finally, in section 5, we have the numerical example for proving the accuracy of this prposed VIM. With some of the well known conclusions on section 6, the paper work has been ended.

2. PRELIMINARIES

In this section, we represented the most basic ideas, definitions and worthful results, those are worned throughout this research paper.

2.1. Definition

Let the universal set be X, each individual allocates a merit of either one or zero for the characteristic function of a crisp set. A fuzzy number $a : X \rightarrow [0, 1]$ which is the regular and real number such that it need not refer to any one single value but rather to a connected set of possible values and each value has its own weight between 0 and 1 which weight is said to be the membership function. The most efficient property of fuzzy sets is their convexity and real line normalized fuzzy set[18].

For all $0 \leq \alpha \leq 1$, that the α-level set $[a]_\alpha$ is a non-empty compact interval. Here the family of all fuzzy numbers are denoted by R. In [15] if $a : I \to [0, 1]$ with α-cut representation $[\underline{a}(\alpha), \overline{a}(\alpha)]$, then the set a satisfies the following conditions:

- $a + b = [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$.
- $a - b = [\underline{a} - \underline{b}, \overline{a} - \overline{b}]$.
- $\underline{a} : [0, 1] \to I$ is a bounded increasing function.
- $\overline{a} : [0, 1] \to I$ is a bounded decreasing function.

2.2. Definition

Let the non-empty set be X and and a fuzzy set $\overline{A} \in X$ is characterized by membership function $\mu_{\overline{A}} : X \to [0, 1]$ and the degree of membership function in the element x can be denoted as $\mu_{\overline{A}(x)}$ in fuzzy set $\overline{A}$ for every $x \in X$. Thus $\overline{A}$ is governed by the set of tuples[5].

$$\overline{A} = (x, \mu_{\overline{A}})|x \in X$$

2.3. Definition

For arbitrary fuzzy numbers $a = [\underline{a}, \overline{a}]$ and $b = [\underline{b}, \overline{b}]$, the quantity $U : R \times R \to [0, \infty)$ such that $U(a, b) = sup_{\alpha\in[0,1]} max\{|\underline{a} - \underline{b}|, |\overline{a} - \overline{b}|\}$ is the Hausdorff distance between a and b[17].

2.4. Definition

A fuzzy function $f : R \to E$ is said to be continuous if for arbitrary fixed $x_0 \in R$ and $\epsilon > 0$ there exists $\delta > 0$ such that

$|x - x_0| < \delta$, then $U(f(x), f(x_0)) < \epsilon$

2.5. Definition

Let $u : [x, y] \to R$ and $x_0 \in (x, y)$. Then we say that u is strongly generalised differentiable at x_0, if there is some element $u'(x_0) \in R$ such that for all $h > 0$ sufficiently close to 0, then there exists $u(x_0 + h)u(x_0), u(x_0)u(x_0 - h)$ and also the limits exists [2].

$$lim_{h\to 0^+} \frac{u(x_0+h)-u(x_0)}{h} = lim_{h\to 0^+} \frac{u(x_0)-u(x_0-h)}{h} = u'(x_0)$$

Thus the $u'(x_0)$ is denoted by $D'_1 u(x_0)$ and u is said to be (1)-differentiable at x_0 for all $h < 0$ which is abundantly close to 0, then there exists $u(x_0 + h) - u(x_0), u(x_0) - u(x_0 - h)$ and the limits.

$$lim_{h\to 0^-} \frac{u(x_0+h)-u(x_0)}{h} = lim_{h\to 0^-} \frac{u(x_0)-u(x_0-h)}{h} = u'(x_0)$$

In this above equation, the $u'(x_0)$ is denoted as $D_2'u(x_0)$ and is said to be (2)-differentiable at x_0

Theorem :Assume that $u : [x, y] \to R$ is a fuzzy -valued function, where $[u(t)]_\alpha = [\underline{u}(x_0), \overline{u}(x_0)]$ for each $\alpha \in [0, 1]$

(i) If x is (1)-differentiable, then $\underline{u}$ and $\overline{u}$ are differentiable functions and $[D_1^1 u(x_0)]_\alpha = [\underline{u'}(x_0), \overline{u'}(x_0)]$

(ii) If x is (2)-differentiable, then $\underline{u}$ and $\overline{u}$ are differentiable functions and $[D_2^1 u(x_0)]_\alpha = [\overline{u'}(x_0), \underline{u'}(x_0)]$

2.6. Definition

Given a fuzzy set $\overline{A}$ defined on the universal set which is X and a number $\alpha \in [0, 1]$, the α-cut, $\overline{A}^\alpha$ and the strong α-cut, $\overline{A}^{\alpha+}$, which are the crisp sets[6].

$$\overline{A}^\alpha = \{x | A(x) \geq \alpha\}$$

$$\overline{A}^{\alpha+} = \{x | A(x) > \alpha\}$$

Unequal in the conventional set theory, the convexity of fuzzy sets introduced the properties of the membership function instead than the approval of a fuzzy set.

3. FUZZY VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS(FVIDE)

In this section, we examine the system of fuzzy Volterra Integro-Differential equations[1] which is the first order of the second kind and is given by

$$x_1'(r) = f_1(r, x_1) + \int_0^t K_1(r, s)x_1(s)ds, \; x_1(r_0) = [\underline{x_1}, \overline{x_1}]$$

$$x_2'(r) = f_2(r, x_2) + \int_0^t K_2(r, s)x_2(s)ds, \; x_2(r_0) = [\underline{x_2}, \overline{x_2}] \tag{3}$$

With the initial conditions, we have

$$x(r_0) = x_0 \in R$$

Also in a brief notation which has been given as

$$x_1'(r) = f_1(r, x_1, \int_0^t K_1(r, s)x_1(s)ds) \; ,$$

$$x_2'(r) = f_2(r, x_2, \int_0^t K_2(r, s)x_2(s)ds) \; , \tag{4}$$

For $0 < r < 1$, the fuzzy Volterra Integro-Differential Equations can be converted into the following fuzzy conditions:

$$\underline{x_1'}(r) = \underline{f_1}(r, x_1) + \int_0^t K_1(r, s)\underline{x_1}(s)ds$$

$$\overline{x_1'}(r) = \overline{f}(r, x_1) + \int_0^t K_1(r, s)\overline{x_1}(s)ds \tag{5}$$

$$\underline{x_2'}(r) = \underline{f_2}(r, x_2) + \int_0^t K_2(r, s)\underline{x_2}(s)ds$$

$$\overline{x_2'}(r) = \overline{f}(r, x_2) + \int_0^t K_2(r, s)\overline{x_2}(s)ds \tag{6}$$

With the initial conditions, we have

$$\underline{x'}(r) = \overline{x'}(r) = 0$$

The fuzzy Volterra Integro-Differential equations has been applied to various method for calculating linear and nonlinear problems. It is applied in real life problems in engineering and applied sciences.

4. VARIATIONAL ITERATION METHOD(VIM)

In this part of our research work, we manifest the most basic concept for the study of Variational Iteration method [19]. In early 1997, J.H. He, who is the scientist suggested the Variational Iteration method for the moderation of the general Lagrange multiplier. Here the Lagrange Multiplier is denoted as λ. This method has been used because of their efficiency and accuracy, which helps to converge approximate solutions very fastly to accurate solutions[15].

Consider the general differential equation as follows:

$$Lu(s) + Nu(s) = g(s) \tag{7}$$

Here the linear and nonlinear operators are denoted as L and N respectively. Then g(s) has been defined as the inhomogeneous term. Assume that $u_0(s)$ be the appropriate solution for the linear and homogeneous equation[13], then we have

$$Lu_0(s) = 0 \tag{8}$$

According to the Variational Iteration Method we have to construct the correction functional[9], that is as follows:

$$u_{n+1}(s) = u_n(s) + \int_0^s \lambda(Lu_n(\tau) + N\tilde{u_n}(\tau) - g(\tau))d\tau \tag{9}$$

Thus λ be denoted as the general Lagrange multiplier which can be recognised by the variational theory optimally. Here in the above equation n be denoted as the nth approximation and finally $\tilde{u_n}$ is said to be the restricted variation.

Therefore, $\delta\tilde{u_n} = 0$ (10)

From the above correction functional, we can obtain

$$\delta u_{n+1}(s) = \delta u_n(s) + \delta \int_0^s \lambda(Lu_n(\tau) + N\tilde{u_n}(\tau) - g(\tau))d\tau \tag{11}$$

Since the restricted variation be zero, then we can identify the Lagrange multiplier from eq.(11) as

$$\delta u_{n+1}(s) = \delta u_n(s) + \delta \int_0^s \lambda(Lu_n(\tau) - g(\tau))d\tau \tag{12}$$

More generally, by the spectacular stationary condition $\delta u_{n+1} = 0$ the Lagrange multiplier λ can be obtained. Now after finding the λ, Lagrange multiplier we can choose an appropriate initial condition u_0. Then next the successive approximations u_n can be get from the solution u[11]. In consequence, we get the solution for the equation(7) as

$$u(x) = lim_{u\to\infty} u_n(x)$$

The correction functional helps to get the successive approximations and we can obtain the exact solutions from the limits of the resulting successive approximations.

4.1. VIM for solving SFVIDE's

Consider the system of fuzzy Volterra Integro-Differential equation as

$$x_1'(r) = f_1(r) + \lambda \int_0^t K_1(r,s)x_1(s)ds$$

$$x_2'(r) = f_2(r) + \lambda \int_0^t K_2(r,s)y_2(s)ds \tag{13}$$

The upcoming iteration sequences be

$$u_{n+1}(r) = u_n(r) + \int_0^t \lambda(\tau)\{u_n'(\tau) - f(\tau) - \int_0^s K(r,s)\tilde{u_n}(s)ds\}d\tau \tag{14}$$

The stationary condition obtained is

$$\lambda = 0 \text{ and } \lambda + 1 = 0$$

The identified general Lagrange multiplier is $\lambda = -1$

Applying the Lagrange multiplier to equation(14), we get the iteration formula for $n \geq 0$ is

$$u_{n+1}(r) = u_n(r) - \int_0^t \{u'_n(\tau) - f(\tau) - \int_0^s K(r,\tau)\tilde{u_n}(s)ds\}d\tau \tag{15}$$

The iteration formulas for the fuzzy Volterra Integro-Differential equation [14]in the Variational Iteration method be

$$\underline{x}_{n+1}(r) = \underline{x_n}(r) - \int_0^t \{\underline{x}'_n - \underline{f}(r) - \int_0^s K(r,\tau)\underline{F_n}(r,s)ds\}d\tau \tag{16}$$

$$\overline{x}_{n+1}(r) = \overline{x_n}(r) - \int_0^t \{\overline{x}'_n - \overline{f}(r) - \int_0^s K(r,\tau)\overline{F_n}(r,s)ds\}d\tau \tag{17}$$

$$\underline{y'}(x) = \overline{y'}(x) = 0 \tag{18}$$

which is the initial approximations.

4.2. Theorem

Let the exact solution of the fuzzy Volterra-integro differential equations be $u \in (C_1[a,b], ||.||)$ of $y'(x) = f(x) + \lambda \int_0^x K(x,t)y(t)dt$ and $u_n(x) \in C[a,b]$ be the obtained solution of the sequence defined by $u_{n+1}(x) = u_n(x) - \int_0^x (u'_n(\tau) - f(\tau) - \int_0^s K(s,\tau)\tilde{u_n}(s)ds)d\tau$ in Variational Iteration method. If $D_n(x) = u_n(x) - u(x)$ and $|K| < c, 1 > c > 0$ then the sequence of approximate solutions $\{u_n(x)\}, n, 0, 1, ...$ converges to the exact solution $u(x)$[13].

Proof

Consider the fuzzy Volterra Integro-Differential Equations be

$$y'(x) = f(x) + \lambda \int_0^x K(x,t)y(t)dt$$

By using the VIM the approximate solution [13] is obtained as

$$u_{n+1}(x) = u_n(x) - \lambda \int_0^x (u'_n(\tau) - f(\tau) - \int_0^s K(s,\tau)\tilde{u_n}(s)ds)d\tau$$

Since the exact solution for the fuzzy Volterra-Integro Differential equation be u.

$$u_n(x) = u_n(x) - \lambda \int_0^x (u'_n(\tau) - f(\tau) - \int_0^s K(s,\tau)\tilde{u_n}(s)ds)d\tau$$

Subtracting the above two equations,then we get that

$$D_{n+1}(x) = D_n(x) - D_n(x) - D_n(0) - \lambda \int_0^x \int_0^s dsd\tau \tag{19}$$

Since the initial condition be $D_n(0) = u_n(0) - u(0) = 0$ then

$$D_{n+1}(x) = \lambda \int_0^x \int_0^s K(s,\tau)D_n(\tau)dsd\tau \tag{20}$$

Taking the maximum norm on both sides, then we get

$$||D_{n+1}(x)||_\infty = |\lambda|||\int_0^x \int_0^s K(s,\tau)D_n(\tau)dsd\tau||_\infty$$

$$||D_{n+1}(x)||_\infty \leq |\lambda| \int_0^x \int_0^s ||K||_\infty ||D_n(\tau)||_\infty dsd\tau$$

Since K is a bounded function by the constant c and $c \in (0,1)$, then we have

$$||D_{n+1}(x)||_\infty \leq |\lambda| \int_0^x \int_0^s ||K||_\infty ||D_n(\tau)||_\infty dsd\tau$$

$$||D_{n+1}||_\infty \leq \lambda cx \int_0^s ||D_n(\tau)||_\infty \tag{21}$$

$$||D_{n+1}||_\infty = \lambda cx \int_0^s ||D_n(\tau)||_\infty$$

When n=0, then the equation becomes

$$||D_1||_\infty = cx\lambda \int_0^s ||D_0(\tau)||_\infty$$

$$||D_1||_\infty = cx\lambda(s-0)Max|D_0|$$

Similarly for $n = 1$, we obtain the equation as

$$||D_2||_\infty = cx\lambda \tfrac{s^2}{2} Max|D_0|$$

When $n = 2$, then we have the equation as

$$||D_3||_\infty = cx\lambda \tfrac{s^3}{3!} Max|D_0|$$

Generally proceeding the mathematical induction like this, we obtain that

$$||D_n||_\infty = cx\lambda \tfrac{s^n}{n!} Max|D_0| \tag{22}$$

As $n \to \infty$, the RHS of eq.(22) becomes zero. Thus $u_n(x)$ converges to $u(x)$

Therefore, the sequence of solution has been get from VIM and also it converges to the exact $u(x)$.

Now we applying the fuzzy concept with the initial conditons, then we obtain the exact solution as

$$\underline{u_n}(x) = \underline{u_n}(x) - \lambda \int_0^x (\underline{u'_n}(\tau) - \underline{f}(\tau) - \int_0^s \underline{K}(s,\tau)\underline{\tilde{u_n}}(s)ds)d\tau$$

$$\overline{u_n}(x) = \overline{u_n}(x) - \lambda \int_0^x (\overline{u'_n}(\tau) - \overline{f}(\tau) - \int_0^s \overline{K}(s,\tau)\overline{\tilde{u_n}}(s)ds)d\tau$$

The approximate solution be

$$\underline{u_{n+1}}(x) = \underline{u_n}(x) - \lambda \int_0^x (\underline{u_n}'(\tau) - \underline{f}(\tau) - \int_0^s \underline{K}(s,\tau)\underline{\tilde{u_n}}(s)ds)d\tau$$

$$\overline{u_{n+1}}(x) = \overline{u_n}(x) - \lambda \int_0^x (\overline{u_n}'(\tau) - \overline{f}(\tau) - \int_0^s \overline{K}(s,\tau)\overline{\tilde{u_n}}(s)ds)d\tau$$

The initial condition be $\underline{u_n}(x) = \overline{u_n}(x) = [\underline{0}, \overline{0}]$

Hence the proof.

5. NUMERICAL EXAMPLES

In this content, we solve the system of fuzzy Volterra Integro-Differential in the Variational Iteration Method (VIM). We apply our introduced iterative technique by assessing the SFVIDE and finally our solution has been revealed. For a perfect sight of this methodology, the following examples are presented. The calculations are executed by a computational software Mathematica 8.

5.1. Algorithm

Step 1 : Choose an initial condition and find the Lagrange multiplier.

Stpe 2 : Calculate approximate equations with the initial conditions.

Step 3 : Substitute the values for the Lagrange multiplier.

Step 4 : Also apply initial value to the correction functional.

Step 5 : Find out the successive iterations.

5.2. Example

Consider the following system of fuzzy Volterra integro-differential equations[5]

$$[\underline{x_1}'(t), \overline{x_1}'(t)] = [(\alpha-1)t^2 + \int_0^t e^{t-\tau}(x_1(\tau)+x_2(\tau))d\tau, (1-\alpha)t^2 + \int_0^t e^{t-\tau}(x_1(\tau)+x_2(\tau))d\tau]$$

$$[\underline{x_2}'(t), \overline{x_2}'(t)] = [\alpha e^t + \int_0^t e^{t-\tau}(x_1(\tau)+x_2(\tau))d\tau, (2-\alpha)e^t + \int_0^t e^{t-\tau}(x_1(\tau)+x_2(\tau))d\tau] \tag{23}$$

subject to the fuzzy initial conditions

$$x_1(0) = [\underline{x_1}, \overline{x_1}]$$

$$x_2(0) = [\underline{x_2}, \overline{x_2}] \tag{24}$$

Here variational iteration method has been used.

The general formula for VIM is

$$\underline{x_{n+1}}(t) = \underline{x_n}(t) + \int_0^t \lambda[\underline{x_n}'(t) - (\alpha-1)t^2 - \int_0^s [\underline{x_1}(t) + \underline{x_2}(t)]e^{t-\tau}]d\tau]ds$$

$$\overline{x_{n+1}}(t) = \overline{x_n}(t) + \int_0^t \lambda[\overline{x_n}'(t) - (\alpha-1)t^2 - \int_0^s [\overline{x_1}(t) + \overline{x_2}(t)]e^{t-\tau}]d\tau]ds$$

$$\underline{x_1}(t) = t^2 - \alpha t^2 + \alpha - \alpha e^t + \alpha e^t t$$
$$\overline{x_1}(t) = t^3 - \alpha t^3 + t^2 - \alpha t^2 - \alpha t e^t + \alpha e^t - 2e^t + 2te^t - \alpha + 2 \quad (25)$$

$$\underline{x_2}(t) = t^2 - \alpha t^2 + 2\alpha t e^t - \alpha e^t + \alpha$$
$$\overline{x_2}(t) = t^2 - \alpha t^2 + 4te^t - 2\alpha t e^t - 2e^t + \alpha e^t + 2 - \alpha \quad (26)$$

Table 1 : The approximate error value of SFVIDE for VIM is

$t = 5, \alpha$	$\underline{x_1}$	$\overline{x_1}$	$\underline{x_2}$	$\overline{x_2}$
$\alpha = 0.25$	0.5634×10^3	0.1887×10^4	0.9754×10^3	0.9926×10^4
$\alpha = 0.5$	0.0648×10^3	0.2192×10^4	0.6327×10^3	0.8581×10^4

For this numerical calculations for this system of fuzzy Volterra Integro-Differential equations we insert the VIM for α on $[0, 1]$ and $t = 5$.

6. CONCLUSION

In this research paper work, we used the Variational Iteration method to solve the system of fuzzy Volterra Integro-Differential equations of first kind. The consequence revealed the convergence and correctness of this VIM for the numerical computations. When comparing with the traditional methods, that VIM supply a we grounded method that only need very small amount of work. The results acquired from the VIM are compared with the exact solutions. All numerical calculations are computed by using the software Mathematica 8. In this work with VIM has been successfully applied for getting the approximate solution of system of fuzzy Volterra Integro-differential Equations. Here we get the final result for this method very nearly to our exact solutions. Moreover this VIM will also be executed for all nonlinear system of integro and integral differential equations of first and second kind respectively.

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