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A Comparative Study on Nonlinearity, Volatility, Chaos and Causal Relationship between Prime Stock Exchanges of E7 Countries in Last Two Decades

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ABSTRACT

E7 countries are of great importance with respect to its emerging economic prominence. In the present work, a study is made over the prime stock markets of E7 countries in decade 2001-2010 and decade 2011-2020. The objective of the present analysis is to understand the nonlinearity, volatility, chaos by different techniques and also to find mutual linear and nonlinear Granger causal association between them, decade wise. Present study shows that though all the prime markets in E7 exhibit nonlinearity and volatility throughout, there is no confirm evidence of chaos in the period under consideration. Remarkably linear Granger causal relation decreases and nonlinear Granger causal relation increases in 2011-2020 with Chinese stock market as exogenous stock market and Brazil stock market as endogenous stock market in last decade.

Keywords: E7; nonlinearity; EGARCH; TGARCH; chaos; correlation dimension; Lyapunov exponent; BDS; Granger causality

1. INTRODUCTION

E7 (Emerging 7) counties, coined by Hawksworth & Cookson at PricewaterhouseCoopers in 2006 [1], are seven major countries with emerging economies, viz. China, India, Brazil, Mexico, Russia, Indonesia and Turkey. Though aggregate GDP of E7 group was approximately 55% in compare to G7 (Group of 7) listing Canada, France, Germany, Italy, Japan, UK and USA, it is predicted to be almost double, that of G7, by 2050 [2]. E7 has suppressed G7 in terms of purchasing power parity (PPP) by 2014 [3] and may exaggerate PPP 75% larger than G7 by 2050 [4]. Six E7 countries (except Mexico) is estimated to be in top ten largest economies by nominal GDP, using PPT exchange rates, by 2030 [5]. That is why financial markets of E7 countries are regarded as a major and significant field of study to the researchers.

Due to complex dynamics of financial time series, a great area of interest lies behind analysing its nonlinear dynamics. There are several studies in this regard. Pandey et al. [6] worked on deterministic nonlinearity in the stock returns of major European equity markets and the United States. Lim & Brooks [7] tested efficiency of Chinese stock markets using nonlinearity. Setianto and Manap [8] showed that Indonesian stock market exhibit nonlinearity. Lahmiri & Bekiros [9] did a comparative study on nonlinear behaviour of Casablanka Stock Exchange, Dow Jones and S&P Industrial sectors. There are several tools to check nonlinearity of a time series. Tsay test [10], which is improved version of Keenan test [11] is a popular nonlinearity test [12]. BDS Test is another very efficient and powerful test of data independency(iid), widely used in various literature ([6], [13-17]). Another idea to examine nonlinearity of data is to find linear dependence of square residuals [18-19] which is the base of McLeod and Li test [20].

If the financial time series shows nonlinearity, there may be presence of volatility, characterized by various models such as autoregressive conditional heteroskedasticity (ARCH) [21], generalized ARCH(GARCH) [22], exponential GARCH(EGARCH) [23], threshold GARCH(TGARCH) [24]. Presence of volatility make prediction of stock market more uncertain and fluctuation in volatility make the dynamics of the system even more complex. Sharma et al. [25] compared linear and nonlinear GARCH models for forecasting selected emerging counties and observed that China is the most volatile market whereas Indonesia is least volatile. Irshad et al. [26] concluded significant volatility spillovers from US to E7 markets after global financial crisis.

Chaos is another important aspect of nonlinear analysis. A deterministic nonlinear system may sometimes behave in completely unpredictable behaviour. Small changes in starting state may produce enormous changes in final state. Though it may look like the system is behaving randomly, being deterministic, system's future dynamics is completely regulated by initial conditions with no randomness. It is termed as deterministic chaos. Searching for existence of chaos in the financial markets remains a relevant issue due to the fact that a chaotic system leads to instability of the system. While some past studies found presence of chaos in some financial markets ([12-13], [27]), other studies support non-chaotic and sometimes stochastic behaviour of the system ([6], [16], [28]).

Another interesting and exiting topic of research is investigation of coherence of E7 countries. Causality analysis is an important and standard tool to determine causal relationship among different time series. Granger causality analysis [29] plays a pivotal role in this aspect with the null hypothesis that one time series does not cause, or in other word, is not helpful to predict behaviour of another time series. Linear granger causality ([30-32]) captures only linear causal relationship whereas for time series showing nonlinear behaviour, in addition, nonlinear Granger causality analysis ([33-35]) may be performed for detection of nonlinear causal relationship between them.

Though several individual studies are conducted on individual characteristics like nonlinearity, volatility, chaos, causality of various financial markets, comparative study of a group of emerging countries in two different time frames are limited.

In this backdrop, the present study aims to explore the nonlinear dynamics of the prime stock exchanges of the countries belonging to E7 with the help of different methods; viz.Tsay test [10], BDS test ([6], [13-17]) and McLeod and Li test [20]. Runs test [36] is used to check the randomness nature of these stock exchanges. As it is indicated that the exchanges are governed by some underlying nonlinear dynamics, volatile nature of the markets are explored using EGARCH [23] and TGARCH model [24], taking leverage effect in to the consideration. Another necessary, but not sufficient cause of nonlinearity may be deterministic chaos. Hence, different chaos tests were implied on these markets by means of 0-1 chaos test ([37), correlation dimension [38] and Lyapunov exponent [39]. Finally, both linear [29] and nonlinear Granger causal relationship [33-35] between the financial series are examined and changing association of these markets are analyzed.

2. DATA AND METHODOLOGY

2.1 DATA

The study is based on daily return data of prime stock exchanges of E7 countries, namely, IBOVESPA (Brazil), IPC (Mexico), JKSE (Indonesia), RTSI (Russia), SENSEX (India), SSE (China) & XU100 (Turkey) [40]. Return series is taken into consideration due to the fact that return data has higher chance to be stationary which is assumption of Vector Autoregressive (VAR) time series model. The data set is grouped in two time frames; (i) Decade-1 (from January 1, 2001 to December 31, 2010), (ii) Decade-2 (from January 1, 2011 to December 31, 2020). As the holidays in each country differ, data set is not uniform in length. Data only with common dates are taken into consideration to maintain uniformity and unbiasedness of the study. Decade-1 and Decade-2 have 1979 and 1947 data points respectively. Comparative analysis of nonlinear, volatile, chaotic and causal behaviour of the stock exchanges in these two decades was obtained.

2.2 NONLINEARITY TEST

2.2.1 TSAY TEST

Tsay test [10] is the generalization of Keenan test [11]. A time series $X_t, t = 1, 2, ..., n$, is assumed to take the form

$$X_t = \mu + \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_u a_{t-u} + \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_{uv} a_{t-u} a_{t-v} \quad (1)$$

Clearly, $\sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_{uv} a_{t-u} a_{t-v} \approx 0$ in (1) implies linearity of the series. Idea of Keenan's test is to firstly detect optimal lag p using standard information criterion like, Akaike Information Criterion (AIC), Schwartz Information Criterion (SIC), Hannan-Quinn

Information Criterion (HQIC), FPE Information Criterion, etc. Next, X_t is regressed on $\left(1, X_{t-1}, X_{t-2}, ..., X_{t-p}\right)$ to get a fitted value $\hat{X}_t$, the corresponding residuals $\hat{a}_t$ and residual sum of squares, *r*. Then, $\hat{X}_t^{\,2}$ is regressed on $\left(1, X_{t-1}, X_{t-2}, ..., X_{t-p}\right)$ and corresponding residuals $\hat{b}_t$ is calculated. Next,

$\hat{\eta}_t = \dfrac{\sum_{t=p+1}^{n} \hat{a}_t \hat{b}_t}{\sum_{t=p+1}^{n} \hat{b}_t^{\,2}}$ is and the test statistics $\hat{F} = \dfrac{(n-2p-2)\hat{\eta}^2}{\left(r-\hat{\eta}^2\right)}$ is generated. Under null hypothesis of linearity condition given by (1) for Gaussian iid of $\hat{a}_t$'s, $\hat{F} \sim F_{1,n-2p-2}$.

Tsay [10] has modified Keenan test [11] by including general quadratic terms of the form $X_{t-i}X_{t-j}$, i,j=1,2,...,p,i<j instead of only $\hat{X}_t^{\,2}$. So, $X_{t-i}X_{t-j}$, i,j=1,2,...,p,i<j are regressed on $\left(1, X_{t-1}, X_{t-2}, ..., X_{t-p}\right)$ instead of $\hat{X}_t^{\,2}$. As there are *m*= $\dfrac{\mathrm{p}(p-1)}{2}$ possible combinations of *i* and *j*, corresponding test statistic $\hat{F} \sim F_{\mathrm{m},n-m-p-1}$ under null hypothesis.

2.2.2 BDS TEST

BDS test [15] is one of the most popular and powerful test to detect nonlinearity in a time series using correlation integral. Though BDS test is a nonparametric test originally developed to detect a independent and identically distributed (iid) series, it has statistical power to detect nonlinearity of the process [41]. When BDs test is applied on the residuals of a linear model, rejection of null hypothesis of iid series implies presence of nonlinearity in the data. In this method, from a time series $X_t, t = 1, 2, ..., n$, $X_t^{\,m} = \left(x_t, x_{t-1}, x_{t-2,...,x_{t-m+1}}\right)$ is generated where *m* denotes the embedding dimension. The correlation integral at *m* is calculated as

$$C_{m,n}(\varepsilon) = \sum_{t<s} I_{\varepsilon}\left(X_t^{\,m}, X_s^{\,m}\right) \frac{2}{n_m\left(n_m - 1\right)} \tag{2}$$

where $n_m = n-(m-1)$ and *I* is indicator function, i,e.,

$$I_{\varepsilon}\left(X_t^{\,m}, X_s^{\,m}\right) = \begin{cases} 1 & \text{when } \sup\left\|X_t^{\,m} - X_s^{\,m}\right\| < \varepsilon \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Basically, $C_{m,n}(\varepsilon)$ estimates

$$\mathrm{P}\left(\left|X_t - X_s\right| < \varepsilon, \left|X_{t-1} - X_{s-1}\right| < \varepsilon, \left|X_{t-2} - X_{s-2}\right| < \varepsilon, ..., \left|X_{t-m+1} - X_{s-m+1}\right| < \varepsilon\right) \tag{4}$$

Now, for independent and identically distributed X_t's, probability described in (4) should be equal to $C_{1,n}(\varepsilon)^m = P\left(\left|X_t - X_s\right| < \varepsilon\right)^m$ (5)

Hence, under null hypothesis of iid X_t's, BDS statistic is given by

$$V(n,m,\varepsilon) = \sqrt{n}\,\frac{C_{m,n}(\varepsilon) - C_{1,n}(\varepsilon)^m}{s_{m,n}} \tag{6}$$

where $s_{m,n}$ is the corresponding standard deviation. Asymptotically, $V(n,m,\varepsilon) \sim N(0,1)$, standard normal distribution.

2.2.3 MCLEOD AND LI TEST

Mcleod and Li test [20] is based on autocorrelation function of the squared residual $\hat{a}_t^2$ obtained from an ARMA(*p,q*) model fitted in the time series $X_t, t = 1, 2, ..., n$. Autocorrelation function of order *k* is calculated by

$$r_k = \frac{\sum_{t=k+1}^{n}\left(\hat{a}_t^2 - \hat{\sigma}^2\right)\left(\hat{a}_{t-k}^2 - \hat{\sigma}^2\right)}{\sum_{t=1}^{n}\left(\hat{a}_t^2 - \hat{\sigma}^2\right)^2},\ \text{k=1,2,...,m} \tag{7}$$

where $\hat{\sigma}^2 = \frac{\sum_{t=1}^{n}\hat{a}_t^2}{n}$.

Test statistic to check nonlinearity uses Lijung-Box statistic applied to the squares of r_k's

$$\text{Q=n}(n+2)\sum_{k=1}^{m}\frac{r_k^2}{n-k} \tag{8}$$

$Q \sim \chi_m^2$ under the null hypothesis of first *m* autocorrelations of squared residuals are 0 (i.e. the series is linear)

2.3 VOLATILITY TESTS

2.3.1 EGARCH MODEL

EGARCH (exponential generalized autoregressive conditional heteroskedasticity) model is used not only to detect volatility of a data, but also to measure the asymmetric nature of the volatility w.r.t positive and negative shock, known as leverage effect. It uses exponential function. Nelson [23] derived that negative shock has more impact to forecast volatility compared to positive shock in his proposed EGARCH (*p*, *q*) model. Let a_t be the residual or shock obtained from the return series $X_t, t = 1, 2, ..., n$ after fitting a ARMA model.

EGARCH (p, q) model is given by

$$a_t = \sigma_t \varepsilon_t$$

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \left\{ \frac{|a_{t-i}|}{\sigma_{t-i}} \right\} + \sum_{i=1}^{q} \gamma_i \left\{ \frac{a_{t-i}}{\sigma_{t-i}} \right\} + \sum_{j=1}^{p} \beta_j \ln\left(\sigma_{t-j}{}^2\right) \tag{9}$$

where $\sigma_t{}^2$ is the implied volatility, $\{\varepsilon_t\}$ is a iid's with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 1$, a_t's are not serially correlated with $E(a_t) = 0$, $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$; $\beta_j \geq 0$ and $\sum_{k=1}^{\max(p,q)} (\alpha_k + \beta_k) < 1$.

Clearly, (9) captures effect of positive and negative a_t's separately, as, for positive and negative a_t, (9) can be rewritten as

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i + \gamma_i) \frac{a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \ln\left(\sigma_{t-j}{}^2\right) \tag{10}$$

and

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i - \gamma_i) \frac{a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \ln\left(\sigma_{t-j}{}^2\right) \tag{11}$$

respectively.

As γ_i's are negative in general, it is derived that negative shocks have more impact on volatility of the series compared to positive shock. γ_i is termed as leverage effect.

2.3.2 TGARCH MODEL

In TGARCH (threshold generalized autoregressive conditional heteroskedasticity) model, proposed by Glosten et al. [24], a threshold 0 is used to capture asymmetric behaviour of the volatility present in the GARCH model. TGARCH(p,q) is formulated as

$$a_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i N_{t-i}) a_{t-i}{}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}{}^2 \tag{12}$$

where $N_{t-i} = \begin{cases} 1 \text{ if } a_{t-i} < 0 \\ 0 \text{ if } a_{t-i} \geq 0 \end{cases}$, α_i and β_j are nonnegative integer maintaining same restriction in EGARCH model. γ_i's are positive in general. Asymmetry can be established as for positive and negative a_t, (12) can be rewritten as

$$\sigma_t^{\,2} = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^{\;\;2} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\;\;2} \tag{13}$$

and

$$\sigma_t^{\,2} = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i + \gamma_i\right) a_{t-i}^{\;\;2} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\;\;2} \tag{14}$$

respectively.

Hence, for positive γ_i's, negative shock contributes larger component $\left(\alpha_i + \gamma_i\right) a_{t-i}^{\;\;2}$ on $\sigma_t^{\,2}$ compared to smaller component $\alpha_i a_{t-i}^{\;\;2}$ for positive shock.

2.4 CHAOS TEST

2.4.1 0-1 CHAOS TEST

0-1 chaos test [37] is a robust test, easy to implement to detect deterministic chaotic nature in a data. The final output of this test is binary, 0 (absence of chaos) or 1 (presence of chaos). In this method, a time series $x(t), t = 1, 2, ..., N$, is transformed to a Fourier series p_n developed as

$$p_n = \sum_{t=1}^{n} x(t) e^{ikc} \text{ where } 1 \le \text{n} \le \text{N} \tag{15}$$

c, being a random number.

The smoothed mean square displacement $\text{D}_c(n)$ is formulated as

$$\text{D}_c(n) = \frac{1}{N-m} \sum_{t=1}^{N-m} \left| p_{t+n} - p_t \right|^2 - \langle x \rangle^2 \frac{1-\cos nc}{1-\cos c} \tag{16}$$

where $\langle x \rangle = (1/N) \sum_{t=1}^{N} x(\text{t})$ and $n \le m \le N/10 \ll N$.

Presence of noise is taken cared by modifying $\text{D}_c(n)$ to $\text{D}_c^*(n)$ as

$$\text{D}_c^*(n) = \text{D}_c(n) + \alpha V_{damp}(n) \tag{17}$$

where $V_{damp}(n) = \langle x \rangle^2 \sin(\sqrt{2n})$

The asymptotic growth rate K_c for different *c*'s are calculated as

$$K_c = corr(n, \text{D}_c^*(n)) \tag{18}$$

and binary output *K* is given by

$$\text{K} = \text{median}(K_c) \tag{19}$$

K=0 for non-chaotic data and *K=1* for chaotic data.

2.4.2 LYAPUNOV TEST

Lyapunov test is another important method to measure sensitivity of data by initial condition. Largest Lyapunov exponent characterizes the rate of separation of infinitesimal close trajectories in attracting manifold [42]. For chaotic attractor, trajectories diverge by exponential rate [43]. There are several methods available to calculate Lyapunov exponent ([39], [44-45]). In this study, method improvised by Rosenstein *et al.* [46] is used as it is advantageous for a small data set.

Given a time series $\{x_t\}$, t=1,2,...N , a trajectory $X = [X_1\ X_2\ ...\ X_M]^T$ be reconstructed where $X_i = [x_i\ x_{i+j}\ ...\ x_{i+(m-1)j}]$ is the state of the system at time i for lag j and embedding dimension m.

So, M = N - (m -1)J (20)

Next, nearest neighbour of X_j, noted as $\mathrm{X}_{\hat{j}}$ is found by taking that point whose distance from X_j is minimum, i.e.

$$d_j(0) = \min_{\mathrm{X}_{\hat{j}}} \left\| X_j - \mathrm{X}_{\hat{j}} \right\| \tag{21}$$

where $d_j(0)$ is the initial distance from X_j to its nearest neighbour $\mathrm{X}_{\hat{j}}$. Here nearest neighbours must have a temporal separation greater than the mean period of the time series in order to consider each pair of neighbours as almost same initial conditions for different trajectories.

After that, largest Lyapunov exponent λ_1 is estimated as described by Sato et al. [47]

$$\lambda_1(i) = \frac{1}{i\Delta t} \cdot \frac{1}{M-i} \sum_{j=1}^{M-i} \ln \frac{d_j(i)}{d_j(0)} \tag{22}$$

where Δt is the sample time period, $d_j(i)$ is the distance between the j th pair of nearest neighbours after time $i\Delta t$.

As Largest Lyapunov exponent follows the power law

$$d(t) = Ce^{\lambda_1 t} \tag{23}$$

where $d(t)$ is the average distance at time t and $C = d(0)$,

$$d_j(i) = C_j e^{\lambda_1 (i\Delta t)} \tag{24}$$

where C_j is the initial separation.

Applying logarithm to both sides of (24), we get

$$\ln\left(d_j(i)\right) = \lambda_1(i\Delta t) + \ln\left(C_j\right) \tag{25}$$

So, largest Lyapunov exponent is estimated applying a least square fit to

$$y(i)=\frac{1}{\Delta t}\left\langle \ln\left(d_j(i)\right)\right\rangle \tag{26}$$

where $\langle . \rangle$ denotes the averages over all values of *j*.

System is chaotic for $\lambda_1 > 0$ and non-chaotic for $\lambda_1 < 0$.

2.4.3 CORRELATION DIMENSION TEST

Correlation dimension test measures correlation dimension of a data and identifies a chaotic data depending on the nature of its correlation dimension for different embedding dimensions. Correlation integral $C_{m,n}(\varepsilon)$ of a derived time series $X_t^m = \left(x_t, x_{t-1,} x_{t-2,\ldots,} x_{t-m+1}\right)$ at embedding dimension *m*, obtained from a time series $X_t, t = 1,2,\ldots,n$ is given in (2). As ε increases, $C_{m,n}(\varepsilon)$ increases by definition, as number of included near points increase. According to Grassberger and Procaccia [38], $C_{m,n}(\varepsilon)$ obeys the power law

$$C_{m,n}(\varepsilon) \sim \varepsilon^{\nu} \tag{27}$$

Estimate of ν when $m \to \infty$ is called correlation dimension (CD).

$C_{m,n}(\varepsilon) \sim \varepsilon^{\nu}$ is calculated for several small values of ε and ν can be estimated by the slope of regression of $\ln\left(C_{m,n}(\varepsilon)\right)$ on $\ln(\varepsilon)$. For purely stochastic data, CD is approximately equal to $m\ \forall m$. On contrary, for chaotic data, CD stabilizes gradually. So, saturation of CD with increase of *m* indicates chaotic behaviour of the data.

2.5 GRANGER CAUSALITY ANALYSIS

2.5.1 LINEAR GRANGER CAUSALITY ANALYSIS

Linear Granger causality analysis [29] is a vital and effective tool to measure degree of linear association of *n* variables $y_1, y_2, \ldots y_n$ comprising a Multivariate Vector autoregression (VAR) model. Let the VAR (*p*) model is described by

$$Y_t = \varphi_0 + \sum_{k=1}^{p} \varphi_k Y_{t-k} + \varepsilon_t \tag{28}$$

where $Y_t = \left(y_1, y_2, \ldots y_n\right)^T$, $\varphi_l = \left(\varphi_{ij}^{(l)}\right)_{n\times n}$ for $l = 1,2,\ldots,p$, $\varepsilon_t = \left(\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{nt}\right)^T$, ε_{mt} being the residual of y_m, m=1,2,...n .

Lag order *p* can be calculated taking minimum value of AIC, SIC and HQIC [48-49].

y_i Granger causes y_j if y_i provides statistically significant contribution in the prediction of y_j. Wald test is one popular method used to detect linear Granger causality for non-normal distribution. Here

$$\text{Test statistics } W = \frac{(SSE_R - SSE_U)}{SSE_U/(T-2p-1)} \sim \chi_p^{\,2} \tag{29}$$

$$\text{under null hypothesis } \varphi_{ji}^{(l)} = 0,\ \text{l=1,2,...,p} \tag{30}$$

(i.e., y_i does not linearly Granger cause y_j).

2.5.2 NONLINEAR GRANGER CAUSALITY ANALYSIS

Linear Granger causality analysis may not be sufficient for the nonlinear time series as nonlinear causal association is overlooked [50-51]. Hence, nonlinear Granger causality analysis is performed to measure contribution of nonlinear part of causality between the variables.

Diks-Panchenko Test ([33-35) is modification of Himestra-Jones Test [51-52]. The idea of Himestra-Jones Test is to use conditional probability and correlation integral on the random variables X_t and Y_t corresponding to the time series x_t and y_t. Under null hypothesis H_0, i.e., assumption of non-Granger causality of y_t by x_t,

$$Y_{t+1} \mid \left(X_t^{\,l_x}; \mathrm{Y}_t^{\,l_y}\right) \sim Y_{t+1} \mid \mathrm{Y}_t^{\,l_y} \tag{31}$$

where $X_t^{\,l_x} = \left(X_{t-l_x+1},...,\mathrm{X}_t\right)$ and $Y_t^{ly} = \left(Y_{t-l_y+1},...,\mathrm{Y}_t\right)$, l_x and are lag of x_t and y_t respectively.

Taking $Z_t = Y_{t+1}$ and denoting $W_t = (\mathrm{X}_t^{\,l_x}, \mathrm{Y}_t^{\,l_y}, \mathrm{Z}_t)$, (31) is relooked as

$$\mathrm{Z} \mid ((\mathrm{X},\mathrm{Y}) = (x,y) \sim Z \mid (Y = y) \tag{32}$$

(32) indicates that under H_0,

$$\frac{f_{X,Y,Z}(x,y,z)}{f_{X,Y}(x,y)} = \frac{f_{Y,Z}(y,z)}{f_Y(y)} \tag{33}$$

f 's being the corresponding probability density functions.

(33) is same as

$$\frac{f_{X,Y,Z}(x,y,z)}{f_Y(y)} = \frac{f_{X,Y}(x,y)}{f_Y(y)} \frac{f_{Y,Z}(y,z)}{f_Y(y)} \tag{34}$$

Himnestra and Jones estimated pdfs by correlation-integral estimator, of the form

$C_{W,n}(\varepsilon) = \frac{2}{n(n-1)} \sum\sum_{i<j} I_{ij}^{W}$ where

$I_{ij}^{W} = I(\|W_i - W_j\| \leq \varepsilon)$, $\|.\|$ being maximum norm and obtained

$$\frac{C_{X,Y,Z}(\varepsilon)}{C_Y(\varepsilon)} = \frac{C_{X,Y}(\varepsilon)}{C_Y(\varepsilon)} \frac{C_{Y,Z}(\varepsilon)}{C_Y(\varepsilon)} \tag{35}$$

Due to this estimation, Himnestra and Jones Test result incurs severe size distortion problem often. To overcome this problem, Diks and Panchenko added a conditional dependence measure by introducing a local weighting function *g(x,y,z)* and modified (34) as

$$H_0 : q = E\left[\left(\frac{f_{X,Y,Z}(x,y,z)}{f_Y(y)} - \frac{f_{X,Y}(x,y)}{f_Y(y)} \frac{f_{Y,Z}(y,z)}{f_Y(y)}\right) g_{X,Y,Z}(x,y,z)\right] = 0 \tag{36}$$

$g(X,Y,Z)$ is not unique. Using $g_{X,Y,Z}(\text{x, y, z}) = f_Y^{\,2}(y)$, as it is normally distributed for the corresponding estimator and asymptotic distribution of the test statistics is obtained, (36) becomes

$$H_o : \text{q} = \text{E}\left[f_{X,Y,Z}(x,y,z) f_Y(y) - f_{X,Y}(x,y) f_{Y,Z}(y,z) \right] = 0 \tag{37}$$

An estimator of *q* based on indicator function is

$$T_n(\varepsilon) = \frac{(2\varepsilon)^{-d_x - 2d_y - d_z}}{n(n-1)(n-2)} \sum_i \left[\sum_{k,k\neq i} \sum_{\text{j,j}\neq i} \left(I_{ik}^{XYZ} I_{ij}^{Y} - I_{ik}^{XY} I_{ij}^{YZ} \right) \right] \tag{38}$$

If we denote local density estimators of a d_W-variate random variable W at W_i by $f_W(W_i) = \frac{(2\varepsilon)^{-d_W}}{n-1} \sum_{j,\text{j}\neq i} I_{ij}^{W}$, $T_n(\varepsilon)$ is altered as

$$T_n(\varepsilon) = \frac{(n-1)}{n(n-2)} \sum_i (f_{X,Y,Z}(X_i, Y_i, Z_i) f_Y(Y_i) - f_{X,Y}(X_i, Y_i) f_{Y,Z}(Y_i, \text{Z}_i)) \tag{39}$$

For a sequence of bandwidths $\varepsilon_n = Cn^{-\beta}, C > 0$ and $\beta \in \left(\frac{1}{4}, \frac{1}{3}\right)$ and under suitable

mixing conditions [53], considering the covariances between the local density estimators, under H_0

$$\sqrt{n}\, \frac{T_n(\varepsilon_n) - \text{q}}{S_n} \rightarrow N(0,1) \tag{40}$$

where S_n^2 is a consistent estimator of asymptotic variance of $T_n(\varepsilon_n)$.

H_o is rejected at preassigned significance level α if $\sqrt{n}\dfrac{T_n(\varepsilon_n)-\text{q}}{S_n} > z_{1-\alpha}$.

3. RESULT

3.1 ADF UNIT ROOT TEST RESULT

ADF Unit Root Test is applied on each individual return series of Decade-1 and Decade-2 to check the stationarity of data and the result is provided in Table 1, confirming that all the series exhibit stationary at 1% level of significance. Selected optimal lag length was minimum between Akaike Information Criterion (AIC), Schwartz Information Criterion (SIC), Hannan-Quinn Information Criterion and FPE Information Criterion.

Table 1: ADF Unit Root test result

Stock Exchange	Decade 1			Decade 2			
	Lag	*p*-value	*t*-stat value	Lag	*p*-value	*t*-stat value	Conclusion
IBOVESPA	12	0.00*	-11.33	28	0.00*	-7.94	Stationary
IPC	36	0.00*	-6.75	1	0.00*	-31.29	Stationary
JKSE	10	0.00*	-11.55	10	0.00*	-13.37	Stationary
RTSI	10	0.00*	-11.72	4	0.00*	-19.97	Stationary
SENSEX	10	0.00*	-12.01	10	0.00*	-12.38	Stationary
SSE	16	0.00*	-9.95	25	0.00*	-8.39	Stationary
XU100	29	0.00*	-9.57	17	0.00*	-9.77	Stationary
* denotes rejection of the hypothesis at the 0.01 level							

3.2 RUNS TEST RESULT

Next, runs test is performed to check the randomness of the stock market returns and produced in Table 2. Table 2 indicates that all the data are random in nature for both the decades.

Table 2: Runs test result

	IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
Decade-1	995 (0.84)	966 (0.27)	969 (0.33)	975 (0.48)	959 (0.16)	965 (0.25)	1000 (0.67)
Decade-2	994 (0.38)	961 (0.54)	980 (0.80)	960 (0.51)	955 (0.38)	967 (0.73)	961 (0.54)
* denotes rejection of the hypothesis of randomness at the 0.01 level							

3.3 NONLINEARITY TESTS

Nonlinear nature of the stock exchanges is tested by three tools; namely, BDS test, Tsay test and McLeod and Li test. BDS test is applied after removing linear dependence i.e. on residuals of AR models. The purpose of this is to make BDS test feasible to analyze nonlinearity. Confirmation of stationarity of residuals are obtained using ADF unit root test in Table 3 and nonlinearity test results are summarized in Table 4. For BDS test, result for different threshold values and difference embedding dimensions is given in Supplementary Material. All stock exchanges show existence of nonlinearity by all three techniques for both decades.

Table 3: ADF Unit Root test result on residual of AR model (with corresponding lag)

Stock Exchange	Decade 1			Decade 2			
	Lag	p-value	t-stat value	Lag	p-value	t-stat value	Conclusion
IBOVESPA	1	0.00*	-31.37	1	0.00*	-31.21	Stationary
IPC	1	0.00*	-31.24	1	0.00*	-31.92	Stationary
JKSE	1	0.00*	-31.51	1	0.00*	-31.15	Stationary
RTSI	1	0.00*	-31.37	1	0.00*	-31.13	Stationary
SENSEX	1	0.00*	-31.41	1	0.00*	-31.19	Stationary
SSE	1	0.00*	-31.45	1	0.00*	-31.17	Stationary
XU100	1	0.00*	-31.36	1	0.00*	-31.27	Stationary
* denotes rejection of the hypothesis at the 0.01 level							

Table 4: Nolinear Analysis test results

Type of test	Decade	Stock Exchange Test statistic (p value)						
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
Tsay	Decade-1	4.24 (0.00)*	2.13 (0.00) *	4.76 (0.00) *	8.73 (0.00) *	4.60 (0.00) *	1.78 (0.00) *	2.42 (0.00) *
	Decade-2	2.23 (0.00) *	8.48 (0.00) *	4.90 (0.00) *	17.29 (0.00) *	8.59 (0.00) *	3.38 (0.00) *	1.83 (0.00) *
BDS	Decade-1	*p*-value= 0.00*	*p*-value= 0.00*	*P* value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*
	Decade-2	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*
McLeod and Li test	Decade-1	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*
	Decade-2	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*	*p*-value= 0.00*
* denotes rejection of the hypothesis of linearity at the 0.01 level								

3.4 VOLATILITY TEST

As nonlinearity is detected in all the stock markets, volatility cluster may be present in them. EGARCH (1, 1) and TGARCH (1, 1) models are fitted to the markets to detect conditional variance as well as leverage effect (if exists). The result, stated in Table 5 points out that all the markets are volatile with the presence of leverage effect in both Decade-1 and Decade-2. Sum of $\alpha+\beta$, i.e. combined effect of ARCH and GARCH is more than 1 in EGARCH model and near to 1 in TGARCH model for all the markets in both time frames interpreting strong combined existence of ARCH and GARCH effect. Negative γ in EGARCH model and positive γ in TGARCH model confirms the leverage effect in all exchanges for both periods. So, E7 markets are more sensitive to negative information compared to positive news.

Table 5a: EGARCH (1, 1) test results

	Decade	Test statistic						
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
ω (constant)	Decade-1	-0.27	-0.36	-1.07	-0.40	-0.35	-0.40	-0.28
	Decade-2	-0.49	-0.33	-0.44	-0.26	-0.30	-0.25	-0.55
	Decade-1	0.19	0.22	0.37	0.29	0.26	0.18	0.20

α (ARCH effect)	Decade-2	0.22	0.17	0.26	0.18	0.17	0.19	0.24
β (GARCH effect)	Decade-1	0.98	0.98	0.91	0.98	0.98	0.97	0.98
	Decade-2	0.96	0.98	0.97	0.98	0.98	0.99	0.96
$\alpha+\beta$	Decade-1	1.17	1.20	1.28	1.27	1.24	1.15	1.18
	Decade-2	1.18	1.15	1.23	1.16	1.15	1.18	1.20
γ (leverage effect)	Decade-1	-0.09	-0.09	-0.12	-0.08	-0.11	-0.04	-0.05
	Decade-2	-0.06	-0.08	-0.11	-0.07	-0.11	-0.06	-0.09

p value=0.00 for all test statistics denotes rejection of the hypothesis of nonvolatility at the 0.01 level

Table 5b: TGARCH (1, 1) test results

	Decade	Test statistic						
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
ω (constant)	Decade-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Decade-2	0.00	0.00	0.00	0.00	0.00	0.00	0.00
α (ARCH effect)	Decade-1	0.02	0.02	0.09	0.08	0.05	0.06	0.07
	Decade-2	0.07	0.03	0.07	0.03	0.01	0.04	0.06
β (GARCH effect)	Decade-1	0.90	0.90	0.71	0.84	0.85	0.89	0.89
	Decade-2	0.83	0.89	0.83	0.89	0.89	0.91	0.80
$\alpha+\beta$	Decade-1	0.92	0.92	0.80	0.92	0.90	0.95	0.96
	Decade-2	0.90	0.92	0.90	0.92	0.90	0.95	0.86
γ (leverage effect)	Decade-1	0.14	0.15	0.25	0.17	0.21	0.04	0.06
	Decade-2	0.10	0.12	0.17	013	0.18	0.10	0.18

p value=0.00 for all test statistics denotes rejection of the hypothesis of nonvolatility at the 0.01 level

3.5 CHAOS TEST

Another important feature of some nonlinear time series is existence of chaos. Chaotic nature of E7 markets is studied using 3 techniques, namely, 0-1 chaos test, Lyapunov test and correlation dimension test. Table 6a describes 0-1 chaos test and Lyapunov test. 0-1 chaos test yields binary output close to 1 for all the time series inferring the possibility of chaotic component in all the series or all the markets may have lots of noise. But, negative Largest Lyapunov exponent for all markets confirms that E7 market is not chaotic and correlation dimension test supports this. Table 6b briefs correlation dimension for different embedding dimension. Though CD increases whenever *m* increases, CD is not equal to *m*, so markets are not purely stochastic. Again, CD does not stabilize gradually when *m* grows suggesting that there is not sufficient evidence of presence of chaos in the data. Hence E7 markets exhibit deterministic non-chaotic behaviour.

Table 6a: 0-1 Chaos test and Lyapunov test results

	Decade							
0-1 Chaos test value								
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
0-1 chaos test	Decade-1	0.998	0.998	0.998	0.996	0.997	0.997	0.996
	Decade-2	0.998	0.998	0.997	0.997	0.997	0.998	0.998
Largest Lyapunov Exponent (optimal embedding dimension *m*) Test Statistic (*p* value)								
Lyapunov test	Decade-1	-0.48 (*m*=6) Z=-81.21 (0.00)	-2.58 (*m*=1) Z=-235.97 (0.00)	-2.63 (*m*=1) Z=-204.18 (0.00)	-0.26 (*m*=8) Z=-64.85 (0.00)	-1.30 (*m*=2) Z=-240.74 (0.00)	-3.82 (*m*=1) Z=-634.46 (0.00)	-1.52 (*m*=2) Z=-226.33 (0.00)
	Decade-2	-0.80 (*m*=3) Z=-182.79 (0.00)	-3.41 (*m*=1) Z=-300.22 (0.00)	-3.64 (*m*=1) Z=-271.98 (0.00)	-3.00 (*m*=1) Z=-218.56 (0.00)	-0.31 (*m*=11) Z=-76.03 (0.00)	-3.60 (*m*=1) Z=-242.42 (0.00)	-1.53 (*m*=2) Z=-231.33 (0.00)

p value denotes rejection of the hypothesis of chaoticness at the 0.01 level

Table 6b: Correlation dimension results

Embedding dimension m	Decade	Correlation dimension						
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
1	Decade-1	0.27	0.13	0.15	0.29	0.19	0.21	0.34
	Decade-2	0.16	0.05	0.07	0.20	0.06	0.11	0.14
2	Decade-1	0.64	0.31	0.36	0.64	0.44	0.49	0.77
	Decade-2	0.39	0.14	0.16	0.47	0.14	0.25	0.36
3	Decade-1	1.08	0.52	0.61	1.03	0.71	0.82	1.24
	Decade-2	0.69	0.24	0.27	0.79	0.26	0.42	0.63
4	Decade-1	1.57	0.77	0.90	1.44	1.01	1.18	1.74
	Decade-2	1.04	0.37	0.39	1.14	0.40	0.60	0.94
5	Decade-1	2.10	1.04	1.23	1.89	1.32	1.55	2.27
	Decade-2	1.44	0.51	0.54	1.53	0.57	0.81	1.30
6	Decade-1	2.66	1.33	1.60	2.35	1.64	1.94	2.84
	Decade-2	1.89	0.67	0.70	1.94	0.76	1.03	1.70
7	Decade-1	3.24	1.63	1.99	2.82	1.98	2.34	3.42
	Decade-2	2.36	0.85	0.87	2.37	0.98	1.27	2.12
8	Decade-1	3.83	1.96	2.40	3.30	2.34	2.76	4.03
	Decade-2	2.85	1.05	1.06	2.82	1.21	1.51	2.57

9	Decade-1	4.42	2.30	2.85	3.80	2.71	3.17	4.67
	Decade-2	3.36	1.27	1.25	3.28	1.45	1.76	3.03
10	Decade-1	5.03	2.66	3.31	4.33	3.10	3.58	5.31
	Decade-2	3.90	1.49	1.47	3.75	1.70	2.02	3.53

Initial threshold=0.02, final threshold=0.1, increment=0.0001

3.6 LINEAR GRANGER CAUSALITY ANALYSIS

After analyzing important features of individual stock exchange of E7 group, we study linear causal relationship (unidirectional or bidirectional) between them by the means of Granger causality-Wald test. A Vector Autoregressive Model (VAR) is created for data using optimal lag, which is found to be 2 (for decade-1) and 1 (for decade-2), based on minimum of Akaike Information Criterion (AIC), Schwartz Information Criterion (SIC), Hannan-Quinn Information Criterion and Granger causality-Wald test is conducted on it. The outcome is produced in Table 7.

Table 7: Granger causality-Wald test based on VAR (2) model for return data (decade-1) and VAR (1) model for return data (decade-2)

Dependent stock exchange		Independent stock exchange χ^2 Statistic (p value)						
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
IBOVESPA	Decade-1	-	1.68 (0.44)	73.85 * (0.00)	37.40* (0.00)	7.84* (0.02)	7.40* (0.02)	19.38* (0.00)
	Decade-2	-	6.20* (0.01)	75.78* (0.00)	11.42* (0.00)	6.35* (0.01)	7.95* (0.00)	0.88 (0.35)
IPC	Decade-1	26.10* (0.00)	-	63.38* (0.00)	40.70* (0.00)	26.73* (0.00)	12.16* (0.00)	9.34* (0.01)
	Decade-2	7.90* (0.00)	-	90.92* (0.00)	3.87* (0.05)	12.81* (0.00)	2.90 (0.09)	0.75 (0.38)
JKSE	Decade-1	3.74 (0.15)	0.72 (0.70)	-	6.93* (0.03)	22.03* (0.00)	1.29 (0.53)	4.24 (0.12)
	Decade-2	1.14 (0.28)	0.00 (0.96)	-	0.29 (0.59)	0.27 (0.60)	0.44 (0.50)	0.97 (0.32)
RTSI	Decade-1	11.85* (0.00)	4.25 (0.12)	22.65* (0.00)	-	1.71 (0.43)	3.83 (0.15)	3.79 (0.15)
	Decade-2	5.93* (0.01)	1.65 (0.20)	65.02* (0.00)	-	1.41 (0.23)	1.87 (0.17)	0.05 (0.82)
SENSEX	Decade-1	3.43 (0.18)	0.24 (0.89)	6.88 (0.03)	15.04* (0.00)	-	2.14 (0.34)	5.69 (0.06)
	Decade-2	2.88 (0.09)	0.15 (0.70)	20.52* (0.00)	5.75* (0.02)	-	3.17 (0.07)	0.81 (0.37)

SSE	Decade-1	2.85 (0.24)	3.52 (0.17)	1.42 (0.49)	3.44 (0.18)	0.79 (0.69)	-	0.03 (0.98)
	Decade-2	0.54 (0.46)	0.03 (0.86)	0.00 (0.95)	0.29 (0.59)	3.86* (0.05)	-	0.04 (0.85)
XU100	Decade-1	14.38* (0.00)	7.46* (0.02)	8.20* (0.02)	5.20 (0.07)	2.36 (0.30)	6.28* (0.04)	-
	Decade-2	2.39 (0.12)	0.50 (0.48)	51.14* (0.00)	0.08 (0.77)	0.15 (0.70)	3.16 (0.07)	-

* denotes rejection of null hypothesis of linear Granger causality at 5% level of significance

It is evident from Table 7 that, there are 20 linear causal relationships (out of possible 42) between E7 markets in Decade-1. IPC, caused by all other markets, behaves as endogenous market. Moreover, IPC does not influence any other market except XU100. SSE, not being affected by any other market, is established as exogenous stock market in Decade-1. Bidirectional linear causality is observed between IBOVESPA $\leftrightarrow$ RTSI, IBOVESPA $\leftrightarrow$ XU100, IPC $\leftrightarrow$ XU100 and JKSE $\leftrightarrow$ RTSI whereas unidirectional linear causality is evident from JKSE $\rightarrow$ IBOVESPA, SENSEX $\rightarrow$ IBOVESPA, SSE $\rightarrow$ IBOVESPA, IBOVESPA $\rightarrow$ IPC, JKSE $\rightarrow$ IPC, RTSI $\rightarrow$ IPC, SENSEX $\rightarrow$ IPC, SSE $\rightarrow$ IPC, SENSEX $\rightarrow$ JKSE, RTSI $\rightarrow$ SENSEX, JKSE $\rightarrow$ XU100 and SSE $\rightarrow$ XU100.

Notably, number of linear causal relationship decreases in Decade-2 (15 linear causal relationship) indicating possible shifting to nonlinear causal association. IBOVESPA emerges as endogenous market in Decade-2 as it is linearly impacted by all other than XU100. XU100 does not have any influence on rest of the exchanges. JKSE is observed as exogenous market in decade-2. In this time frame, bidirectional linear causality occurs between IBOVESPA $\leftrightarrow$ IPC and IBOVESPA $\leftrightarrow$ RTSI. Unidirectional linear causality is prominent from JKSE $\rightarrow$ IBOVESPA, SENSEX $\rightarrow$ IBOVESPA, SSE $\rightarrow$ IBOVESPA, JKSE $\rightarrow$ IPC, RTSI $\rightarrow$ IPC, SENSEX $\rightarrow$ IPC, JKSE $\rightarrow$ RTSI, JKSE $\rightarrow$ SENSEX, RTSI $\rightarrow$ SENSEX, SENSEX $\rightarrow$ SSE and JKSE $\rightarrow$ XU100.

3.7 NONLINEAR GRANGER CAUSALITY ANALYSIS: DP TEST

Finally, as all the markets are nonlinear, we investigate nonlinear causal relationship using Diks-Panchenko test for nonlinear pairwise bivariate Granger causality analysis. Embedding dimension is taken as lag length increased by 1 where lag length is obtained using the same procedure described in 2.6. The choice of β, in the bandwidth $\varepsilon = Cn^{-\beta}$ is considered as $-\frac{2}{7}$, as mean squared error of the estimator is asymptotically least for this selection. Covariance between conditional concentrations for a bivariate series arises mainly to volatility and to estimate the Autoregressive Heteroskedasticity model, C is considered as 8. The result is summarized in Table 8.

Table 8: DP test result for return data

Dependent stock exchange		Independent stock exchange χ^2 Statistic (p value)						
		IBOVESPA	IPC	JKSE	RTSI	SENSEX	SSE	XU100
IBOVESPA	Decade-1	-	1.40 (0.08)	0.83 (0.20)	0.17 (0.43)	0.57 (0.28)	0.16 (0.56)	2.91* (0.00)
	Decade-2	-	0.92 (0.18)	-0.40 (0.65)	1.78* (0.04)	0.12 (0.45)	-0.47 (0.68)	0.70 (0.24)
IPC	Decade-1	0.92 (0.18) Lag=10	-	-0.22 (0.59)	0.67 (0.25)	1.35 (0.09)	-1.25 (0.89)	0.55 (0.71)
	Decade-2	1.91* (0.03) Lag=6	-	3.94* (0.00)	1.66* (0.05)	0.67 (0.25)	1.09 (0.14)	1.57 (0.06)
JKSE	Decade-1	0.41 (0.65) Lag=12	0.16 (0.44) Lag=10	-	0.41 (0.64)	0.44 (0.33)	-0.46 (0.67)	1.02 (0.15)
	Decade-2	-0.27 (0.60) Lag=15	2.89* (0.00) Lag=1	-	1.79* (0.04)	1.03 (0.15)	1.10 (0.14)	0.41 (0.34)
RTSI	Decade-1	1.17 (0.12) Lag=12	0.41 (0.34) Lag=12	-0.30 (0.62) Lag=10	-	0.52 (0.30)	-1.11 (0.87)	-0.29 (0.61)
	Decade-2	2.24* (0.01) Lag=6	2.11* (0.02) Lag=3	1.99* (0.02) Lag=4	-	-0.39 (0.65)	0.77 (0.22)	-0.01 (0.50)
SENSEX	Decade-1	0.93 (0.17) Lag=10	0.74 (0.23) Lag=10	0.54 (0.29) Lag=10	1.05 (0.15) Lag=10	-	0.99 (0.16)	0.91 (0.18)
	Decade-2	0.06 (0.52) Lag=8	0.17 (0.43) Lag=8	0.15 (0.44) Lag=13	-1.00 (0.84) Lag=10	-	1.24 (0.11)	-0.57 (0.71)
SSE	Decade-1	0.91 (0.82) Lag=12	0.28 (0.39) Lag=10	0.40 (0.34) Lag=11	-1.17 (0.88) Lag=10	0.89 (0.19) Lag=2	-	1.27 (0.10) Lag=10
	Decade-2	0.61 (0.27) Lag=13	0.93 (0.18) Lag=7	0.81 (0.21) Lag=17	0.39 (0.35) Lag=17	0.37 (0.36) Lag=12	-	-0.09 (0.53)
XU100	Decade-1	1.70 (0.44) Lag=4	-0.38 (0.65) Lag=36	0.75 (0.23) Lag=10	0.89 (0.19) Lag=10	1.27 (0.10) Lag=10	1.20 (0.11) Lag=1	-
	Decade-2	0.19 (0.42) Lag=9	1.80* (0.04) Lag=0	1.73 (0.08) Lag=11	0.56 (0.28) Lag=9	0.41 (0.66) Lag=11	0.53 (0.30) Lag=17	-

* denotes rejection of null hypothesis of nonlinear Granger causality at 5% level of significance

It is evident from Table 8 that, in 2001-2010, interestingly, there is only 1 nonlinear Granger causality impact; from XU100 $\rightarrow$ IBOVESPA. But, in 2011-2020, number of nonlinear causal relationship significantly increases to 10 which clearly emphasises nonlinear degree of association in later half of the period of the study. In this decade, Bidirectional nonlinear Granger causality is identified between IBOVESPA $\leftrightarrow$ RTSI, IPC $\leftrightarrow$ JKSE, IPC $\leftrightarrow$ RTSI and JKSE $\leftrightarrow$ RTSI. In addition, unidirectional Granger causality is detected from IBOVESPA $\rightarrow$ IPC and IPC $\rightarrow$ XU100.

4. DISCUSSION AND CONCLUSSION

The study examines and compares some important underlying characteristics of prime stock exchanges belonging to E7 group, in 2001-2010 and 2011-2020. Return series of all the exchanges, as well as the residual series after fitting AR model are found to be stationary in both the decades. But all of them shows random behaviour which makes the prediction elusive. All the three techniques, which are used to detect nonlinearity, confirm nonlinear behaviour for all the markets. Volatility is prominent always in each market with the presence of leverage effect. So, bad or negative information causes more panic to the investors and makes more fluctuation in E7 markets compared to good or positive information. Though E7 markets are nonlinear and 0-1 test enhances the chance of presence of chaos, chaotic nature is not supported by Lyapunov test and correlation dimension. Hence, cause of detection of chaos in 0-1 test may be due to noise in the data. In absence of chaos, reliable forecasting can be performed which is encouragement for stock predictors of E7 countries. Finding of the present study, in this regard, agrees with views of Hsieh [54] in the favour of autoregressive conditional heteroskedasticity compared to low dimensional chaos in stock market data. Other important aspect of our study is understanding both linear and nonlinear causal relationship between stock exchanges of E7. Evidently, linear Granger causal relationship decreases and switches towards nonlinear Granger causal relationship significantly in decade-2. It may be due to affinity of E7 markets towards nonlinearity and increased complexity as time goes on. IPC was endogenous market in Decade-1, but IBOVESPA is endogenous in Decade-2, interpreting increasing linear dependency of Brazil stock exchange in recent decade. According to the linear Granger causal analysis, JKSE emerges as exogenous stock exchange in Decade-2 compared to SSE in Decade-1. But nonlinear Granger causality in Decade-2 exhibits the nonlinear causal dependency of JKSE on RTSI and IPC although SSE is independent in this period. Hence, combining both linear and nonlinear Granger causality result, SSE remains the most independent market in Decade-2. Overall nonlinear association between E7 countries is much lesser till the end of 2020, which advises investors for possible diversification of investment, with Chinese stock market as a prime part of it.

In conclusion, our study is helpful for the stock market investors, predictors, brokers, analysts and other related people to understand some of the important features of leading countries with emerging economies. Nonlinearity and chaos analysis is crucial part to model and predict the market. Causal analysis is helpful for international investors to analyse possible diversification. Our study will be useful in this aspect.

Further research includes building more comprehensive framework for the analysis to understand the above-mentioned underlying behaviour of the markets more prominently. In addition, further research may be incorporated for other group of countries like G7, G20, BRICS etc.

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SUPPLEMENTARY MATERIAL

BDS Test (Dedcade-1)

```
data:  BOVESPA Residual

Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0096 0.0192 0.0288 0.0384

Standard Normal =
       [ 0.0096 ] [ 0.0192 ] [ 0.0288 ] [ 0.0384 ]
[ 2 ]      2.9868     3.7827     4.9118     5.5049
[ 3 ]      4.6106     5.4573     6.8633     8.2130
[ 4 ]      6.0511     7.1509     8.2894     9.5526
[ 5 ]      7.3623     8.5736     9.4871    10.5826
[ 6 ]      8.9392    10.0290    10.5310    11.3244
[ 7 ]     11.2253    11.5738    11.6262    12.0153
[ 8 ]     13.7461    13.2743    12.6641    12.6416
[ 9 ]     17.3119    15.1065    13.6210    13.1201
[ 10 ]    24.0649    17.0070    14.5566    13.5434

p-value =
       [ 0.0096 ] [ 0.0192 ] [ 0.0288 ] [ 0.0384 ]
[ 2 ]      0.0028      2e-04          0          0
[ 3 ]      0.0000      0e+00          0          0
[ 4 ]      0.0000      0e+00          0          0
[ 5 ]      0.0000      0e+00          0          0
[ 6 ]      0.0000      0e+00          0          0
[ 7 ]      0.0000      0e+00          0          0
[ 8 ]      0.0000      0e+00          0          0
[ 9 ]      0.0000      0e+00          0          0
[ 10 ]     0.0000      0e+00          0          0


data:  IPC Residual

Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0069 0.0138 0.0206 0.0275

Standard Normal =
       [ 0.0069 ] [ 0.0138 ] [ 0.0206 ] [ 0.0275 ]
[ 2 ]      6.7069     6.9693     7.0020     6.8809
[ 3 ]      9.0435     9.7430     9.9852     9.8223
[ 4 ]      9.9399    10.7813    11.1048    10.9415
[ 5 ]     11.3869    12.2950    12.5161    12.1722
[ 6 ]     12.8834    13.7496    13.7473    13.2415
[ 7 ]     15.0273    15.4097    14.9431    14.2809
[ 8 ]     18.0429    17.3078    16.0431    15.0705
[ 9 ]     20.8777    19.3121    16.9975    15.6231
[ 10 ]    23.9804    21.4832    17.8933    16.0784

p-value =
       [ 0.0069 ] [ 0.0138 ] [ 0.0206 ] [ 0.0275 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
```

```
[ 8 ]          0          0          0          0
[ 9 ]          0          0          0          0
[ 10 ]         0          0          0          0
```

data: JKSE Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0076 0.0153 0.0229 0.0305

Standard Normal =
       [ 0.0076 ] [ 0.0153 ] [ 0.0229 ] [ 0.0305 ]
[ 2 ]      7.2087     7.6607     7.9055     8.4230
[ 3 ]      9.3189    10.0014    10.2385    10.6213
[ 4 ]     10.6045    10.9652    11.1435    11.6790
[ 5 ]     11.7090    11.7685    11.6608    12.0139
[ 6 ]     13.1512    12.4172    11.9570    12.2076
[ 7 ]     14.1219    13.1782    12.4014    12.5915
[ 8 ]     15.8716    13.9721    12.6938    12.7554
[ 9 ]     17.6421    14.7451    12.9983    12.9130
[ 10 ]    21.9670    15.5198    13.3306    13.0833

p-value =
       [ 0.0076 ] [ 0.0153 ] [ 0.0229 ] [ 0.0305 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0
```

data: RTSI Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0114 0.0227 0.0341 0.0454

Standard Normal =
       [ 0.0114 ] [ 0.0227 ] [ 0.0341 ] [ 0.0454 ]
[ 2 ]      8.1870     9.7347    10.7955    11.7307
[ 3 ]     10.7027    12.4845    13.5962    14.1589
[ 4 ]     12.5730    14.5401    15.4940    15.5774
[ 5 ]     15.2138    16.3546    16.8068    16.3751
[ 6 ]     18.1581    18.0782    17.9321    17.0333
[ 7 ]     22.7223    20.4360    19.1596    17.6438
[ 8 ]     28.3458    22.7352    20.1814    18.0668
[ 9 ]     35.2270    25.3455    21.3023    18.4472
[ 10 ]    44.5811    28.3021    22.5039    18.8597

p-value =
       [ 0.0114 ] [ 0.0227 ] [ 0.0341 ] [ 0.0454 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
```

```
[ 8 ]          0           0           0           0
[ 9 ]          0           0           0           0
[ 10 ]         0           0           0           0
```

data: SENSEX Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0084 0.0169 0.0253 0.0337

Standard Normal =
       [ 0.0084 ] [ 0.0169 ] [ 0.0253 ] [ 0.0337 ]
[ 2 ]     10.4119    10.6180    11.2074    11.6929
[ 3 ]     13.8236    13.8567    14.0887    14.1520
[ 4 ]     16.0975    16.4595    16.3710    16.0868
[ 5 ]     18.3705    18.6706    17.9887    17.2050
[ 6 ]     22.4056    21.5129    19.8808    18.5363
[ 7 ]     27.2201    24.7088    21.7611    19.6896
[ 8 ]     33.1970    28.0592    23.4306    20.5529
[ 9 ]     40.8617    31.9200    25.1625    21.3782
[ 10 ]    53.7678    36.5801    27.0223    22.1269

p-value =
       [ 0.0084 ] [ 0.0169 ] [ 0.0253 ] [ 0.0337 ]
[ 2 ]          0           0           0           0
[ 3 ]          0           0           0           0
[ 4 ]          0           0           0           0
[ 5 ]          0           0           0           0
[ 6 ]          0           0           0           0
[ 7 ]          0           0           0           0
[ 8 ]          0           0           0           0
[ 9 ]          0           0           0           0
[ 10 ]         0           0           0           0
```

data: SSE Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0087 0.0173 0.0260 0.0346

Standard Normal =
       [ 0.0087 ] [ 0.0173 ] [ 0.026 ] [ 0.0346 ]
[ 2 ]      2.4666     3.3955    4.5023     4.5843
[ 3 ]      5.7448     6.4435    7.0507     6.4114
[ 4 ]      7.8215     8.2620    8.5489     7.6184
[ 5 ]      9.9001    10.1083   10.1488     9.0203
[ 6 ]     12.4219    12.1466   11.7243    10.2529
[ 7 ]     15.5559    14.2220   13.0525    11.1153
[ 8 ]     19.0404    16.4396   14.3170    11.8870
[ 9 ]     23.9361    19.1577   15.6771    12.6255
[ 10 ]    30.9260    22.5217   17.1184    13.2901

p-value =
       [ 0.0087 ] [ 0.0173 ] [ 0.026 ] [ 0.0346 ]
[ 2 ]      0.0136      7e-04         0          0
[ 3 ]      0.0000      0e+00         0          0
[ 4 ]      0.0000      0e+00         0          0
[ 5 ]      0.0000      0e+00         0          0
[ 6 ]      0.0000      0e+00         0          0
[ 7 ]      0.0000      0e+00         0          0
[ 8 ]      0.0000      0e+00         0          0
[ 9 ]      0.0000      0e+00         0          0
```

```
[ 10 ]     0.0000      0e+00          0           0
```

data: XU100 Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0114 0.0229 0.0343 0.0458

Standard Normal =
       [ 0.0114 ] [ 0.0229 ] [ 0.0343 ] [ 0.0458 ]
[ 2 ]      6.3585     7.0757     7.3058     7.7948
[ 3 ]      9.9533    10.4823    10.2874    10.3659
[ 4 ]     12.8601    13.1253    12.5305    12.1381
[ 5 ]     15.6368    15.3846    14.4206    13.5644
[ 6 ]     18.3609    17.2417    15.8153    14.5427
[ 7 ]     21.1552    18.9823    17.0563    15.3914
[ 8 ]     23.7642    20.7148    18.1653    16.0319
[ 9 ]     25.6278    22.6482    19.2901    16.6546
[ 10 ]    23.8781    24.9074    20.3672    17.1593

p-value =
       [ 0.0114 ] [ 0.0229 ] [ 0.0343 ] [ 0.0458 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0
```

BDS Test (Dedcade-2)

data: BOVESPA Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0078 0.0156 0.0234 0.0312

Standard Normal =
       [ 0.0078 ] [ 0.0156 ] [ 0.0234 ] [ 0.0312 ]
[ 2 ]      3.7999     5.7104     8.0979     9.9828
[ 3 ]      5.2345     6.8895     9.4227    11.5246
[ 4 ]      5.6767     7.4612    10.0934    12.2367
[ 5 ]      6.4182     7.9758    10.5091    12.4525
[ 6 ]      6.5100     8.4842    10.9520    12.7684
[ 7 ]      6.3873     9.3317    11.5229    13.0656
[ 8 ]      6.8586    10.1758    12.1545    13.4312
[ 9 ]      7.3861    11.1566    12.8540    13.7766
[ 10 ]     7.6863    12.2443    13.6472    14.2416

p-value =
       [ 0.0078 ] [ 0.0156 ] [ 0.0234 ] [ 0.0312 ]
[ 2 ]       1e-04          0          0          0
[ 3 ]       0e+00          0          0          0
[ 4 ]       0e+00          0          0          0
[ 5 ]       0e+00          0          0          0
[ 6 ]       0e+00          0          0          0
[ 7 ]       0e+00          0          0          0
[ 8 ]       0e+00          0          0          0
```

```
[ 9 ]      0e+00          0          0          0
[ 10 ]     0e+00          0          0          0

data:  IPC Residual

Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0049 0.0099 0.0148 0.0198

Standard Normal =
       [ 0.0049 ] [ 0.0099 ] [ 0.0148 ] [ 0.0198 ]
[ 2 ]      8.1188     8.6205     8.5873     8.4120
[ 3 ]     11.0585    11.8384    12.3225    12.5859
[ 4 ]     12.7401    13.6763    14.3693    14.9246
[ 5 ]     13.7271    14.5980    15.2754    15.8535
[ 6 ]     14.7620    15.4162    15.9356    16.5088
[ 7 ]     16.6727    16.4644    16.6334    17.0599
[ 8 ]     19.9407    17.6197    17.3476    17.5086
[ 9 ]     21.0718    18.9646    18.0720    17.9084
[ 10 ]    20.8389    20.8897    19.0144    18.3204

p-value =
       [ 0.0049 ] [ 0.0099 ] [ 0.0148 ] [ 0.0198 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0

data:  JKSE Residual

Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0054 0.0108 0.0162 0.0217

Standard Normal =
       [ 0.0054 ] [ 0.0108 ] [ 0.0162 ] [ 0.0217 ]
[ 2 ]      8.8229    10.5732    11.7522    11.5146
[ 3 ]     11.0589    12.5103    13.4311    13.6281
[ 4 ]     12.6613    13.8880    14.6966    15.0627
[ 5 ]     14.5413    15.1580    15.6389    15.8837
[ 6 ]     17.0400    16.6132    16.5205    16.6341
[ 7 ]     19.6219    18.1300    17.2901    17.1470
[ 8 ]     22.1121    20.0461    18.1380    17.5838
[ 9 ]     25.6193    22.2923    19.0140    17.9381
[ 10 ]    31.9239    25.0764    19.9545    18.3060

p-value =
       [ 0.0054 ] [ 0.0108 ] [ 0.0162 ] [ 0.0217 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0
```

```
data:  RTSI Residual

Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0091 0.0182 0.0273 0.0364

Standard Normal =
        [ 0.0091 ] [ 0.0182 ] [ 0.0273 ] [ 0.0364 ]
[ 2 ]      6.4942     7.2630     7.4294     7.6520
[ 3 ]      8.6920     9.7731    10.4240    10.9966
[ 4 ]     10.7101    11.7580    12.2424    12.5772
[ 5 ]     13.6117    13.5832    13.5078    13.5753
[ 6 ]     16.5000    15.2464    14.4008    14.1849
[ 7 ]     19.3233    17.0464    15.3683    14.8998
[ 8 ]     22.3575    18.7810    16.2084    15.4010
[ 9 ]     25.9441    20.8234    17.1399    15.8557
[ 10 ]    31.5481    23.2867    18.0543    16.1935

p-value =
        [ 0.0091 ] [ 0.0182 ] [ 0.0273 ] [ 0.0364 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0

data:  SENSEX Residual

Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0054 0.0108 0.0163 0.0217

Standard Normal =
        [ 0.0054 ] [ 0.0108 ] [ 0.0163 ] [ 0.0217 ]
[ 2 ]      4.2479     4.8343     5.9411     7.2880
[ 3 ]      6.6075     7.0566     7.6771     9.1615
[ 4 ]      8.5152     8.9549     9.2575    10.6135
[ 5 ]     10.0614    10.3826    10.2637    11.3531
[ 6 ]     11.3817    11.5696    11.1640    11.9294
[ 7 ]     12.9663    12.9157    12.0899    12.5692
[ 8 ]     15.5887    14.4838    12.9857    13.0946
[ 9 ]     18.6798    16.4043    13.9784    13.5778
[ 10 ]    22.0101    18.4996    14.9948    14.0251

p-value =
        [ 0.0054 ] [ 0.0108 ] [ 0.0163 ] [ 0.0217 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0

data:  SSE Residual
```

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0066 0.0132 0.0199 0.0265

Standard Normal =
       [ 0.0066 ] [ 0.0132 ] [ 0.0199 ] [ 0.0265 ]
[ 2 ]      4.8310     5.9426     6.9200     7.6058
[ 3 ]      7.7380     8.9206     9.9730    10.6443
[ 4 ]      9.6195    10.9350    11.7759    12.4005
[ 5 ]     11.4051    12.1453    12.7323    13.2466
[ 6 ]     14.8204    13.8861    13.8410    13.9398
[ 7 ]     18.3025    15.5629    14.6848    14.4067
[ 8 ]     22.9339    17.4562    15.5620    14.9233
[ 9 ]     29.7784    19.7988    16.5864    15.4195
[ 10 ]    39.6139    22.5189    17.6326    15.8830

p-value =
       [ 0.0066 ] [ 0.0132 ] [ 0.0199 ] [ 0.0265 ]
[ 2 ]           0          0          0          0
[ 3 ]           0          0          0          0
[ 4 ]           0          0          0          0
[ 5 ]           0          0          0          0
[ 6 ]           0          0          0          0
[ 7 ]           0          0          0          0
[ 8 ]           0          0          0          0
[ 9 ]           0          0          0          0
[ 10 ]          0          0          0          0
```

data: Xu100 Residual

```
Embedding dimension =   2  3  4  5  6  7  8  9 10

Epsilon for close points =  0.0071 0.0142 0.0214 0.0285

Standard Normal =
       [ 0.0071 ] [ 0.0142 ] [ 0.0214 ] [ 0.0285 ]
[ 2 ]      2.2126     2.8011     3.5190     4.1000
[ 3 ]      3.6439     4.5206     5.6135     6.5041
[ 4 ]      3.9977     5.3630     6.6975     7.6665
[ 5 ]      4.4657     6.0467     7.4406     8.3674
[ 6 ]      4.7070     6.6482     8.0689     8.9079
[ 7 ]      5.6754     7.5537     8.7474     9.3308
[ 8 ]      6.1277     8.4865     9.4101     9.7507
[ 9 ]      6.4307     9.5006    10.0104    10.0606
[ 10 ]     9.3598    10.2255    10.4966    10.2874

p-value =
       [ 0.0071 ] [ 0.0142 ] [ 0.0214 ] [ 0.0285 ]
[ 2 ]      0.0269     0.0051      4e-04          0
[ 3 ]      0.0003     0.0000      0e+00          0
[ 4 ]      0.0001     0.0000      0e+00          0
[ 5 ]      0.0000     0.0000      0e+00          0
[ 6 ]      0.0000     0.0000      0e+00          0
[ 7 ]      0.0000     0.0000      0e+00          0
[ 8 ]      0.0000     0.0000      0e+00          0
[ 9 ]      0.0000     0.0000      0e+00          0
[ 10 ]     0.0000     0.0000      0e+00          0
```