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Mean Time-To-Failure (MTTF) Analysis of A Cold Standby System Using Rayleigh Distribution

RK Bhardwaj
Department of Statistics, Punjabi University,
Patiala-147002, India
E-mail: rkb_mstates@rediffmail.com

Abstract

The cold standby systems are widely used when continued operation is required despite the failure of an operating unit. The time at which such a system fails depends on the lifetimes of the successive units placed in operation. This paper investigates the time-to-failure behaviour of a two-unit cold standby system in which the lifetimes of the primary and standby units are assumed to follow Rayleigh distributions. Under the assumptions of independent component lifetimes, perfect switching, and no repair, the system remains operational until both units have failed. The system lifetime is therefore formulated in terms of the successive lifetimes of the two units. Analytical expressions are derived for the probability density function, cumulative distribution function, reliability function, hazard rate, and mean time to failure of the system. The numerical results demonstrate that the MTTF decreases with increasing unit failure rate, while variations in switch, service-facility, and inspection parameters significantly influence the overall reliability performance of the proposed system. The model offers a simple analytical framework for studying standby reliability when the component failure rate increases with operating time and provides a basis for extending the analysis to more complex standby configurations and maintenance policies.

Keywords: *Cold Standby, Probabilistic Model, Time-to-Failure, Rayleigh Distribution.*

1. Introduction

The length of time for which a system can continue to operate before failure is one of the central concerns in reliability analysis. In many real-life systems, failure of an operating unit does not necessarily mean that the entire system must stop. A reserve unit may be kept available and brought into service when the operating unit fails. Such an arrangement is especially useful when continuity of operation is important. A cold standby system is one of the simplest forms of such a reserve arrangement. It consists of an operating unit together with one or more units that remain inactive until they are required. In a two-unit cold standby system, the first unit performs the required function while the second unit remains in reserve. When the operating unit fails, the standby unit is activated and takes over its function. If the switching mechanism is assumed to work perfectly and no repair is carried out, the system remains operational for the lifetime of the first unit followed by the lifetime of the standby unit. Consequently, the time at which the entire system fails is determined by the combined lifetimes of the two units. This feature makes cold standby systems interesting from a probabilistic point of view. The reliability of the complete system cannot be described simply by considering the lifetime of either unit separately. Instead, the lifetime distribution of the system must account for the successive operation of the two units. The choice of the component lifetime distribution therefore has a direct influence on the resulting time-to-failure characteristics of the system.

Among the various lifetime distributions used in reliability studies, the Rayleigh distribution provides a useful model when the likelihood of failure increases with time. Its increasing hazard rate can represent situations in which an operating component becomes progressively more vulnerable to failure as it ages. At the same time, the mathematical form of the distribution is sufficiently simple to permit explicit derivations of several reliability measures. These properties make the Rayleigh distribution a natural choice for examining the effect of an increasing failure tendency in a standby system. The use of a Rayleigh lifetime model also provides an alternative to the commonly used exponential assumption. Under an exponential model, the failure rate remains constant throughout the operating period, whereas the Rayleigh model allows the failure rate to vary with time. This distinction is important when the failure behaviour of a component is influenced by ageing. For a cold standby system, it is therefore worthwhile to examine how this time-dependent failure behaviour carries over from individual units to the lifetime of the system as a whole.

In this work, a two-unit cold standby system is considered, with the lifetimes of the operating and standby units described by Rayleigh distributions. The time to failure of the system is formulated in terms of the successive lifetimes of these units. From this formulation, the principal characteristics of the system lifetime are obtained, including its probability density function, cumulative distribution function and mean time to failure. The analysis provides a direct way of examining how the Rayleigh failure behaviour of individual units is reflected in the overall lifetime of a cold standby system.

The model considered here is deliberately simple, but it provides a useful starting point for reliability analysis under an increasing failure-rate assumption. The same framework can subsequently be extended to systems with more standby units, imperfect switching, dependent component lifetimes, repair facilities, or other lifetime distributions. Thus, the analysis of the Rayleigh-based cold standby model offers both a specific time-to-failure result and a foundation for more general standby-system investigations.

2. Review Of Literature

The standby redundancy has been widely used in reliability modelling to improve the operational continuity of systems in which failure of an active unit may interrupt the required service. The literature on standby systems has gradually moved from relatively simple redundant configurations towards models incorporating repair, inspection, imperfect switching, maintenance, degradation, and more flexible failure-time assumptions. The studies considered in this section reflect this development and provide the background for the time-to-failure analysis of a cold standby system.

El-Said and El-Sherbeny (2010) [1] considered a two-unit cold standby system with a two-stage repair process and an intervening waiting period. Using regenerative point-process techniques, the authors obtained expressions for time-dependent and steady-state availability, reliability, mean time to failure (MTTF), and profit. The results demonstrated that repair characteristics and waiting time can have a substantial effect on system performance. The study also shows that cold standby models can be extended beyond simple failure-only assumptions by incorporating realistic repair processes.

Lu et al. (2012) [2] examined a cold standby system from the perspective of periodic inspection. Rather than treating inspection just as an operational detail, the authors used the delay-time concept to determine an appropriate inspection interval. This work linked the inspection schedule with system reliability and maintenance decisions and demonstrated the importance of selecting inspection times appropriately in cold standby systems. Gupta et al. (2013) [3] studied a two-unit cold standby system with dissimilar units and Weibull failure and repair distributions. The model incorporated a single repair facility, priority in operation and repair, preventive maintenance, inspection, and replacement. System measures such as MTSF, availability, and profit were obtained using stochastic modelling techniques. The use of Weibull failure and repair laws is particularly relevant to the development of non-exponential standby models because it permits the failure behaviour of the units to vary with time rather than assuming a constant failure rate.

Mendes et al. (2017) [4] addressed the determination of an appropriate interval between periodic inspections for a two-component cold standby multistate system. The research considered the effects of reliability, maintenance costs, and production losses and used discrete-time Markov-chain modelling to represent transitions among system states. The study demonstrates that cold standby reliability analysis can be closely connected with maintenance scheduling and economic considerations.

In another study, Sadeghi and Roghanian (2017) [5] investigated a warm standby repairable system with two different forms of an imperfect switching mechanism. The active and standby units were allowed to have different operating roles, while the switching mechanism was not assumed to be perfectly reliable. Markov-process and Laplace-transform techniques were used to derive MTTF and steady-state availability. Although the study concerns a warm rather than a cold standby configuration, it is relevant because it highlights the influence of switching reliability on standby-system performance.

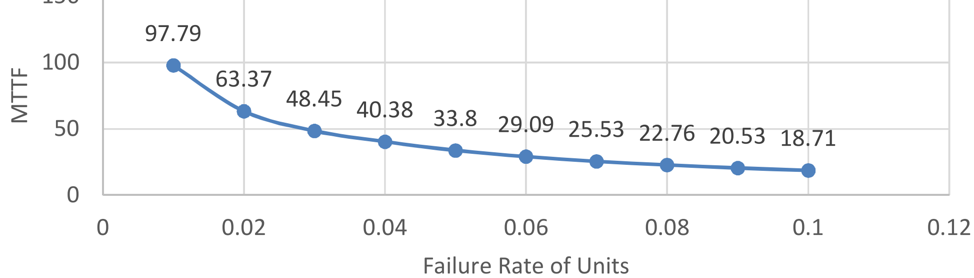
Wang et al. (2018) [6] considered redundancy optimization for cold standby systems under periodic inspection and preventive maintenance. Their model incorporated degradation of standby components during prolonged storage and jointly considered component selection, the number of redundant components, and the inspection interval. An optimization procedure was developed to maximize system reliability subject to resource constraints. The study is significant because it treats the standby unit as a component whose condition may change even before it is called into operation.

Hu et al. (2019) [7] extended redundant-system reliability analysis by considering imperfect switching together with uncertainty in lifetime-distribution parameters. Their framework covered both cold and warm redundant systems and considered different representations of switch lifetime. Reliability and MTTF were derived for the resulting models. This work emphasizes that the switching mechanism and uncertainty in lifetime parameters can both influence the estimated reliability of a redundant system.

Recently, the work of Behboudi et al. (2021) [8] further developed the analysis of two-unit cold standby systems by introducing a periodic switching policy based on the concept of virtual age. The proposed approach allows the active and standby units to be exchanged periodically rather than waiting exclusively for failure. The authors derived the survival function in terms of a general baseline time-to-failure distribution and also considered imperfect switching. Their results showed that, for increasing-failure-rate distributions, periodic switching can influence system performance. The study therefore provides an important connection between the shape of the component lifetime distribution and the operational policy adopted for a cold standby system.

Akhavan Niaki and Yaghoubi (2021) [9] considered another standby system with imperfect switching and non-repairable components. Closed-form expressions for system reliability and MTTF were derived using a Markov approach. The model assumed constant component failure rates while allowing the switch failure rate to increase with repeated use. Their work provides an analytical treatment of larger cold standby configurations and explicitly demonstrates the effect of imperfect switching on reliability and MTTF. The existing research on cold standby systems has largely focused on repair, inspection, maintenance, degradation, redundancy optimization, and steady-state performance. Although non-exponential lifetime distributions have also been considered, comparatively limited attention has been given to the direct analysis of the time-to-failure behaviour of a basic two-unit cold standby system with Rayleigh-distributed component lifetimes**.** This study addresses this gap by developing an analytical framework for the system lifetime and deriving its principal reliability characteristic under the Rayleigh framework.

Figure 1: MTTF behaviour with Failure rate of Unit



3. Notations

Notation	Description
$d1(\mathrm{t}) = 2\lambda t e^{-\lambda t^2}$	: Failure time distribution of Unit
$d2(\mathrm{t}) = 2\alpha t e^{-\alpha t^2}$	: Repair time distribution of Unit
$d3(\mathrm{t}) = 2\mu t e^{-\mu t^2}$	: Failure time distribution of Switching Facility
$d4(\mathrm{t}) = 2\beta t e^{-\beta t^2}$	: Repair time distribution of Service Facility
$d5(\mathrm{t}) = 2\gamma t e^{-\gamma t^2}$	: Repair time distribution of Switching Facility
$d6(\mathrm{t}) = 2\xi t e^{-\xi t^2}$	: Inspection time distribution of Switching Facility
$d7(\mathrm{t}) = 2\theta t e^{-\theta t^2}$	: Inspection time distribution of Service Facility
$\psi i(t)$	: Cdf of first passage time from i^{th} regenerative state
a/b	: Chances of post-inspection repair or replacement of Unit
p/q	: Chances of Switching Facility be up or down
c/d	:Chances of post-inspection repair or replacement of Switching Facility
R/NR	: Regenerative/ Non Regenerative

4. Development Of System Model

4.1. System Description

The system under consideration consists of two identical units arranged in a cold standby configuration. System starts operation with one unit operative, while the other in standby. As operating unit fails, the switching facility attempts to bring the standby unit into operation.

The switching facility is assumed to be imperfect and may fail during the switching process. A repair facility is provided to carry out the required remedial activities. Following the failure of a unit or the switching facility, the failed component is inspected to determine whether it can be repaired or needs to be replaced. Repair requires a finite amount of time, whereas replacement is considered to be instantaneous. When both the unit and switching facility require remedial attention, priority is assigned to the switching facility. The repair facility is assumed to be subject to failure while engaged in repairing a failed unit or switching facility, but it does not fail while remaining in idle state. Whenever the server facility fails, it immediately undergoes remedial activity and becomes available again after completion of the remedial activity. The system remains operational whenever an operative unit can be successfully connected to the system through the switching facility.

Table 1: Description of possible current and future States of the System Model

CURRENT STATE	STATE TYPE	STATE DESCRIPTION	FUTURE STATES
1.	R	One unit Up, another Standby	2,6
2.	R	One Up, another under inspection	1,3,4,8
3.	R	One unit Up, another under repair	1,5,11
4.	R	One in inspection waiting, Service facility under remedy	12
5.	R	One in repair waiting, Service facility under remedy	10
6.	NR	One in repair waiting, Switching facility under inspection	7,14
7.	NR	All actions waiting, service facility under repair	13
8.	NR	All waiting for inspection activity	2,9,17
9.	NR	Inspection waiting, service facility undergoes repair	15, 17
10.	NR	Waiting continued, repair continued	15
11.	NR	One unit under repair, another waiting for inspection	7,13
12.	NR	Continued Service facility repair and unit wait for inspection	2, 16
13.	NR	System down, switching facility under inspection	2, 9, 17
14.	NR	Previous actions continued, Inspection	2,16
15.	NR	Continued Inspection, another waiting inspection	2,9,17
16.	NR	Inspections, Repairs continued waiting, service facility undergoes repair	14
17.	NR	Units continued waiting Inspection, repair	2,18
18.	NR	Inspection continued waiting, Repair waiting, service facility under repair	17

4.2. Transition Probabilities and Mean Sojourn Times

The system is represented by a finite set of states (1st-18th) as given in Table 1, where each state describes the operating, failed, repair, replacement, or treatment condition of the system components. The state transitions are governed by the failure, switching, repair, replacement, and server-treatment mechanisms defined for the system. Since the time spent in a state need not follow an exponential distribution, the system is formulated as a semi-Markov process. Let (S=S1, S2,………S18) denote the state space of the system. The process enters a particular state according to the occurrence of a failure or completion of a remedial activity and remains in that state for a random sojourn time before making a transition to another state. The transition probabilities and corresponding sojourn-time distributions are determined from the failure and repair characteristics of the units, switching mechanism, and repair server. The semi-Markov kernel of the model is given equation (1).

$$\mathrm{Qij}\ (\mathrm{t}) = \mathrm{P(System\ transits\ from\ Si\ to\ Sj\ with\ holding\ time}\ \leq \mathrm{t)} \quad (1)$$

The jump process observed immediately at the epochs of state transitions forms an embedded discrete-state continuous time Markov chain with transition probabilities:

$$Pij = Qij(\infty); i, j = 1, 2, \ldots .18 \qquad (2)$$

The mean sojourn time is defined as follows:

$$\mu i = \int_0^\infty P(T > t)dt; i = 1,2,3,4 \qquad (3)$$

5. Mean Time-To-Failure (Mttf) Analysis

To derive the expression for the Mean Time-to-Failure (MTTF), the failed states are treated as absorbing states, yielding the following set of recursive relations:

$$\begin{aligned}
&\psi 1(\mathrm{t}) = \mathrm{Q1,2(t)} * \psi 2(\mathrm{t}) + \mathrm{Q1,6(t)}\\
&\psi 2(\mathrm{t}) = \mathrm{Q2,1(t)} * \psi 1(\mathrm{t}) + \mathrm{Q2,3(t)} * \psi 3(\mathrm{t}) + \mathrm{Q2,4(t)} * \psi 4(\mathrm{t}) + \mathrm{Q2,8(t)}\\
&\psi 3(\mathrm{t}) = \mathrm{Q3,1(t)} * \psi 1(\mathrm{t}) + \mathrm{Q3,5(t)} * \psi 5(\mathrm{t}) + \mathrm{Q3,11(t)}\\
&\psi 4(\mathrm{t}) = \mathrm{Q4,2(t)} * \psi 2(\mathrm{t}) + \mathrm{Q4,12(t)}\\
&\psi 5(\mathrm{t}) = \mathrm{Q5,3(t)} * \psi 3(\mathrm{t}) + \mathrm{Q5,10(t)} \qquad (4)
\end{aligned}$$

$$MTTF\ (T) = \frac{\begin{array}{c}\mu 1(1 - p2,4.\,p4,2)(1 - p3,5.\,p5,3)\\ +p1,2(\mu 2 + p2,4.\,\mu 4) + p1,2.\,p2,3(\mu 3 + p3,4.\,\mu 5)\end{array}}{(1 - p2.4.\,p4,2 - p2,1.\,p1,2)(1 - p3,5.\,p5,3) - p2,1.\,p1,2.\,p2,3} \qquad (5)$$

6. Results And Discussion

Considering the data values for different parameters of Raleigh distribution, we obtained the results in graphical forms demonstrating the trends of MTTF of system model. Based on the numerical values shown in the figures 1-4, the results can be described more precisely as follows: Considering the parameter values of the Raleigh distribution, namely μ=0.4, γ=0.3, α=0.6, β=0.7,p/q=0.88/0.12, θ=0.5, a/b=0.7/0.3, ξ=0.6, and c/d=0.6/0.), the MTTF of the proposed system was evaluated for different values of the failure rate of the unit.

Figure 2: MTTF behaviour at Failure rate of Switch (μ=0.02)

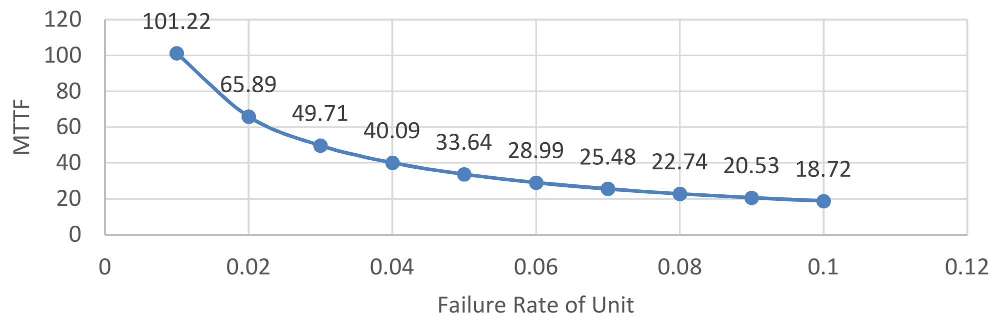


Figure 3: MTTF behaviour at Failure rate of Service Facility (β=0.99)

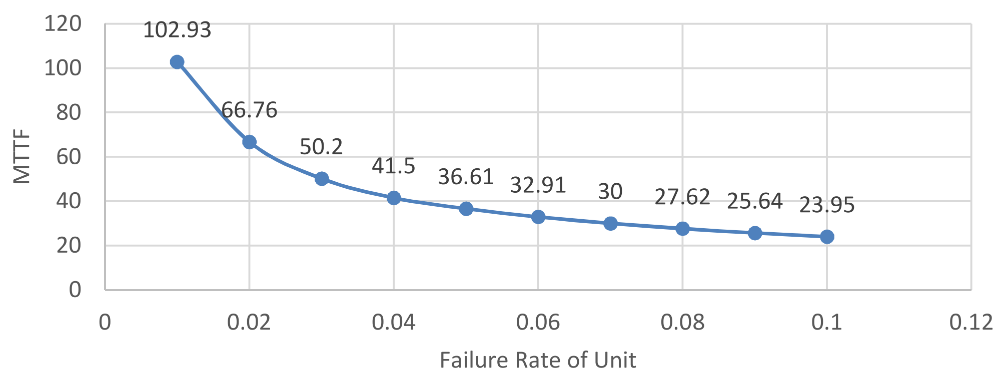
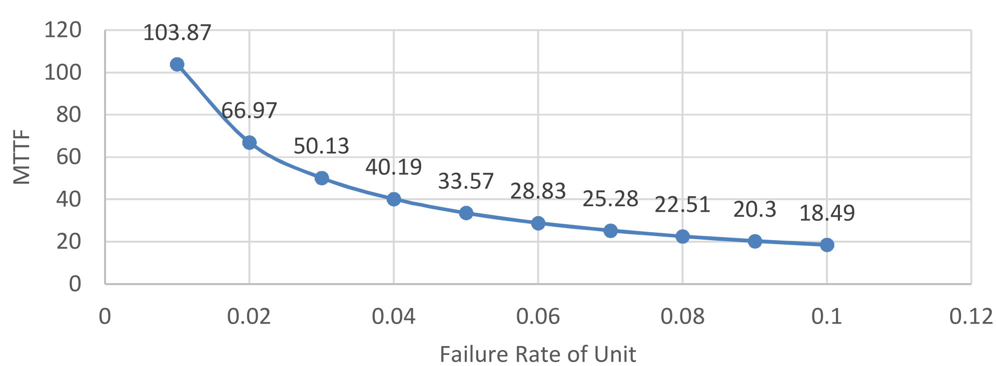


Figure 4: MTTF behaviour at Inspection rate of Service Facility (θ= 0.7)



The corresponding trends are presented in Figures 1–4. Figure 1 shows the variation of MTTF with the failure rate of the unit. A clear decreasing trend is observed. The MTTF decreases from 97.7 at a low failure rate to 18.71 at a failure rate of 0.10. The decline is relatively steep at lower failure rates and becomes progressively gradual as the failure rate increases. Thus, an increase in the unit failure rate adversely affects the expected operating lifetime of the system. Figure 2 presents the MTTF behaviour with respect to the unit failure rate when the failure rate of the switch is $\mu=0.02$. The MTTF decreases from 101.22 to 18.72 as the unit failure rate increases to 0.100.10. Compared with Figure 1, the MTTF values are slightly higher over most of the considered range. The reduction is again more pronounced at lower failure rates, followed by a relatively flatter trend at higher failure rates. This indicates that the system MTTF remains strongly dependent on the failure rate of the operating unit even when the switch failure parameter is varied. Figure 3 depicts the variation of MTTF with the unit failure rate for $\beta=0.99$, representing the specified service-facility failure-rate condition. The MTTF decreases from 102.93 at the lower unit failure rate to 23.95 at a unit failure rate of 0.10. The curve follows the same general decreasing pattern, with a relatively sharp reduction initially and a gradual decline thereafter. The higher MTTF values compared with Figure 1 indicate that the selected value of β has a noticeable effect on system performance. Figure 4 illustrates the MTTF behaviour for an inspection rate of the service facility, $\theta=0.7$. The MTTF decreases from 103.87 to 18.49 as the unit failure rate increases to 0.10. The figure shows that increasing the unit failure rate consistently reduces the expected lifetime of the system. The initial portion of the curve exhibits a sharper decline, whereas the rate of reduction becomes smaller at higher failure rates. The plots, in figures 1-4, consistently demonstrate an inverse relationship between the unit failure rate and MTTF. In all cases, the MTTF decreases substantially with an increase in the unit failure rate. The reduction is particularly pronounced in the lower failure-rate region, while the curves tend to flatten as the failure rate approaches 0.10. The comparison also shows that changes in the switch failure rate, service-facility failure parameter, and inspection rate modify the magnitude of MTTF, although the fundamental decreasing trend remains unchanged. These results confirm the sensitivity of the system's expected lifetime to the failure behaviour of its operating unit.

Statements and Declarations

Data availability statement

All data supporting the research is found within this research paper.

Conflicts of interest

The author declares no conflicts of interest.

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