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Proof of the Collatz Conjecture

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Abstract

The Collatz conjecture proposed in 1937 by German mathematician Lothar Collatz remains unsolved. This paper sets out to prove the Collatz conjecture by exploring the function of n, of the conjecture and applying this function of n, to a system of two linear equations. From the function of n, in the Collatz conjecture where $f(\mathrm{n})$ = 3n + 1, for odd positive integers and $f(\mathrm{n})$ = n / 2, for even positive integers, once any positive integer other than two has converged to four, the conjecture tells us that any positive integer will converge to the number two which will inevitably converge to the number one.

Keywords: Collatz conjecture, 3n + 1 conjecture, Syracuse problem.

MSC: 11-xx

1. Introduction

The Collatz conjecture proposed in 1937 by Lothar Collatz states that any positive integer n will converge to the number one by applying the function of n in the conjecture where, $f(\mathrm{n})$ = 3n + 1, for odd positive integers, and $f(\mathrm{n})$ = n / 2, for even positive integers. That is if the positive integer is even the next term will be half of the previous term and if the positive integer is odd the next term will be three times the previous term plus one [1] [3] [4]. The following is the Collatz conjecture [1].

Let ℕ be the set of all positive integers and n an element of (n ϵ ℕ).

The following map defined by Collatz:

$$f(\mathrm{n}) = \begin{cases} n/2, & if\ \mathrm{n}\ is\ even, \\ 3\mathrm{n} + 1, & if\ \mathrm{n}\ is\ odd. \end{cases} \qquad (1)$$

From the map defined by Collatz it can be determined that every positive integer will be subject to the function of n, of the conjecture. Therefore it is envisaged that the conjecture is acting on every positive integer as part of a system of two equations [5]. The conjectures function of n, acting on any given integer demonstrates that a system of two linear equations needs to be used to prove the conjecture. For any positive integer n, converging to one implies that n + y will equal two due to the $f(\mathrm{n})$ = n / 2, of the conjecture. For any positive integer n, being subject to the conjecture, $f(\mathrm{n})$ = 3n + 1, and, $f(\mathrm{n})$ = n / 2, will converge to one via the four, two, one loop and therefore must converge to four first. By using a system of two linear equations and mathematical induction it is proven that any positive integer will converge to four then two and ultimately converge to one.

2. Method

The following system of two linear equations is created from the function of n, in the Collatz conjecture and the proposition that every positive integer n, will converge to one via the four, two, one loop.

Creating a system of two linear equations from the function of n, $f(\mathrm{n})$ = 3n + 1, and, $f(\mathrm{n})$ = n / 2, in the Collatz conjecture.

Finding equation 2 from the $f(\mathrm{n})$ = n / 2.

(∀n ϵ ℕ)

n / 2 + y / 2 = 1

n / 2 = 1 – y / 2

n = 2(1 – y / 2)

n = 2 – 2y / 2

n = 2 – 2y / 2

n = 2 – y

n + y = 2

Equation 2: n + y = 2 (2)

Finding equation 3 from $f(\mathrm{n})$ = 3n + 1, where y is equivalent to the integer being questioned, and in this first case, equivalent to one where the conjecture converges to four before converging to one.

Equation 3: 3n + y = 4 (3)

Let ℕ be the set of all positive integers and (n ϵ ℕ). For all positive integers n, there exists a y element of ℕ, where n = 2 – y.

(∀n ϵ ℕ) (∃y ϵ ℕ) (n = 2 – y).

From equation 2:

$n + y = 2$

$n = 2 – y$

Substituting equation 2 into equation 3:

Let ℕ be the set of all positive integers and (n ϵ ℕ). For all positive integers n, there exists a y element of ℕ, where 3n + y = 4.

$(\forall n \in \mathbb{N})\ (\exists y \in \mathbb{N})\ (3n + y = 4).$

$3n + y = 4$

$3(2 – y) + y = 4$

$6 – 3y + y = 4$

$6 – 2y = 4$

$– 2y = 4 – 6$

$– 2y = – 2$

$y = – 2 / –2$

$y = 1.$

Using Mathematical Induction to prove P(n) is true for all positive integers n = 1, 2,…

$$\left.\begin{array}{r} P(1)\ is\ true \\ (\forall n \in \mathbb{N})[P(n)\ true \implies P(n + y + 2)\ \text{true} \end{array}\right\} (\forall n \in \mathbb{N})\ P(n) \text{ is true.} \qquad [2].$$

Proof by Induction.

From equation 2 where y equals one:

Basic proposition states: P(n): P(n + y + 2) is even.

Base step: P(1): (n + y) = 1 + 1 = 2, where 2 is even.

P(1) ⟹ 2 where 2 is even.

Induction step:

P(n): P(n + y + 2) is even.

Through the addition of two, P(n + y + 2) ⟹ n even, as the addition of two to an even number will result in an even number.

∴

From equation 2 where y equals one:

$n + y = 2$

P(n + y + 2) ⟹ n even

n + y + 2 ⟹ n even

1 + 1 + 2 = 4,

Where four is even.

P(n) ⟹ P(n + y + 2).

As the function of n, $f(n) = 3n + 1$, and, $f(n) = n / 2$, applies to all positive integers in the Collatz conjecture, the second equation in the system of two linear equations is proven as follows.

From equation 3, where y equals one:

The basic proposition states, P(n): (3n + y) is even.

Base step: P(1): (3(1) + y) = 4 where 4 is even.

P(n): 3n + y = 4

P(1): 3(1) + 1 = 4

= 3 + 1 = 4 where 4 is even.

Therefore through the addition of two, P(n + y + 2) ⟹ n even, as the addition of two to an even number will result in an even number.

From equation 3, where y equals one and substituting equation 2 into equation 3:

Induction step: P(n) ⟹ P(n + y + 2).

P(n + y + 2) ⟹ n even.

3n + y = 4

Substituting equation 2 into equation 3 and adding two.

3(2 – y) + y + 2 = 4

6 – 3y + y + 2 = 4

8 – 2y = 4

– 2y = 4 – 8

– 2y = – 4

y = – 4 / – 2

y = 2 where 2 is even.

y ≡ n

P(n + y + 2) ⟹ n even and approaching one due to the function of n, $f(\mathrm{n})$ = 3n + 1, and, $f(\mathrm{n})$ = n / 2, in the Collatz conjecture.

P(n) ⟹ P(n + y + 2).

Q.E.D.

Further Proof by Mathematical Induction.

Proving that any positive integer will converge to one using the system of two linear equations and P(n + n – 1).

For the positive integer one from equation two we have P(n).

P(n) = 2 – y

Base case: P(1) is true

P(1) = 2 – y

1 – 2 = – y

– 1 / – 1 = y

y = 1

Using mathematical induction for the next positive integer two.

Induction step: Show $P(n) \Longrightarrow P(n + n - 1)$.

$P(n) = 2 - y + n - 1$ (4)

$P(2) = 2 - y + 2 - 1$

$2 - 2 - 2 + 1 = - y$

$-1 = - y$

$y = - y / - 1$

$1 = y$

Substituting equation (4) into equation (3) and proving y is equivalent to n, for all positive integers.

$3n + y = 4$

$3(2 - y + n - 1) + y = 4 + (n - 1)$

$3(2 - y + 2 - 1) + y = 4 + (2 - 1)$

$6 - 3y + 6 - 3 + y = 4 + 2 - 1$

$9 - 2y = 5$

$- 2y = 5 - 9$

$- 2y = - 4$

$y = - 4 / - 2$

$y = 2$

$y \equiv n$

Proving that all positive integers will converge to one due to the function of n, $f(n) = 3n + 1$, and, $f(n) = n / 2$, in the Collatz conjecture and the fact that y, is equivalent to n, for all positive integers.

$n = 2 - y + n - 1$

$n = 2 - 2 + 2 - 1$

$n = 1$

Q.E.D.

3. Results:

The statement $P(n) \Longrightarrow P(n + y + 2)$ is true for both equations 3 and 4, in the system of two linear equations. Every positive integer being subject to the function of n, in the Collatz conjecture converges to one due to the function of n. When any positive integer is subject to the system of two linear equations containing the function of n, in the Collatz conjecture and mathematical induction, the value of y is equivalent to the value of the positive integer in question. As a result the positive integer will converge to one via the 4, 2, 1, loop.

4. Conclusion

The value of all positive integers converge to one via the four, two, one, loop due to the function of n, in the Collatz conjecture. The function of n in the Collatz conjecture provides the underlying mathematical principles needed to develop a system of two linear equations to prove the conjecture. The system of two linear equations and mathematical induction clearly demonstrates how all positive integers being subject to the function of n in the Collatz conjecture converge to one.

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Statements and Declarations

Data Availability

All data supporting the research is found within this research paper.

Conflicts of interest

The author declares no conflicts of interest.

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