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Regarding the Gap Between Prime Numbers

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Abstract

Until now there has been no efficient formula for determining the prime numbers. I show an efficient formula that takes only prime number values or multiples of 5, 7 and 11, and results in the successive prime number or a multiple of 5, 7 and 11. It is shown that by performing the same operation on a number that is a multiple of a prime number when using the formula will result in the next successive prime number value. The formula also identifies a pattern allowing mathematicians to begin at any random prime number and find the next successive prime number without fail.

Keywords: Prime number gap formula,
MSC: 11-xx

INTRODUCTION

There are numerous proofs to show that prime numbers are infinite [1][2]. Unfortunately there is no formula to determine the successive prime number [2]. The following is a demonstration of a formula where the input is either a prime number or a multiple of the prime numbers 5, 7 and 11, and results in the next successive prime number or intermittent values that are multiples of 5, 7 and 11. Given the first two prime numbers the third, fourth and fifth are found by the summation of the highest prime number with the previous and subtracting 1 from the result. For all prime numbers greater than 11 the formula created takes the summation of the highest prime number with the previous prime number and subtracts the next previous prime number. Considering that the gap between each prime number varies and follows the prime number difference function, $d_k = \left| P_a - P_{-b} \right|$, the formula provided in this paper provides not only a mathematical formula to determine the next prime number but also represents the gap between each successive prime number or multiples of 5, 7 and 11, [3].

METHOD

For all prime numbers.

$(\forall n \in \mathbb{N})\ [n|p \Longrightarrow [(n = 1) \vee (n = p)]].$

For all ($n \in \mathbb{N}$) there exists a k element of $\mathbb{N}$ that satisfies the equation, $n = 2k + 1$, where n is odd and is not divisible by 2.
$(\forall n \in \mathbb{N})\ (\exists k \in \mathbb{N}),\ n = 2k + 1$, n is odd $\Longrightarrow (2 \nmid n)$.
For all ($n \in \mathbb{N}$) there exists a k element of $\mathbb{N}$ that satisfies the equation, $n = 2k + 1$, where n divides 5n or 7n or 11n.
$(\forall n \in \mathbb{N})\ (\exists k \in \mathbb{N}),\ n = 2k + 1, \Longrightarrow (n | 5n \vee 7n \vee 11n)$.
Firstly starting at the first prime number of 2 the third prime is found by adding the first two prime numbers 2 and 3. The next successive prime is found by adding the resulting prime number 5 to the previous prime number 3 and taking the difference between the previous two prime numbers away from the summation of the higher two.

Example 1:
Finding the third prime number P_3 given the first two prime numbers P_1 and P_2.
$P_3 = P_1 + P_2$
$= 2 + 3$
$= 5$
$n|p = 5|5 = 1$ and $1|5 = 5$
Finding the fourth prime number P_4.
$P_4 = (P_3 + P_2) – (P_2 - P_1)$
$= (5 + 3) – (3 – 2)$
$= 8 – 1$
$= 7$
$n|p = 7|7 = 1$ and $1|7 = 7$
Finding the fifth prime number P_5.
$P_5 = (P_4 + P_3) – (P_2 - P_1)$
$= (7 + 5) – (3 – 2)$
$= 12 – 1$
$= 11$
$n|p = 11|11 = 1$ and $1|11 = 11$.

Example 2:
The following is a formula to determine the successive prime number after P_x.
Finding each successive prime number > 11 using the formula.
$P = (P_x + P_{-2}) – P_{-3}$
Where;
P = the successive prime number.
P_x = the highest prime number known.
P_{-2} = the first previous prime number or multiple of $5 \vee 7 \vee 11 < P_x$.
P_{-3} = the next previous prime number or multiple of $5 \vee 7 \vee 11 < P_{-2}$.
This is the theoretical gap between all primes > 11.
$P = (P_x + P_{-2}) – P_{-3}$
$= (11 + 7) – 5$
$= 18 – 5$
$= 13$
$n|p = 13|13 = 1$ and $1|13 = 13$

Finding the next Successive Prime number or multiple of 5 ∨ 7 ∨ 11.
$P = (P_x + P_{-2}) - P_{-3}$
$= (13 + 11) - 7$
$= 24 - 7$
$= 17$
n|p = 17|17 = 1 and 1|17 = 17.
Demonstrating a number that is a multiple of 5.
$P = (P_x + P_{-2}) - P_{-3}$
$= (23 + 19) - 17$
$= 25$
n|p = 25|25 = 1 and 1|25 = 25 and 5|25 = 5.
Finding the successive prime number or multiple of 5 ∨ 7 ∨ 11.
$P = (P_x + P_{-2}) - P_{-3}$
$= (25 + 23) - 19$
$= 48 - 19$
$= 29$
n|p = 29|29 = 1 and 1|29 = 29.
Demonstrating a number that is a multiple of 7.
$P = (P_x + P_{-2}) - P_{-3}$
$= (47 + 43) - 41$
$= 49$
n|p = 49|49 = 1 and 1|49 = 49 and 7|49 = 7.
Finding the successive prime number or multiple of 5 ∨ 7 ∨ 11.
$P = (P_x + P_{-2}) - P_{-3}$
$= (49 + 47) - 43$
$= 96 - 43$
$= 53$
n|p = 53|53 = 1 and 1|53 = 53.
Demonstrating a number that is a multiple of 11.
$P = (P_x + P_{-2}) - P_{-3}$
$= (73 + 71) - 67$
$= 77$
n|p = 77|77 = 1 and 1|77 = 77 and 11|77 = 7.
Finding the successive prime number or multiple of 5 ∨ 7 ∨ 11.
$P = (P_x + P_{-2}) - P_{-3}$
$= (77 + 73) - 71$
$= 150 - 71$
$= 79$
n|p = 79|79 = 1 and 1|79 = 79.
QED.

Example 3:
Finding the difference between each successive prime number using the following formula.
$d_k = \left| P_a - P_b \right|$

Where
$P_{a} = (P_{x} + P_{-2}) - P_{-3})$, for the highest prime number or multiple of $5 \vee 7 \vee 11$.
$P_{b} = (P_{a} + P_{-x}) - P_{-2})$, for the prime number previous to P_{a}.
$d_{k} = \left| \{[(P_{x} + P_{-2}) - P_{-3}] - [(P_{a} + P_{-x}) - P_{-2}]\} \right| = \left| P_{a} - P_{b} \right|$
The following demonstrates the gap between prime numbers or a multiple of $5 \vee 7 \vee 11$ starting at any random prime number.
Starting at the prime number 73.
$P_{a} = (P_{x} + P_{-2}) - P_{-3})$
$= (73 + 71) - 67$
$= 77$
$P_{b} = (P_{a} + P_{-x}) - P_{-2})$
$= (77 + 73) - 71$
$= 79$
$d_{k} = \left| P_{a} - P_{-b} \right|$
$= \left| 77 - 79 \right|$
$= \left| -2 \right|$
$= 2$
Continuing from 79.
$P_{a} = (P_{x} + P_{-2}) - P_{-3})$
$= (79 + 77) - 73$
$= 83$
$d_{k} = \left| P_{a} - P_{-b} \right|$
$= \left| 83 - 79 \right|$
$= \left| 4 \right|$
$= 4$
$P_{b} = (P_{a} + P_{-x}) - P_{-2})$
$= (83 + 79) - 77$
$= 85$
$d_{k} = \left| P_{a} - P_{-b} \right|$
$= \left| 83 - 85 \right|$
$= \left| -2 \right|$
$= 2$
This formula also demonstrates that the difference between P_{x} and P_{-2} added to the final answer P_{a} will give the answer to the next P_{b} or multiple of $5 \vee 7 \vee 11$.
QED.

Example 4:
$P_{a} = (P_{x} + P_{-2}) - P_{-3})$
$= (79 + 77) - 73$
$= 83$
$P_{b} = (P_{x} - P_{-2}) + P_{a}$
$= (79 - 77) + 83$
$= 2 + 83$
$= 85$
This shows the next successive number is 85 which is the next value for P_{b}.

Continuing this procedure the difference between P_x and P_{-2} alternates between a value of two and four and does not change.
QED.

Example 5:
Demonstrating finding the successive prime starting at any random prime number.
First start by adding either two or four to the starting prime number P.
P + 2 or P + 4
Then add this number to the starting prime number (P + 2) + P = n
Then subtract the initial prime number added to either two or four depending on what was added to the initial prime number i.e. if two was added first then four is added second.
n – (P + 2) + 4) = number to subtract from (P + 2) + P to give the successive prime number or multiple of 5 or 7 or 11.
Then once the initial prime plus either two or four is subtracted from n, this number is subtracted from the initial prime number plus two or four plus the initial prime number to find the successive prime number or multiple of 5 or 7 or 11, and the process is continued the same as in example 2.
i.e. Starting with prime number 1747
(P + 2)
1747 + 2 = 1749 which is a multiple of 11.
Addition of (P + 2) to the starting prime number to give n.
(P + 2) + P = n
1749 + 1747 = 3496
Subtracting ((P + 2) + 4) from n to give the number to subtract from (P + 2) + P.
n – (P + 2) + 4) = number to subtract from (P + 2) + P to give the successive prime number or multiple of 5 or 7 or 11.
= 3496 – (1749 + 4)
= 3496 – 1753
= 1743
∴
$\{[(P + 2\vee4) + P] – [n – [(P + 2\vee4) + 2\vee4]]\}$ = the next prime number or multiple of 5 or 7 or 11.
1749 + 1747 – 1743 = P, the next prime number or multiple of 5 or 7 or 11 and in this case is 1753.
QED.

RESULTS
By following the formula $P = (P_x + P_{-2}) – P_{-3}$ in example 2, the next prime number or multiple of 5, 7 and 11, is found by the summation of the previous two primes or multiple of 5, 7 and 11 and the subtraction of the third previous prime or multiple of 5, 7 and 11, for all primes greater than 11. This formula works in sequence when you begin at eleven and continue on from that point. To start at any random prime number and determine where the next successive prime number will appear the value of 2 or 4 is added to yield either a prime number or multiple of 5, 7 and 11 as demonstrated in

example 3. If the addition of two results in a non-prime or non-multiple of 5, 7 and 11 then the addition of four will be used instead. The reason is that the difference $d_k = |P_a - P_b|$, between the two previous primes or multiples of 5, 7 and 11 alternates between the value of two and four for each successive prime or multiple of 5, 7 and 11. This is demonstrated in example 3 and example 4.

CONCLUSION

The formula $P = (P_x + P_{-2}) - P_{-3}$, demonstrates that the next successive prime greater than eleven, will always be the summation of the highest prime number and the previous prime number or multiple of 5, 7 and 11, then subtracting the next previous prime number or multiple of 5, 7 and 11. Although this formula produces the next successive prime, the number is not always a prime number. The successive number maybe a multiple of 5, 7 and 11. However when the same formula is applied to these results the next successive prime number is always found. Therefore the formula takes prime numbers or multiples of 5, 7 and 11, as input and finds the successive prime number. When the formula is applied to the multiples of 5, 7 and 11, the successive prime number is always found. There is no deviation from the next prime number other than finding a multiple of 5, 7 and 11, in between primes. Also the formula can be used when starting at any random prime number and adding either two or four to that number to yield either the next prime number or a multiple of 5, 7 and 11. If the addition of two yields a non-prime, non-multiple of 5, 7 and 11, then the addition of four is used instead as the difference d_k, between the two previous prime numbers or multiple of 5, 7 and 11, alternates between the value of two and four and does not change. The formula has shown to result in not only an efficient way of determining the next prime number but also represents the ever changing gap between prime numbers. In conclusion the formula $P = (P_x + P_{-2}) - P_{-3}$, used in sequence from the number 11 will always find the next prime number or multiple of 5, 7 or 11. The formula $\{[(P + 2 \vee 4) + P] - [n - [(P + 2 \vee 4) + 2 \vee 4]]\}$ = the next prime number or multiple of 5, 7 or 11, when starting at any random number.

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Statements and Declarations

Data Availability

All data supporting the research is found within this research paper.

Conflicts of interest

The author declares no conflicts of interest.

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