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Twin Prime Conjecture Proof

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Abstract

The Twin Prime conjecture first proposed by French mathematician Alphonse de Polignac in the year 1846 remains unproven until now. This paper sets out to prove the Twin Prime conjecture by creating a system of equations and finding both a y element of ℕ, ($y \in \mathbb{N}$) and a y element of k ($y \in k$), to find the twin prime values and then two further values where each will differ by an exact amount. From this proof the method demonstrates that the elements of these phenomena prove that there will be an infinite number of twin primes that differ by two.

Keywords: Number theory, Twin Prime conjecture and Polignac's conjecture.

MSC: 11-xx

INTRODUCTION

The Twin Prime conjecture also referred to as the Polignac's conjecture proposes that there are infinitely many pairs of twin primes that differ by two. This statement was first made by the French mathematician Alphonse de Polignac, who first stated that any even number can be expressed as the difference between two primes and when the even number is two it is the twin prime conjecture [1][2][4][5].

It has been proven in the proof of Goldbach conjecture that all even numbers can be expressed as the sum of two primes [3]. The following demonstrates that there will be infinitely many twin primes that differ by two. Firstly, the y element of ℕ, ($y \in \mathbb{N}$) of all natural numbers n of ℕ, is found. Secondly a system of two equations is created since all even numbers, one greater than, ($1 >$) and one less than, ($1 <$) all twin prime numbers is a multiple of 6, [5]. These equations produce a ($y \in \mathbb{N}$) and an n element of ℕ, ($n \in \mathbb{N}$) for all even numbers between twin primes. Thirdly the twin prime numbers are found by substituting the ($n \in \mathbb{N}$) into two equations where 6n represents the even number between twin primes, which will result in two primes with the characteristic difference of 2. Fourthly these twin prime values are substituted into the two equations where the y element of k, (y ϵ k) for all even numbers produced by the addition of two prime numbers replaced by the (y ϵ ℕ) of the (n ϵ ℕ) for the even number between the twin primes. The result is two numbers (q_1, q_2) that differ by 12 as the twin primes, 1 > and 1 <, 6(n) differs by 2 giving 6(2) = 12. The manipulation of these equations proves that all twin primes placed into these equations will produce two numbers that differ by 12 hence proving that twin primes are infinite as the even number between two primes produced by the addition of two primes is infinite, thus proving the Twin Prime conjecture.

METHOD

Proof of infinitely many twin primes.

Finding the statement of (2 – y), for all positive natural numbers of ℕ.

$(\forall n \in \mathbb{N})\ (\exists y \in \mathbb{N})\ (n = 2 - y)$.

$(\forall n \in \mathbb{N})$.

$n/2 + y/2 = 1$

$n/2 = 1 - y/2$

$n = 2(1 - y/2)$

$n = 2 - 2y/2$

$n = 2 - y$

For all twin prime numbers (n ϵ ℕ), n divides p.

$$(\forall n \in \mathbb{N})\ [n|p \Longrightarrow [(n = 1) \vee (n = p)]] \Longrightarrow 6n - 1 = p_1 \quad (1)$$

For all twin prime numbers (n ϵ ℕ), n divides p.

$$(\forall n \in \mathbb{N})\ [n|p \Longrightarrow [(n = 1) \vee (n = p)]] \Longrightarrow 6n + 1 = p_2 \quad (2)$$

If all twin primes differ by 2 and the number between two primes is a multiple of 6 then the following system of two equations are as follows [4].

Equation (3). $6n + 1 = 2 - y$

Equation (4). $6n + 1 = 2 - y$

From equation (3), finding the (y ϵ ℕ),

$6n + 1 = 2 - y$

$6n + 1 - 2 = - y$

$6n - 1 = - y$

From equation (4), finding n.

$6n + 1 = 2 - y$

Substituting ($y \in \mathbb{N}$), from equation (3), into equation (4) to find the ($n \in \mathbb{N}$).

$6n + 1 = 2 + y$

$6n + 1 – 2 = y$

$6n = y + 1$

$n = (y + 1) / 6$

$n = y/6 + 1/6$

Using equation (1), (2) and the ($n \in \mathbb{N}$) to find the first pair of twin primes (p_1, p_2), > 2.

From equation (1) and let y equal the first prime number 3 > 2.

$n|p = 3|3 = 1$ and $1|3 = 3$

$6n – 1 = p_1$

$p_1 = 6(y/6 + 1/6) – 1$

$p_1 = 6(3/6 + 1/6) – 1$

$p_1 = (18/6 + 6/6) – 1$

$p_1 = 4 – 1$

$p_1 = 3$

Finding the twin prime of three using equation (2) and the ($n \in \mathbb{N}$).

$6n + 1 = p_2$

$p_2 = 6(y/6 + 1/6) + 1$

$p_2 = 6(3/6 + 1/6) + 1$

$p_2 = (18/6 + 6/6) + 1$

$p_2 = 4 + 1$

$p_2 = 5$

Where

$n|p = 5|5 = 1$ and $1|5 = 5$

$\therefore 2 = p_2 – p_1$

$2 = 5 – 3$

As 6n represents the even number between twin primes and that even number is the result of the addition of two primes, 6n represents the even number between twin primes [4].

As proven by the Goldbach conjecture proof, all even numbers can be expressed as the addition of two primes [3]. As the even number is generated by the addition of two primes a ($y \in k$) for the resulting even number is created as follows.

Adding two prime numbers together will result in a ($y \in k$) value for the resulting even number.

p + p = n, even.

2k + 1 + 2k + 1 = 0

2(1/2 – y/2) +1 + 2(1/2 – y/2) + 1 = 0

(1 – 2y/2) + 1 + (1 – 2y/2) + 1 = 0

1 – y + 1 + 1 – y + 1 = 0

–2y + 4 = 0

–2y = – 4

y = – 4/– 2

y = 2

Substituting this (y ϵ k) into the (y ϵ ℕ) equation 2k + 1 = 2 – y for the odd number, 1 > and 1 <, the even number between twin primes, will equate to zero.

2k + 1 = 2 – y

2k + 1 = 2 – 2

2k + 1 = 0

Taking this (y ϵ k) and substituting into the equation to find the prime values, 1 > and 1 <, the even number between twin primes will result in two odd numbers (q_1, q_2), that differ by 12. The reason these two numbers differ by 12 is due to 6n representing the even number between the twin primes and twin primes differ by 2 which gives.

6(n) = 12

n = 12/6

n = 2

6(2) = 12

∴

From equation (1), and substituting the (y ϵ k), of the odd numbers, 1 < and 1 > the even number between twin primes, the first odd number is found by evaluating the twin prime number 3 as the (y ϵ k), represented as the (y ϵ ℕ), which is substituted for n in the first equation.

$6n – 1 = q_1$

$6(2k + 1)) – 1) = q_1$

$6(2(y/2) + 1)) – 1 = q_1$

$q_1 = 6(2(3/2) + 1)) – 1$

$q_1 = 6(3 + 1)) – 1$

$q_1 = 6(4) – 1$

$q_1 = 24 – 1$

$q_1 = 23$

Where;

n|q = 23|23 = 1 and 1|23 = 23

Finding the second odd number using the same equation and the (y ϵ k) represented as the (y ϵ ℕ) which is substituted for n equation (1), of the odd numbers, 1 < and 1 > the even number between twin primes. Let y denote the second twin prime 5.

$6n – 1 = q_2$

$6(2k + 1)) – 1) = q_2$

$6(2(y/2) + 1) – 1 = q_2$

$q_2 = 6(2(5/2) + 1) – 1$

$q_2 = 6(5 + 1) – 1$

$q_2 = 6(6) – 1$

$q_2 = 36 – 1$

$q_2 = 35$

Finding the difference between the values produced by the prime numbers, 1 > and 1 <, the even number between two primes.

$q_2 – q_1 = 6n$

35 – 23 = 6(2)

35 – 23 = 12

12 = 12

QED

RESULT

It can be seen that as for all natural numbers n there exists a statement of (n = 2 – y), which gives a (y ϵ ℕ), as (y/6 + 1/6). When this (y ϵ ℕ) is substituted into the equation where the natural even number between the twin primes is a multiple of 6 an (n ϵ ℕ) is found. This (n ϵ ℕ) then produces the twin primes where their difference is 2. To prove that these two numbers are twin primes the (y ϵ k) for the even number between two primes is found. When this (y ϵ k) is substituted into the equation (1) as the (y ϵ ℕ) and the twin primes evaluated as y, two new odd numbers, q_1, q_2 are found. These two numbers differ by 12 due to the (y ϵ k) found from the addition of two primes resulting in an even number and that this even number is a multiple of 6.

CONCLUSION

Due to the fact that prime numbers are infinite, and it has been proven that all even numbers can be expressed by the addition of two primes in the Goldbach conjecture, the Twin Prime conjecture is proven [3]. The twin primes not only differ by 2 but the even number created from the addition of two primes produces a ($y \in k$) that proves that if two numbers are twin primes then their evaluation as y for the ($y \in k$) represented as the ($y \in \mathbb{N}$) for the two odd numbers, 1 < and 1 > the even number between twin primes, will differ by 12. This is due to the odd numbers ($y \in k$) along with the even number being a multiple of 6, and the twin primes differing by 2.

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Statements and Declarations

Data availability statement

All data supporting the research is found within this research paper.

Conflicts of interest

The author declares no conflicts of interest.

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