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A Direct Proof of Fermat's Last Theorem

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Abstract

Pierre de Fermat was a French mathematician and considered to be the founder of modern theory of numbers including differential calculus, analytic geometry and co-founder of probability. It is believed that he once scribbled that he had found a proof on the margin of his copy of Arithmetica by Diophantus of Alexandria, but the proof was too large for the margin. The proof that he is said to have mentioned was to that of the Diophantine equation, $x^n + y^n = z^n$. The conjecture derived from this equation has come to be known as Fermat's last theorem and classed as a theorem based on his statement despite never being proven.

Keywords: Fermat's Last Theorem, Fermat's Conjecture.

Introduction

Pierre de Fermat proposed that the Diophantine equation $x^n + y^n = z^n$, has no nonzero solution for $n > 2$ where x, y and z are integers $\neq 0$ (Wolfram Mathworld, p.1, 2024: Encyclopedia Britannica p.1, 2024). Fermat's statement on this matter reads as follows:

"It is impossible for a cube to be the sum of two cubes, a fourth power to be the sum of two fourth powers, or in general for any number that is a power greater than the second to be the sum of two like powers. I have discovered a truly marvellous demonstration of this proposition that this margin is too narrow to contain." (Wolfram Mathworld, p.1, 2024).

I examine the Diophantine equation $x^n + y^n = z^n$, and investigate the integers n, x, y and z in terms of their definition as integers. Hence a positive even integer represented as 2k, and a positive odd integer represented as 2k+1. Here I prove Fermat's last theorem by demonstrating that all combinations of both even and odd integers represented as x, y and z raised to the power of the integer n, as either odd or even, gives no nonzero solution. By taking the partial derivative of each term in the equation $x^n + y^n = z^n$, the summation of the x and y terms on the left-hand side is divided by the z term on the right-hand side and should equate to one if the rate of change of being raised to a power n is equal on both sides. The method will show the following combinations of the equation $x^n + y^n = z^n$. These combinations are described in detail as follows and will demonstrate that there is no nonzero solution to Fermat's last theorem for $n > 2$.

i) Firstly, an odd integer z raised to the power of an even integer n divides the summation of an odd integer x raised to the power of an even integer n and an even integer y raised to the power of an even integer n.
ii) Secondly, an even integer z raised to the power of an even integer n divides the summation of an even integer x raised to the power of an even integer n and an even integer y raised to the power of an even integer n.
iii) Thirdly, an even integer z raised to the power of an even integer n divides the summation of an odd integer x raised to the power of an even integer n and an odd integer y raised to the power of an even integer n.
iv) Fourthly, an even integer z being raised to the power of an odd integer n divides the summation of an odd integer x raised to the power of an odd integer n and an odd integer y raised to the power of an odd integer n.
v) Fifthly, an even integer z raised to the power of an odd integer n divides the summation of an even integer x raised to the power of an odd integer n and an even integer y raised to the power of an odd integer n.

Finally, an odd integer z raised to the power of an odd integer n divides the summation of an odd integer x raised to the power of an odd integer n and an even integer y raised to the power of an odd integer n.

Method

The following definitions define the odd and even integers.

An integer $n \in \mathbb{Z}$ is even if there is an integer k where $n = 2k$ and it is divisible by 2. An integer is even if and only if $(\exists k \in \mathbb{Z})\ (n = 2k)$.

An integer $n \in \mathbb{Z}$ is an odd integer if there is an integer k where $n = 2k + 1$ and it is not divisible by 2. An integer is odd if and only if $(\exists k \in \mathbb{Z})\ (n = 2k + 1)$, (Farlow 2020, pp.32:54).

In Fermat's last theorem the Diophantine equation in question is as follows.

$x^n + y^n = z^n$

The integers x, y and z can be either even or odd and not equal to zero. The integer n is greater than 2 and can be even or odd.

Using partial differentiation to determine if $x^n + y^n = z^n$. Where $x^n + y^n / z^n$ will equal 1 if the rate of change of the x and y term summation is equal to the rate of change of the z term.

Using partial differentiation to find the first derivative of each term in the equation
$x^n + y^n = z^n$.

Combination 1.

Where n is even, x is odd, y is even and z is odd, the following equation will result.

$x^{2k} + y^{2k} = z^{2k}$

$(2k + 1)^{2k} + (2k)^{2k} = (2k+1)^{2k}$

Taking the partial derivative of x with respect to x where y is constant.

$\partial x/\partial x = (2k)(2k + 0)^{(2k - 2k)} + 0 = 4k^2$

Taking the partial derivative of y with respect to y where x is constant.

$\partial y/\partial y = 0 + (2k)(2k)^{(2k - 2k)} = 4k^2$

Taking the partial derivative of z with respect to z.

$\partial z/\partial z = (2k)(2k + 0)^{(2k - 2k)} = 4k^2$

Adding the partial differentiation terms of x and y together.

$4k^2 + 4k^2 = 8k^2$

Comparing the left-hand side x and y term summation to the right-hand side z term of the equation.

$\partial x/\partial x + \partial y/\partial y = \partial z/\partial z$

$8k^2 \neq 4k^2$

$8k^2 / 4k^2$

$\neq 1$

No odd and even combination of Integers will satisfy this equation. QED.

Using partial differentiation to find the first derivative of each term in the equation

$x^n + y^n = z^n$

Combination 2.

Where n is even, x is even, y is even and z is even, the following equation will result.

$x^{2k} + y^{2k} = z^{2k}$

$(2k)^{2k} + (2k)^{2k} = (2k)^{2k}$

Taking the partial derivative of x with respect to x where y is constant.

$\partial x/\partial x = (2k)(2k)^{(2k-2k)} + 0 = 4k^2$

Taking the partial derivative of y with respect to y where x is constant.

$\partial y/\partial y = 0 + (2k)(2k)^{(2k-2k)} = 4k^2$

Taking the partial derivative of z with respect to z.

$\partial z/\partial z = (2k)(2k)^{(2k-2k)} = 4k^2$

Adding the partial differentiation terms of x and y together.

$4k^2 + 4k^2 = 8k^2$

Comparing the left-hand side x and y term summation to the right-hand side z term of the equation.

$\partial x/\partial x + \partial y/\partial y = \partial z/\partial z$

$8k^2 \neq 4k^2$

$8k^2 / 4k^2$

$\neq 1$

No odd and even combination of Integers will satisfy this equation. QED.

Using partial differentiation to find the first derivative of each term in the equation

$x^n + y^n = z^n$.

Combination 3.

Where n is even, x is odd, y is odd and z is even, the following equation will result.

$x^{2k} + y^{2k} = z^{2k}$

$(2k + 1)^{2k} + (2k + 1)^{2k} = (2k)^{2k}$

Taking the partial derivative of x with respect to x where y is constant.

$\partial x/\partial x = (2k)(2k + 0)^{(2k-2k)} + 0 = 4k^2$

Taking the partial derivative of y with respect to y where x is constant.

$\partial y/\partial y = 0 + (2k)(2k + 0)^{(2k-2k)} = 4k^2$

Taking the partial derivative of z with respect to z.

$\partial z/\partial z = (2k)(2k)^{(2k-2k)} = 4k^2$

Adding the partial differentiation terms of x and y together.

$4k^2 + 4k^2 = 8k^2$

Comparing the left-hand side x and y term summation to the right-hand side z term of the equation.

$\partial x/\partial x + \partial y/\partial y = \partial z/\partial z$

$8k^2 \neq 4k^2$

$8k^2 / 4k^2$

$\neq 1$

No odd and even combination of Integers will satisfy this equation. QED.

Using partial differentiation to find the first derivative of each term in the equation $x^n + y^n = z^n$.

Combination 4.

Where n is odd, x is odd, y is odd and z is even, the following equation will result.

$x^{2k+1} + y^{2k+1} = z^{2k+1}$

$(2k + 1)^{2k+1} + (2k + 1)^{2k+1} = (2k)^{2k+1}$

Taking the partial derivative of x with respect to x where y is constant.

$\partial x/\partial x = (2k + 1)(2k + 0)^{(2k + 1 - 2k + 1)} + 0 = 4k^2 + 2k$

Taking the partial derivative of y with respect to y where x is constant.

$\partial y/\partial y = 0 + (2k + 1)(2k + 0)^{(2k + 1 - 2k + 1)} = 4k^2 + 2k$

Taking the partial derivative of z with respect to z.

$\partial z/\partial z = (2k + 1)(2k)^{(2k + 1 - 2k + 1)} = 4k^2 + 2k$

Adding the partial differentiation terms of x and y together.

$4k^2 + 2k + 4k^2 + 2k = 8k^2 + 4k$

Comparing the left-hand side x and y term summation to the right-hand side z term of the equation.

$\partial x/\partial x + \partial y/\partial y = \partial z/\partial z$

$8k^2 + 4k \neq 4k^2 + 2k$

$8k^2 + 4k / 4k^2 + 2k$

$\neq 1$

No odd and even combination of Integers will satisfy this equation. QED.

Using partial differentiation to find the first derivative of each term in the equation

$\mathbf{x^n + y^n = z^n.}$

Combination 5.

Where n is odd, x is even, y is even and z is even, the following equation will result.

$x^{2k+1} + y^{2k+1} = z^{2k+1}$

$(2k)^{2k+1} + (2k)^{2k+1} = (2k)^{2k+1}$

Taking the partial derivative of x with respect to x where y is constant.

$\partial x/\partial x = (2k + 1)(2k)^{(2k + 1 - 2k + 1)} + 0 = 4k^2 + 2k$

Taking the partial derivative of y with respect to y where x is constant.

$\partial y/\partial y = 0 + (2k + 1)(2k)^{(2k +1 - 2k + 1)} = 4k^2 + 2k$

Taking the partial derivative of z with respect to z.

$\partial z/\partial z = (2k + 1)(2k)^{(2k + 1 - 2k + 1)} = 4k^2 + 2k$

Adding the partial differentiation terms of x and y together.

$4k^2 + 2k + 4k^2 + 2k = 8k^2 + 4k$

Comparing the left-hand side x and y term summation to the right-hand side z term of the equation.

$\partial x/\partial x + \partial y/\partial y = \partial z/\partial z$

$8k^2 + 4k \neq 4k^2 + 2k$

$8k^2 + 4k / 4k^2 + 2k$

$\neq 1$

No odd and even combination of Integers will satisfy this equation. QED.

Using partial differentiation to find the first derivative of each term in the equation

$x^n + y^n = z^n$.

Combination 6.

Where n is odd, x is odd, y is even and z is odd, the following equation will result.

$x^{2k +1} + y^{2k+1} = z^{2k+1}$

$(2k+1)^{2k+1} + (2k)^{2k+1} = (2k+1)^{2k+1}$

Taking the partial derivative of x with respect to x where y is constant.

$\partial x/\partial x = (2k + 1)(2k + 0)^{(2k + 1 - 2k + 1)} + 0 = 4k^2 + 2k$

Taking the partial derivative of y with respect to y where x is constant.

$\partial y/\partial y = 0 + (2k + 1)(2k)^{(2k +1 - 2k + 1)} = 4k^2 + 2k$

Taking the partial derivative of z with respect to z.

$\partial z/\partial z = (2k + 1)(2k + 0)^{(2k + 1 - 2k + 1)} = 4k^2 + 2k$

Adding the partial differentiation terms of x and y together.

$4k^2 + 2k + 4k^2 + 2k = 8k^2 + 4k$

Comparing the left-hand side x and y term summation to the right-hand side z term of the equation.

$\partial x/\partial x + \partial y/\partial y = \partial z/\partial z$

$8k^2 + 4k \neq 4k^2 + 2k$

$8k^2 + 4k / 4k^2 + 2k$

$\neq 1$

No odd and even combination of Integers will satisfy this equation. QED.

Results

i) When the result of an odd integer z being raised to the power of an even integer n divides the summation of an odd integer x raised to the power of an even integer n and an even integer y raised to the power of an even integer n the result is not equal to 1.

ii) When the result of an even integer z being raised to the power of an even integer n divides the summation of an even integer x raised to the power of an even integer n and an even integer y raised to the power of an even integer n the result is not equal to 1.

iii) When the result of an even integer z being raised to the power of an even integer n divides the summation of an odd integer x raised to the power of an even integer n and an odd integer y raised to the power of an even integer n the result is not equal to 1.

iv) When the result of an even integer z being raised to the power of an odd n divides the summation of an odd integer x raised to the power of an odd integer n and an odd integer y raised to the power of an odd integer n the result is not equal to 1.

v) When the result of an even integer z being raised to the power of an odd integer n divides the summation of an even integer x raised to the power of an odd integer n and an even integer y raised to the power of an odd integer n the result is not equal to 1.

vi) When the result of an odd integer z being raised to the power of an odd integer n divides the summation of an odd integer x raised to the power of an odd integer n and an even integer y raised to the power of an odd integer n the result is not equal to 1.

Conclusions

From using the method of partial differentiation, there is no combination of even and odd integers x, y and z raised to the power of either even or odd integer n that satisfy the equation $x^n + y^n = z^n$. When the summation of the partially differentiated x and y terms are compared to the partially differentiated z term, they are not equal. When the summation of the partially differentiated x and y terms is divided by the partially differentiated z term the result is not equal to one. The fact that the result is not equal to one proves that the x and y term summation rate of change is not equal to the rate of change of the z term. Therefore, it is concluded that Fermat's last theorem is proven to be correct as there are no integers that satisfy the equation $x^n + y^n = z^n$., as demonstrated by the method in this paper.

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Statements and Declarations

Data Availability

All data supporting the research is found within this research paper.

Conflicts of interest

The author declares no conflicts of interest.

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