

Proof of the Riemann Hypothesis

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Abstract

Described herein is a proof of the Riemann hypothesis that all non-trivial zeros of $\zeta(s)$ have real part equal to $1/2$ and are real. This proof shows how the real numbers when equated to zero will have an $x/2$ value that is equivalent to zeta function complex number when it is equal to zero. By substitution it will prove that when the real number approaches zero the imaginary part will be equal to the real part and equate to zero. This proof shows how the Riemann zeta function is related to the distribution of the prime numbers as the complex plane becomes equal to b^2 . This finding improves on previous findings related to the distribution of the prime numbers or multiples of 3, 5, 7 and 11.

Keywords

Riemann Hypothesis

Introduction

As stated by Bombieri in the description of the Riemann Hypothesis many mathematicians agree that the Riemann hypothesis is one of the most important open problems in pure mathematics. The Riemann hypothesis states that the nontrivial zeros of $\zeta(s)$ have real part equal to $1/2$, where $s = 1/2 + ib$ is a complex number consisting of real part $1/2$ and an imaginary part ib of the complex number (Bombieri 2024). In 1859 Bernhard Riemann created a formula for the number of primes up to a preassigned limit and is expressed in terms of the number of zeros of the zeta function. Reimann Stated that the zeros found in the preassigned limit are real and it is this statement that describes the Riemann Hypothesis (Bombieri 2024). The Riemann zeta function has trivial zeros at the negative even integers and when the Riemann zeta function is zero, for other values of s these are referred to as the nontrivial zeros. These nontrivial zeros are said to lie on the vertical line equal to $1/2 + ib$. When the zeta function has reached infinity there is a pole at $s = 1$ (Bombieri 2024).

Method

The Riemann hypothesis states that the non-trivial zeros of the zeta function have real part equal to $1/2$.

It is well established that the imaginary number i is equal to $\sqrt{-1}$ and i^2 is equal to -1 .

$$(\forall x \in \mathbb{C}) (\exists y \in \mathbb{C}) (x = 1/2 + iy \text{ or } x = 1/2 - iy).$$

When $\zeta(s) = 0$

$$1/2 + bi = 0$$

$$1/2 = -bi$$

$$1/2 = i^2 b\sqrt{-1}$$

$$1/2 = i^2 bi$$

Substituting into the complex number as x approaches zero.

$$1/2 + bi = 0$$

$$i^2 b\sqrt{-1} + bi = 0$$

$$-1bi + bi = 0$$

$$bi - bi = 0$$

When the zeta function is at a non-trivial zero the real part of the complex number is at $1/2$, and the complex plane becomes as follows.

$$bi - bi = 0$$

$$(bi)(i^2)(bi) = 0$$

$$b^2 i^4 = 0$$

$$b^2 i^2 i^2 = 0$$

$$b^2 (-1)(-1) = 0$$

$$b^2 (1) = 0$$

$$b^2 = 0$$

For all real numbers as x approaches zero there exists the statement of.

$$(\forall x \in \mathbb{R}) (\exists y \in \mathbb{R}) (x = -y).$$

$$x/2 + y/2 = 0$$

$$x/2 = -y/2$$

$$x = 2(-y/2)$$

$$x = -2y/2$$

$$x = -y$$

substituting back into the initial equation when x is at zero.

$$x/2 + y/2 = 0$$

$$y/2 - y/2 = 0$$

When the zeta function is at non-trivial zero the real part is equal to 1/2.

$$(\forall x \in \mathbb{R}) (\exists y \in \mathbb{C}) (x = -y/2 = -bi).$$

$$1/2 = -bi$$

Which is equal to the real part of.

$$1/2 = -y/2$$

∴

$$1/2 = -y/2 = -bi = 0$$

When the zeta function is at a non-trivial zero.

$$1/2 = y/2 - y/2 = bi - bi = b^2 = 0$$

QED.

When the zeta function is at a non-trivial zero the real part is equal to 1/2 and the complex plane becomes equivalent to b^2 . This rule of the real numbers being squared demonstrates the distribution of the primes as each real number squared minus the previous real number squared produces the next successive prime number or multiple of 3, 5, 7 or 11. The following equation demonstrates the distribution of the primes or multiples of 3, 5, 7, and 11.

$$b^2 - b^2_{-1} = \text{The next successive prime number or multiple of } 3 \vee 5 \vee 7 \vee 11.$$

Where,

b^2 = real number squared.

b^2_{-1} = the previous real number squared.

QED.

Results

For the real number when $x/2 + y/2$ equals zero the x component is equal to $-y$. For the complex number at zero, $1/2$ equals, $-bi$. When the imaginary values are substituted back into the original equation the result is zero for both the real number at x equals $1/2$ and the complex number at x equal $1/2$. The complex plain becomes equivalent to b^2 when the zeta function is at a non-trivial zero.

Conclusion

It can be seen from the equations of the real number at zero that $x = -y$ and when substituted back into the original equation equals zero when x equals $1/2$. By substituting -1 as i^2 for the imaginary part i^2bi of the complex number, it equates to zero when at a non-trivial zero. When the complex number of the Riemann hypothesis is equal to zero the real part $1/2$ is equal to $-bi$. The real number part $1/2$ of the complex number equals $-y/2$ and $-bi$ as x approaches zero. Therefore, the non-trivial zeros of $\zeta(s)$ have real part equal to $1/2$ and are real as $1/2 = y/2 - y/2 = bi - bi = b^2 = 0$, when the zeta function is at a non-trivial zero. Due to the complex plane becoming equivalent to b^2 , these findings verify the findings in the manuscript titled Regarding the Gap Between Prime Numbers written by Daniel John Thompson. This finding improves on the formula, $P = (P_x + P_{-2}) - P_{-3}$ and confirms that the Riemann zeta function is related to the distribution of the prime numbers (Thompson, 2024).

Statements and Declarations

Data Availability

All data supporting the research is found within this research paper.

Conflicts of interest

The author declares no conflicts of interest.

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