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Solution of Homogeneous and Non-Homogeneous ODE With Constant Coefficient through Sadik Transform

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Abstract

In this paper we will solve Homogeneous ODE with constant coefficient and non-Homogeneous ODE with constant coefficient by Sadik Transform. In solving linear differential equation of Homogenous and Non Homogenous form Sadik transform is now very powerful tool because after applying the Sadik transform we can get choices whether we precede by Sadik transform or any other transform kernels are of exponential type or similar to the kernel of the Laplace transform just by fixing values of alpha and beta according to a convenience and situation of the problem.

Key words: Sadik Transform, Homogeneous ODE with constant coefficient and non-Homogeneous ODE with constant coefficient

Introduction

In mathematics, an integral transform maps an equation from its domain into another domain of new variable where it might be converted and solved algebraically and easily than in the original domain. Different Integral transform have been successfully used for two centuries in solving many problems in applied mathematics, mathematical physics, Physical chemistry and engineering science. The solution is back to the original domain using the inverse integral transform. There are so many integral transform to solve Differential equations, Partial Differential Equations, integral transform etc. who claim their superiority over each other. Recently a new integral transform named the Sadik transform has been introduced by Sadikali Latif Shaikh in 2018. The Sadik transform is nothing but generalization of the Laplace transform, Sumudu transform, Elzaki transform and all those integral transforms whose kernels are of exponential type or similar to the kernel of the Laplace transform.

Prelimiraries

In this section, we recall some notions and definitions that will be used through this paper.

Sadik Transform: If $u(t)$ piecewise continuous on the interval $0 \le t \le A$ for any $A >$ 0, and $|u(t)| \le K$, where $t \ge M$, for any real and positive constant K and M, then Sadik transform $\mathbb{S}(\upsilon^{\alpha}, \beta) = \mathbb{S}[u(t)]$ is defined by

$$\mathbb{S}(\upsilon^{\alpha}, \beta) = \mathbb{S}[u(t)] = \frac{1}{\upsilon^{\beta}} \int_0^{\infty} e^{-\upsilon^{\alpha} t} u(t) dt$$

where, υ is complex variable, α is any non-zero real numbers, and β is any real number[2].

Properties of Sadik transform

(1) $\mathbb{S}\left(t^n\right) = \dfrac{\upsilon^{-\beta} n!}{\upsilon^{(n+1)\alpha}}$ (2) $\mathbb{S}\left(\sin(at)\right) = \dfrac{a\upsilon^{-\beta}}{\upsilon^{2\alpha} + a^2}$

(3) $\mathbb{S}\left(\cos(at)\right) = \dfrac{\upsilon^{\alpha}.\upsilon^{-\beta}}{\upsilon^{2\alpha} + a^2}$ (4) $\mathbb{S}\left(e^{at}\right) = \dfrac{\upsilon^{-\beta}}{\upsilon^{\alpha} - a}$

(5) $\mathbb{S}\left(\sinh(at)\right) = \dfrac{a\upsilon^{-\beta}}{\upsilon^{2\alpha} - a^2}$ (6) $\mathbb{S}\left(\cosh(at)\right) = \dfrac{\upsilon^{\alpha}.\upsilon^{-\beta}}{\upsilon^{2\alpha} - a^2}$

Sadik transform on differential coefficients

$$\mathbb{S}\left[x'(t)\right] = \upsilon^{\alpha}\mathbb{S}\left(\upsilon^{\alpha}, \beta\right) - \upsilon^{-\beta}x(0),$$

$$\mathbb{S}[x^{(n)}(t)] = \upsilon^{n\alpha}\mathbb{S}\left(\upsilon^{\alpha}, \beta\right) - \sum_{k=0}^{n-1} \upsilon^{k\alpha-\beta} x^{(n-1)-k}(0)$$

3. Main Results

Theorem 1. Consider a homogeneous ordinary differential equation of order n is given by

$$a_1 \frac{d^n x}{dt^n} + a_2 \frac{d^{n-1}x}{dt^{n-1}} + ... + a_{n+1} x = 0 \quad (1)$$

with initial conditions

$$x(0) = C_1,\ x'(0) = C_2, ... x^{(n-1)}(0) = C_n$$

then by Sadik transform show that solution of (1) is given by

$$x(t) = \mathbb{S}^{-1}\left(\frac{\upsilon^{-\beta}\left(\left(a_1\upsilon^{(n-1)\alpha} + \cdots + a_n\right)C_1 + \left(a_1\upsilon^{(n-2)\alpha} + \cdots + a_{n-1}\right)C_2 + \cdots + a_1 C_n\right)}{\left(a_1\upsilon^{n\alpha} + \cdots + a_n\upsilon^{\alpha} + a_{n+1}\right)} \right)$$

Proof: Applying Sadik transform to (1) we get

$$a_1\left[\upsilon^{n\alpha}\mathbb{S}(\upsilon^{\alpha},\beta)-\cdots-\upsilon^{(n-1)\alpha-\beta}x(0)\right]+\cdots+a_n\left[\upsilon^{\alpha}\mathbb{S}(\upsilon^{\alpha},\beta)-\upsilon^{-\beta}x(0)\right]+a_{n+1}\mathbb{S}(\upsilon^{\alpha},\beta)=0 \quad (2)$$

simplifying and applying initial conditions we get

$$\left(a_1\upsilon^{n\alpha}+\cdots+a_n\upsilon^{\alpha}+a_{n+1}\right)\mathbb{S}(\upsilon^{\alpha},\beta)-\upsilon^{-\beta}\left(a_1\upsilon^{(n-1)\alpha}+\cdots+a_n\right)C_1-$$

$$\upsilon^{-\beta}\left(a_1\upsilon^{(n-2)\alpha}+\cdots+a_{n-1}\right)C_2-\cdots\upsilon^{-\beta}\left(a_1\upsilon^{\alpha}+a_2\right)C_{n-1}-\upsilon^{-\beta}a_1C_n=0$$

then Sadik transform of function x (t) is

$$\mathbb{S}(\upsilon^{\alpha},\beta)=\frac{\upsilon^{-\beta}\left(\left(a_1\upsilon^{(n-1)\alpha}+\cdots+a_n\right)C_1+\left(a_1\upsilon^{(n-2)\alpha}+\cdots+a_{n-1}\right)C_2+\cdots+a_1C_n\right)}{\left(a_1\upsilon^{n\alpha}+\cdots+a_n\upsilon^{\alpha}+a_{n+1}\right)} \quad (3)$$

solution x (t) of homogeneous differential equation (1) is inverse sadik transform of (3)

$$x(t)=\mathbb{S}^{-1}\left(\frac{\upsilon^{-\beta}\left(\left(a_1\upsilon^{(n-1)\alpha}+\cdots+a_n\right)C_1+\left(a_1\upsilon^{(n-2)\alpha}+\cdots+a_{n-1}\right)C_2+\cdots+a_1C_n\right)}{\left(a_1\upsilon^{n\alpha}+\cdots+a_n\upsilon^{\alpha}+a_{n+1}\right)}\right) \quad (4)$$

Corollary 1. Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2}+a_2\frac{dx}{dt}+a_3x=0$ with initial conditions $x(0)=C_1, x'(0)=C_2$ then show that by Sadik transform solution is given by

$$x(t)=\mathbb{S}^{-1}\left[\upsilon^{-\beta}\frac{\left(\left(a_1\upsilon^{\alpha}+a_2\right)C_1+a_1C_2\right)}{\left(a_1\upsilon^{2\alpha}+a_2\upsilon^{\alpha}+a_3\right)}\right]$$

Proof: Simply putting n=2 in (3) and (4) we get solution of differential equation in terms of Sadik transform

$$\mathbb{S}(\upsilon^{\alpha},\beta)=\upsilon^{-\beta}\frac{\left(\left(a_1\upsilon^{\alpha}+a_2\right)C_1+a_1C_2\right)}{\left(a_1\upsilon^{2\alpha}+a_2\upsilon^{\alpha}+a_3\right)} \quad (5)$$

$$x(t)=\mathbb{S}^{-1}\left[\upsilon^{-\beta}\frac{\left(\left(a_1\upsilon^{\alpha}+a_2\right)C_1+a_1C_2\right)}{\left(a_1\upsilon^{2\alpha}+a_2\upsilon^{\alpha}+a_3\right)}\right] \quad (6)$$

Corollary 2. Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2}+a_2\frac{dx}{dt}+a_3x=0$ with initial conditions $x(0)=C_1, x'(0)=C_2$ then show that by Laplace transform solution is given by

$$x(t)=L^{-1}\left[\frac{\left((a_1\upsilon+a_2)C_1+a_1C_2\right)}{\left(a_1\upsilon^2+a_2\upsilon+a_3\right)}\right]$$

Proof: Since by [2] Laplace transform is particular case of Sadik transform if $\alpha=1,\ \beta=0$, $\mathbb{S}\left(\upsilon^{\alpha},\beta\right)=F(\upsilon)$ and $\mathbb{S}\left(x(t)\right)=L\left(x(t)\right)$ and if $n=2$ in (3) and (4) we get solution of differential equation in terms of Laplace transform

$$F(\upsilon)=\frac{\left((a_1\upsilon+a_2)C_1+a_1C_2\right)}{\left(a_1\upsilon^2+a_2\upsilon+a_3\right)} \qquad \& \qquad x(t)=L^{-1}\left[\frac{\left((a_1\upsilon+a_2)C_1+a_1C_2\right)}{\left(a_1\upsilon^2+a_2\upsilon+a_3\right)}\right]$$

using partial fraction and inverse Laplace transform we get the exact solution of DE.

Corollary 3. Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2}+a_2\frac{dx}{dt}+a_3x=0$ with initial conditions $x(0)=C_1, x'(0)=C_2$ then show that by Sumudu transform solution is given by

$$x(t)=S^{-1}\left[\frac{\left((a_1+a_2\upsilon)C_1+a_1C_2\upsilon\right)}{\left(a_1+a_2\upsilon+a_3\upsilon^2\right)}\right]$$

Proof: Since by [2] Sumudu transform is particular case of Sadik transform if $\alpha=-1,\ \beta=1$ and $\mathbb{S}\left(\upsilon^{\alpha},\beta\right)=G(\upsilon)$ and $\mathbb{S}\left(x(t)\right)=S\left(x(t)\right)$ and if $n=2$ in (3) and (4) we get solution of differential equation in terms of Sumudu transform

$$G(\upsilon)=\frac{\left((a_1+a_2\upsilon)C_1+a_1C_2\upsilon\right)}{\left(a_1+a_2\upsilon+a_3\upsilon^2\right)} \qquad \& \qquad x(t)=S^{-1}\left[\frac{\left((a_1+a_2\upsilon)C_1+a_1C_2\upsilon\right)}{\left(a_1+a_2\upsilon+a_3\upsilon^2\right)}\right]$$

using partial fraction and inverse Sumudu transform we get the exact solution of DE.

Corollary 4: Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2}+a_2\frac{dx}{dt}+a_3x=0$ with initial conditions $x(0)=C_1, x'(0)=C_2$ then show that by Elzaki transform solution is given by

$$x(t)=T^{-1}\left[\upsilon^2\frac{\left((a_1+a_2\upsilon)C_1+a_1C_2\upsilon\right)}{\left(a_1+a_2\upsilon+a_3\upsilon^2\right)}\right]$$

Proof: Since by [2] Elzaki transform is particular case of Sadik transform if $\alpha=-1,\ \beta=-1$ and $\mathbb{S}\left(\upsilon^{\alpha},\beta\right)=T(\upsilon)$ and $\mathbb{S}\left(x(t)\right)=T\left(x(t)\right)$ and if $n=2$ in (3) and (4) we get solution of differential equation in terms of Elzaki transform

$$T(\upsilon) = \upsilon^2 \frac{\left((a_1 + a_2\upsilon)C_1 + a_1C_2\upsilon\right)}{\left(a_1 + a_2\upsilon + a_3\upsilon^2\right)} \quad \&$$

$$x(t) = T^{-1}\left[\upsilon^2 \frac{\left((a_1 + a_2\upsilon)C_1 + a_1C_2\upsilon\right)}{\left(a_1 + a_2\upsilon + a_3\upsilon^2\right)}\right]$$

using partial fraction and inverse Elzaki transform we get the exact solution of DE.

Corollary 5: Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2} + a_2\frac{dx}{dt} + a_3x = 0$ with initial conditions $x(0) = C_1, x'(0) = C_2$ then show that by Abood transform solution is given by

$$x(t) = A^{-1}\left[\frac{1}{\upsilon}\frac{\left((a_1\upsilon + a_2)C_1 + a_1C_2\right)}{\left(a_1\upsilon^2 + a_2\upsilon + a_3\right)}\right]$$

Proof: Since by [2] Abood transform is particular case of Sadik transform if $\alpha = 1,\ \beta = 1$ and $\mathbb{S}(\upsilon^\alpha, \beta) = K(\upsilon)$ and $\mathbb{S}(x(t)) = A(x(t))$ and if $n = 2$ in (3) and (4) we get solution of differential equation in terms of Abood transform

$$A(\upsilon) = \frac{1}{\upsilon}\frac{\left((a_1\upsilon + a_2)C_1 + a_1C_2\right)}{\left(a_1\upsilon^2 + a_2\upsilon + a_3\right)} \quad \&$$

$$x(t) = A^{-1}\left[\frac{1}{\upsilon}\frac{\left((a_1\upsilon + a_2)C_1 + a_1C_2\right)}{\left(a_1\upsilon^2 + a_2\upsilon + a_3\right)}\right]$$

using partial fraction and inverse Abood transform we get the exact solution of DE.

Corollary 6. Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2} + a_2\frac{dx}{dt} + a_3x = 0$ with initial conditions $x(0) = C_1, x'(0) = C_2$ then show that by Kamal transform solution is given by

$$x(t) = K^{-1}\left[\upsilon\frac{\left((a_1 + a_2\upsilon)C_1 + a_1C_2\upsilon\right)}{\left(a_1 + a_2\upsilon + a_3\upsilon^2\right)}\right]$$

Proof: Since by [2] Kamal transform is particular case of Sadik transform if $\alpha = -1,\ \beta = 0$ and $\mathbb{S}(\upsilon^\alpha, \beta) = G(\upsilon)$ and $\mathbb{S}(x(t)) = K(x(t))$ and if $n = 2$ in (3) and (4) we get solution of differential equation in terms of Kamal transform

$$G(\upsilon) = \upsilon\frac{\left((a_1 + a_2\upsilon)C_1 + a_1C_2\upsilon\right)}{\left(a_1 + a_2\upsilon + a_3\upsilon^2\right)} \quad \&$$

$$x(t) = K^{-1}\left[\upsilon\frac{\left((a_1 + a_2\upsilon)C_1 + a_1C_2\upsilon\right)}{\left(a_1 + a_2\upsilon + a_3\upsilon^2\right)}\right]$$

using partial fraction and inverse Kamal transform we get the exact solution of DE.

Corollary 7. Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2}+a_2\frac{dx}{dt}+a_3x=0$ with initial conditions $x(0)=C_1, x'(0)=C_2$

then show that by Tarig transform solution is given by

$$x(t)=T^{-1}\left[\upsilon\frac{\left(\left(a_1+a_2\upsilon^2\right)C_1+a_1C_2\upsilon^2\right)}{\left(a_1+a_2\upsilon^2+a_3\upsilon^4\right)}\right]$$

Proof: Since by [2] Tarig transform is particular case of Sadik transform if $\alpha=-2,\ \beta=1$ and $\mathbb{S}\left(\upsilon^{\alpha},\beta\right)=F(\upsilon)$ and $\mathbb{S}\left(x(t)\right)=T\left(x(t)\right)$ and if $n=2$ in (3) and (4) we get solution of differential equation in terms of Tarig transform

$$G(\upsilon)=\upsilon\frac{\left(\left(a_1+a_2\upsilon^2\right)C_1+a_1C_2\upsilon^2\right)}{\left(a_1+a_2\upsilon^2+a_3\upsilon^4\right)} \qquad \&$$

$$x(t)=T^{-1}\left[\upsilon\frac{\left(\left(a_1+a_2\upsilon^2\right)C_1+a_1C_2\upsilon^2\right)}{\left(a_1+a_2\upsilon^2+a_3\upsilon^4\right)}\right]$$

using partial fraction and inverse Tarig transform we get the exact solution of DE.

Corollary 8. Consider homogeneous ordinary differential equation of order two $a_1\frac{d^2x}{dt^2}+a_2\frac{dx}{dt}+a_3x=0$ with initial conditions $x(0)=C_1, x'(0)=C_2$

then show that by Mohand transform solution is given by

$$x(t)=M^{-1}\left[\upsilon^2\frac{\left((a_1\upsilon+a_2)C_1+a_1C_2\right)}{\left(a_1\upsilon^2+a_2\upsilon+a_3\right)}\right]$$

Proof: Since by [2] Tarig transform is particular case of Sadik transform if $\alpha=1,\ \beta=-2$ and $\mathbb{S}\left(\upsilon^{\alpha},\beta\right)=R(\upsilon)$ and $\mathbb{S}\left(x(t)\right)=M\left(x(t)\right)$ and if $n=2$ in (3) and (4) we get solution of differential equation in terms of Mohand transform

$$R(\upsilon)=\upsilon^2\frac{\left((a_1\upsilon+a_2)C_1+a_1C_2\right)}{\left(a_1\upsilon^2+a_2\upsilon+a_3\right)} \qquad \&$$

$$x(t)=M^{-1}\left[\upsilon^2\frac{\left((a_1\upsilon+a_2)C_1+a_1C_2\right)}{\left(a_1\upsilon^2+a_2\upsilon+a_3\right)}\right]$$

using partial fraction and inverse Mohand transform we get the exact solution of DE.

Solution of third, fourth…order Homogeneous differential equation with constant coefficient can be easily found by different transform only using (3) and (4).

4. Solution of Non-Homogeneous ODE with Constant Coefficient by Sadik Transform

Theorem 2. Consider a homogeneous ordinary differential equation of order n is given by

$$a_1 \frac{d^n x}{dt^n} + a_2 \frac{d^{n-1} x}{dt^{n-1}} + ... + a_{n+1} x = f(t) \tag{7}$$

with initial conditions $x(0) = C_1,\ x'(0) = C_2, ... x^{(n-1)}(0) = C_n$ then by Sadik transform show that solution of (7) is given by

$$x(t) = \mathbb{S}^{-1}\left(\frac{\mathbb{S}(f(t)) + \upsilon^{-\beta}\left(\left(a_1 \upsilon^{(n-1)\alpha} + \cdots + a_n\right)C_1 + \cdots + a_1 C_n\right)}{\left(a_1 \upsilon^{n\alpha} + \cdots + a_n \upsilon^{\alpha} + a_{n+1}\right)} \right)$$

Proof: Applying Sadik transform to (7) we get

$$a_1\left[\upsilon^{n\alpha}\mathbb{S}(\upsilon^{\alpha},\beta) - \cdots - \upsilon^{(n-1)\alpha-\beta} x(0)\right] + \cdots + a_n\left[\upsilon^{\alpha}\mathbb{S}(\upsilon^{\alpha},\beta) - \upsilon^{-\beta} x(0)\right] + a_{n+1}\mathbb{S}(\upsilon^{\alpha},\beta) = f(t)$$

simplifying and applying initial conditions we get

$$\left(a_1 \upsilon^{n\alpha} + \cdots + a_n \upsilon^{\alpha} + a_{n+1}\right)\mathbb{S}(\upsilon^{\alpha},\beta) - \cdots \upsilon^{-\beta}\left(a_1 \upsilon^{\alpha} + a_2\right)C_{n-1} - \upsilon^{-\beta} a_1 C_n = f(t)$$

then Sadik transform of function x (t) is

$$\mathbb{S}(\upsilon^{\alpha},\beta) = \frac{\mathbb{S}(f(t)) + \upsilon^{-\beta}\left(\left(a_1 \upsilon^{(n-1)\alpha} + \cdots + a_n\right)C_1 + \cdots + a_1 C_n\right)}{\left(a_1 \upsilon^{n\alpha} + \cdots + a_n \upsilon^{\alpha} + a_{n+1}\right)} \tag{8}$$

solution x (t) of homogeneous differential equation (7) is inverse sadik transform of (8)

$$x(t) = \mathbb{S}^{-1}\left(\frac{\mathbb{S}(f(t)) + \upsilon^{-\beta}\left(\left(a_1 \upsilon^{(n-1)\alpha} + \cdots + a_n\right)C_1 + \cdots + a_1 C_n\right)}{\left(a_1 \upsilon^{n\alpha} + \cdots + a_n \upsilon^{\alpha} + a_{n+1}\right)} \right) \tag{9}$$

5. Application of Sadik Transform in ODE

Example 1. Solve the second order differential equation by Sadik Transform $\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 20.\sin(2t)$ with initial conditions $x(0) = -1,\ x'(0) = 2$

Here $n = 2,\ a_1 = 1,\ a_2 = -1,\ a_3 = -2,\ C_1 = -1$ and $C_2 = 2$ from (9)

$$x(t) = \mathbb{S}^{-1}\left[\frac{\mathbb{S}(20\sin(2t)) + \upsilon^{-\beta}\left((\upsilon^{\alpha} - 1)(-1) + (1)(2)\right)}{\upsilon^{2\alpha} - \upsilon^{\alpha} - 2} \right]$$

$$x(t)=\mathbb{S}^{-1}\left[\frac{\frac{40.\upsilon^{-\beta}}{\upsilon^{2\alpha}+4}+\upsilon^{-\beta}\left(-\upsilon^{\alpha}+3\right)}{\upsilon^{2\alpha}-\upsilon^{\alpha}-2}\right]$$

using method of partial fraction

$$x(t)=\mathbb{S}^{-1}\left[2.\frac{\upsilon^{-\beta}}{\upsilon^{\alpha}-2}-4.\frac{\upsilon^{-\beta}}{\upsilon^{\alpha}+1}+\frac{\upsilon^{\alpha-\beta}}{\upsilon^{2\alpha}+2^2}-3.\frac{2\upsilon^{-\beta}}{\upsilon^{2\alpha}+2^2}\right] \quad (10)$$

$$\therefore x(t)=2.e^{2t}-4.e^{-t}+\cos(2t)-3\sin(2t)$$

by fixing values of α and β in (10) for Laplace, Sumudu, Kamal, Tarig, Elzaki, Mohand, Abood transforms and applying its inverse transform we get same solution. For example put $\alpha=-1,\ \beta=0$ and $\mathbb{S}^{-1}$ by K^{-1} then Sadik transform and its inverse will become Kamal and its inverse transform,

$$x(t)=K^{-1}\left[2.\frac{\upsilon}{1-2\upsilon}-4.\frac{\upsilon}{1-\upsilon}+\frac{\upsilon}{1+4\upsilon^2}-3\times\frac{2\upsilon^2}{1+4\upsilon^2}\right]$$

applying inverse Kamal transform formulae from table we get same solution as

$$\therefore x(t)=2.e^{2t}-4.e^{-t}+\cos(2t)-3\sin(2t)$$

Example 2. Solve the differential equation by Sadik Transform $\frac{d^3x}{dt^3}-\frac{d^2x}{dt^3}-\frac{d^2x}{dt^2}-\frac{dx}{dt}+x=8te^{-t}$ with initial conditions $x(0)=0,\ x'(0)=1,\ x''(0)=0$

Here

$n=3,\ a_1=1,\ a_2=-1,\ a_3=-1,\ a_4=1,\ C_1=0,\ C_2=1,\ C_3=0$ and $f(t)=8te^{-t}$

from (9) solution of given different equation by Sadik transform is given by

$$x(t)=\mathbb{S}^{-1}\left[\frac{\left(\mathbb{S}\left(8te^{-t}\right)+\upsilon^{-\beta}\left[\left(\upsilon^{2\alpha}-\upsilon^{\alpha}-1\right)(0)+\left(\upsilon^{\alpha}-1\right)(1)+0\right]\right)}{\upsilon^{3\alpha}-\upsilon^{2\alpha}-\upsilon^{\alpha}+1}\right]$$

by first shifting theorem $\mathbb{S}\left[e^{at}f(t)=f\left(\left(\upsilon^{\alpha}-a\right),\ \beta\right)\right]$

$$x(t)=\mathbb{S}^{-1}\left[\upsilon^{-\beta}\left(\frac{\upsilon^{3\alpha}+\upsilon^{2\alpha}-\upsilon^{\alpha}+7}{\left(\upsilon^{\alpha}-1\right)^2\left(\upsilon^{\alpha}+1\right)^3}\right)\right]$$

by using partial fraction we get

$$x(t) = \mathbb{S}^{-1}\left[\frac{-\upsilon^{-\beta}}{\upsilon^{\alpha}-1} + \frac{\upsilon^{-\beta}}{(\upsilon^{\alpha}-1)^2} + \frac{\upsilon^{-\beta}}{(\upsilon^{\alpha}+1)} + 2\frac{\upsilon^{-\beta}}{(\upsilon^{\alpha}+1)^2} + 2\frac{\upsilon^{-\beta}}{(\upsilon^{\alpha}+1)^3}\right] \quad (11)$$

by inverse Sadik transform and its first shift theorem,

$$x(t) = e^{-t}(1+2t+t^2) + e^{t}(t-1)$$

fixing values of α and β in (11) for Laplace, Sumudu, Kamal, Tarig, Elzaki, Mohand, Abood transforms and applying its inverse transform we get same solution. For example put $\alpha = -1,\ \beta = 1$ and $\mathbb{S}^{-1}$ by S^{-1} then Sadik transform and its inverse will become Sumudu and its inverse transform,

$$x(t) = S^{-1}\left[\frac{-\upsilon^{-1}}{\upsilon^{-1}-1} + \frac{\upsilon^{-1}}{(\upsilon^{-1}-1)^2} + \frac{\upsilon^{-1}}{(\upsilon^{-1}+1)} + 2\frac{\upsilon^{-1}}{(\upsilon^{-1}+1)^2} + 2\frac{\upsilon^{-1}}{(\upsilon^{-1}+1)^3}\right]$$

$$x(t) = S^{-1}\left[\frac{-1}{1-\upsilon} + \frac{\upsilon}{(1-\upsilon)^2} + \frac{1}{(1+\upsilon)} + 2\frac{\upsilon}{(1+\upsilon)^2} + 2\frac{\upsilon^2}{(1+\upsilon)^3}\right]$$

applying inverse Sumudu transform formulae we get same solution as

$$x(t) = e^{-t}(1+2t+t^2) + e^{t}(t-1)$$

Above results shows that solution of differential ordered non-Homogeneous Differential equation with constant coefficient can easily be finding by mentioned transform only using .(9)

Result

Sadik transform is very strongest tool to solve ordinary differential equation it also allow us to take turn to any other integral transform at any stage in this paper we tried to formulate differential equation by Sadik Transform formula also we have presented its application through different examples.

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