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Fractal Reflections of Tamil Nadu coast

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Abstract

In this paper, the authors assessed the fractal dimensions of coast line of Tamil Nadu using 1:50000 and 1:250000 scale SOI toposheets to bring out the scale invariability and randomness of the processes that operate along this coast. Using divider, the number of steps required to trace the TN coast at a given width, i.e. step size and counts were obtained for two different scale images of TN coast. As slope (Richardson's formula) of a double log graph of fractal parameters is equal to fractal dimension, it is figured out as 1.29 for 1:50000 scale and 1.04 for 1:250000 scale map. R-squared values (0.990 and 0.9940) support the significance of the results. t-test necessitated the acceptance of null hypothesis, following that, it proves that Tamil Nadu coast is not any exception with regard to the fractal property. In addition, by deploying fractal data into a QuickBasic program the average fractal dimension was obtained as 1.09. From the graphical plots of step lengths versus coastal length for two different scales of maps, an attempt was made to arrive at an average step length that could reveal a more or less close approximation to the actual coastal length. It has been met through the interpolation of intersecting point of those two trend lines, i.e., average step length S = 1.6 cm and the corresponding coast length is 1078.05 Km for 1:50000 and 998.5 Km for 1:250000. Further interpolation was exercised to know coast length for the supposed step length of 1cm and is found to be 1078.4 Km for 1:50000 and 998.18 for 1:250000 which emphasizes scale invariability property of the coast.

Keyword: Tamil Nadu coast, self-similar, scale invariability, fractal geometry, fractal dimension, divider method, Richardson's slope method

Introduction

Shapes of natural objects, in general, defy precise measurements because of their irregularity. In particular, geometric disposition of coastlines of islands and countries

turns measurement into a difficult task by its inherent roughness introduced by the wind, wave and hydraulic actions. Dimensioning of such jagged shapes required ingenious methods such as the concept of fractals. Fractal dimension is vastly being used to measure roughness of coast, signals, objects, statistical distribution etc.
Mandelbrot is the “father of fractals”. It is derived from Latin “fractus” meaning “broken” (Falconer, 2013) and he was developing the “theory of roughness” in nature. It is (Mandelbrot, 1967) first to say that there exist intermediate dimensions between discrete dimensions; that is, between one and two dimensions there are fractional dimensions. Several natural objects appear similar in outline even at different scales of inquiry or observation; for example, coastline, leaf, rocks, and stars in the sky. The term “*fractal*”, coined by Mandelbrot (1967), refers to a piece of a large system bearing all the parent’s properties, especially shape attributes (Southgate et al., 2000). By analogy, time series data of climate is also said to be in fractal domain. Climatic variables vary within a day, a month, a year and over millennium with ups and downs – a similarity or pattern over the span of time or scale of inquiry and that is the reason it is called a self similar attribute of the Earth. So, the application of fractal concept is vast. There is an infinite variety of fractals and a number of ways to create the fractals. They are Mandel fractals, dynamic fractal, Julia Fractal, lambda fractals, Plasma fractal, Newton fractals, complex Newton fractals, Newton domains of attraction fractals, Barnsley Mandelbrot and Julia fractals, bifurcation fractals, circle fractals, Sierpinski gasket fractals, IFS fractals, inverse Julia fractals, lambda FN fractals, ginger fractals, icon fractal, Lorenz fractals, formula fractals and Julibrot fractals.
It is possible to measure objects mathematically which satisfy the condition of self-similarity. The coastline, a sum total result of waves, tides, and wind actions, acquires irregular shape. Fractal geometry provides a general framework for the study of such irregular set. So measurement of the irregular shape of coastline is estimated using fractal dimension, one of the mathematical models. The fractal dimension of coastline with irregular outline will vary according to the degree of “roughness” and hence the value of fractal dimensions (Radhakrishnan, 2002). The coastline of Britain was the first fractal system studied by mathematicians. Fractal geometry is a valuable tool for describing the shapes of objects (Paszto et al., 2011).

History of Fractal

The fractal was first observed at the end of the 19th century. George Ferdinand Ludwig Philipp Cantor (1845-1918) founded set theory and introduced the concept of infinite numbers with his discovery of cardinal numbers. He also advanced the study of trigonometric series. Niels Fabian Helge Von Koch (1870-1924) described the Koch curve in 1904 in a paper entitled "On a continuous curve without tangents constructible from elementary geometry" (original French title: "*Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire*"). He discovered a set, each of its triangles always facing outward, resulting into a curve resembling a snowflake. So this curve is called the von Koch’s snowflake. Waclaw Sierpinski (1882-1969) work predated Mandelbrot’s discovery of fractals. In 1916, he introduced his Sierpinski triangle (Falconer, 2013). Hausdorff proposed a natural way of “measuring”, in 1918, and proved that the dimension of the middle third cantor set is log 2 / log 3 = 0.631 later turned to be popularly known as “Hausdorff dimension” (Menger, 1928), generated a fractal curve resembling sponge, a three dimensional object (Menger sponge).

Fractal Geometry

The most familiar fractal images are Sierpinski Triangle and Pyramid, the Sierpinski Carpet and Manger Sponge, the Koch Snowflake, and that of the iconic Mandelbrot set and Julia set images. There are two important properties in fractals: (1) Self-similarity and (2) its non-integer dimension. An object is said to possess the property of self-similarity even when it is repeatedly magnified it will look like the same in shape, for example fern leaf. Integer dimensions viz., zero, one, two and three cover point, line, area and volume respectively (Shamsgovara, 2012; Aschwanden, 2011). The linear structures are lines, segments of lines, contours, curved, intermittent, discrete, folded and intertwined. This is what (Peter, 2003) reiterates: self-similarity is geometrically fractal and independent of scale appearing equally detailed at any level of magnification. It interprets to the point that fractal values are not affected by shrinking or enlargements of fractal pattern.

STUDY AREA

When an attempt was made to quantify the length of the coast of Tamil Nadu – a part of Ph.D. thesis work of the first author – it became necessary to resort to the principles of fractal dimensions. It is because the tracing of coastline appeared highly irregular with immense roughness. The coastline considered is from Chennai (Lat. 13° 06’ 04” N and Long. 80° 24’ 95” E) to Kanniyakumari (Cape Comarin, Lat. 8° 08’ 83” N and Long. 77° 53’ 84” E) (Figure 1).

The Tamil Nadu coast is 1067 Km in extent. A coastline is a fractal interface between land and sea (Salingaros, 2008). The Tamil Nadu coastline is adorned by shores of rocks and sand. There are two main activities that render irregularity to the shape of coastline. One is natural activities like: coastal erosion and accretion. This is induced by the ocean wave and wind blow. The second one is manmade activities like: urbanization, beach nourishment, dredging, mining and tourism development. Tamil Nadu coast is not spared from such impacts; and the coastline needs to be uneven. This aspect of irregularity warrants a fractal dimensional study while a measurement of coastline is attempted. Fractal analysis is a very useful tool to find the effects of scale changes on the properties of images (Kamalakannan, 2013).

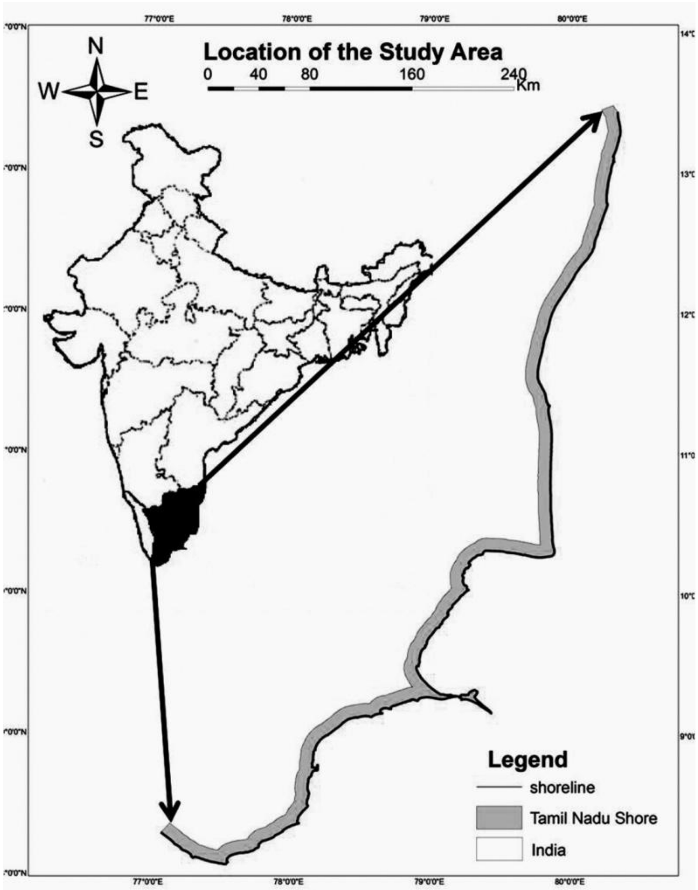

Figure 1: Tamil Nadu coast line

Materials and Methods

For the estimation of fractal dimension of the Tamil Nadu coastline, its trace maps of two different scales (1:50000 and 1:250000) were used. Trace maps were produced by sketching the coastline on overlays-tracing sheets-over Survey of India toposheets (See Table 1). Length of the coastline should be the product of map length of the coastline and the map scale. There are various ways of map length measurement. Cumbersome one is aligning a thread along the traced line of the coast. Coastline length is estimated by multiplying the length of the thread used to cover the trace and scale of the tracing. Its laborious part is removed by another traditional method of using divider.

Divider method is to determine the length of linear features and also it's a very well-known technique (Klinkenberg et al., 1992). However, the result depends on the size of the unit length of the divider.

Self similarity of this curvature impregnated coastline is tested through fractal dimensioning of the coastline. In other words, it is a query of how smooth or rough the coast is. Two variables involved in divider method are: 1. Step length of the divider (S) and 2. Number of steps (N) required covering the entire stretch of the coastline (on trace). Then the fractal dimension (D) becomes the ratio of logarithm of N to logarithm of S. Here the fractal dimension D of a coastline can be derived from equation 2.

$L(S) = N\times S$ ... (1)

Where C is a positive constant.

Based on the fractal dimension the estimated length of the coast L(S) can be solved by

$L(S) = CS^{1-D}$ ... (2)

D must exceed 1 for a linear to be a fractal, because it is obvious that increasing step length decreases number of steps which warrants resorting of power function during the fractal analysis. The complexity of the line increases the value of D (Chattopadhyay et al., 2007). Thus, fractal dimension is estimated using the following equation:

$D = \text{Log}(N)/\text{Log}(S)$ … (3)

D can also be determined through a plot of N vs S on a log-log sheet (Figure 2. A and B); it is the slope of the curve fit of N-S graph. Corresponding equations of plots provide regression coefficients and significance of the fit as R^2. The values are given below Table.2.

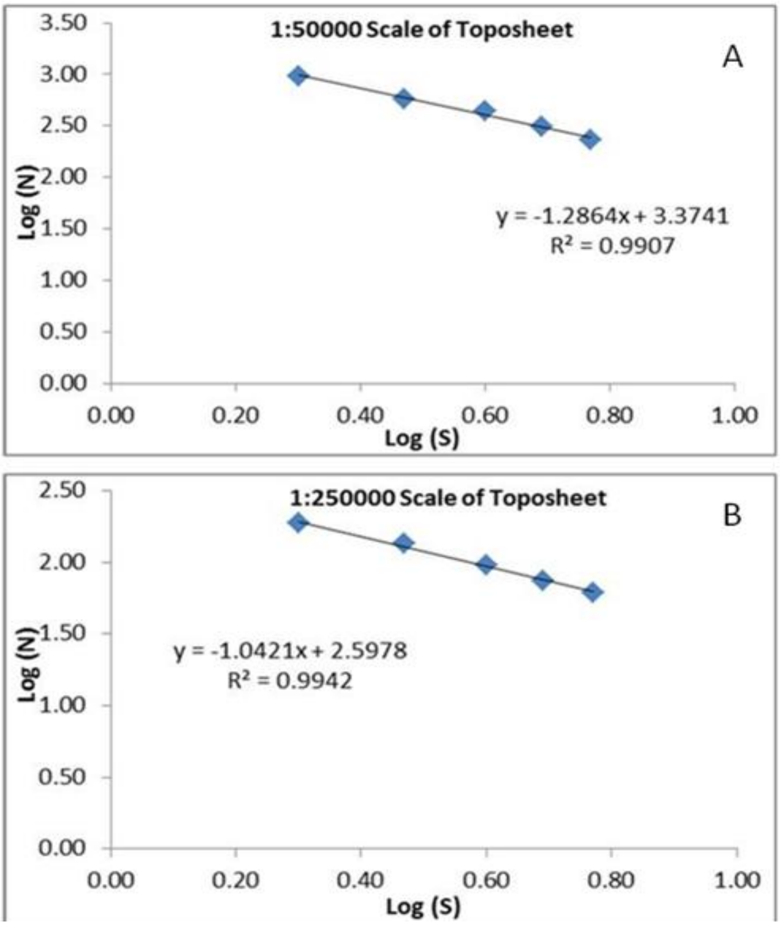

Figure 2: Fractal dimensions of Tamil Nadu coast: A: 1:50000; B: 1:250000

Table 1: Index numbers of Survey of India toposheets used in the analysis in two scales as indicated.

1:50000 scale	1:250000 scale
66C/6, 66 C/7, 66 C/8,	66 C
66 D/1and D/8, 66 D/2, 66 D/4,	66D
58 P/16, 58 M/13, 58 M/14, 58 M/15, 58 M/16	58 M
58 N/13, 58N/14, 58N/15, 58 N/11, 58 N/7 and 8,58 N/4	58 N
58 K/16, 58 K/12, 58 K/8 58 K/14	58 K
58 L/1, 58 L/2, 58 L/3,	58 L
58 H/3, 58 H/4, 58H/8, 58H/12, 58 H/15 and K/16	58 H
58 O/1, 58 O/2, 58 O/3, 58 O/4, 58 O/7, 58 O/8	58 O

Table 2: Data used to plot Fig. 2 A and B

Step Length (S) in cm	No. of steps (N)	Estimated Length of the coast (L) in Km	Log (S)	Log(N)	D= Log N/ Log S
1:50000 scale					
2	956	956.00	0.3010	2.9805	1.28
3	583	874.50	0.4771	2.7657	
4	437	874.00	0.6021	2.6405	
5	312	780.00	0.6990	2.4942	
6	234	702.00	0.7782	2.3692	
1:250000 scale					
2	188	940.00	0.3010	2.2742	1.04
3	136	1020.00	0.4771	2.1335	
4	96	960.00	0.6021	1.9823	
5	75	937.50	0.6990	1.8751	
6	62	930.00	0.7782	1.7924	

Result and Discussion

Fractal dimension of Tamil Nadu coastline is estimated using divider method. It is further determined using the least square method. To the plotted points trend lines are fitted and whose equations are $y = -1.2864x + 3.3741$ and $y = -1.042x + 2.5978$ for 1:50,000 and 1:2,50,000 scales, respectively. Applying Tanner et al., 2006's method D, the fractal dimension is calculated from the log-log sheet by step lengths and number of steps as follows: $D_{1:50000} = 1.29$ and $D_{1:250000} = 1.04$; this confirms fractal nature of the Coast.

Results were tested for statistical significance by T-test (Paired two samples for means Table 3). Finally trend lines are drawn and are obtained the slopes. The R^2 (coefficient of determination) value is nearly 0.99 or 99% for both plots of log N vs log S (Figure 2 A and B). This value demonstrates that the relation is highly linear.

Table 3: t-test statistic for estimated coast lengths at different scales.

Parameter	Map Scale	
	1:50000	1:250000
Mean	883.81	965.29
Variance	12817.83	1076.98
Pearson correlation	0.524	
Hypothesized mean difference	0	
df	6	
t-stat	-2.156	
P(T<=t)one tail	0.037	
T critical one-tail	1.943	
P(T<=t) two-tail	0.075	
T critical two tail	2.447	

Step length 1 (S = 1) was intentionally omitted, at the same is interpreted by looking at the graph. Hence Y-values are obtained by solving equations using S = 1 (Fig 4). Estimated ŷ values do not differ significantly as t-test provides evidence for the acceptance of null hypothesis. As intersection of curves of both scales provides average step length and average estimated coastal lengths and are as 1018.05 for 1:50000 scale and 988.25 for 1:250000 scale.

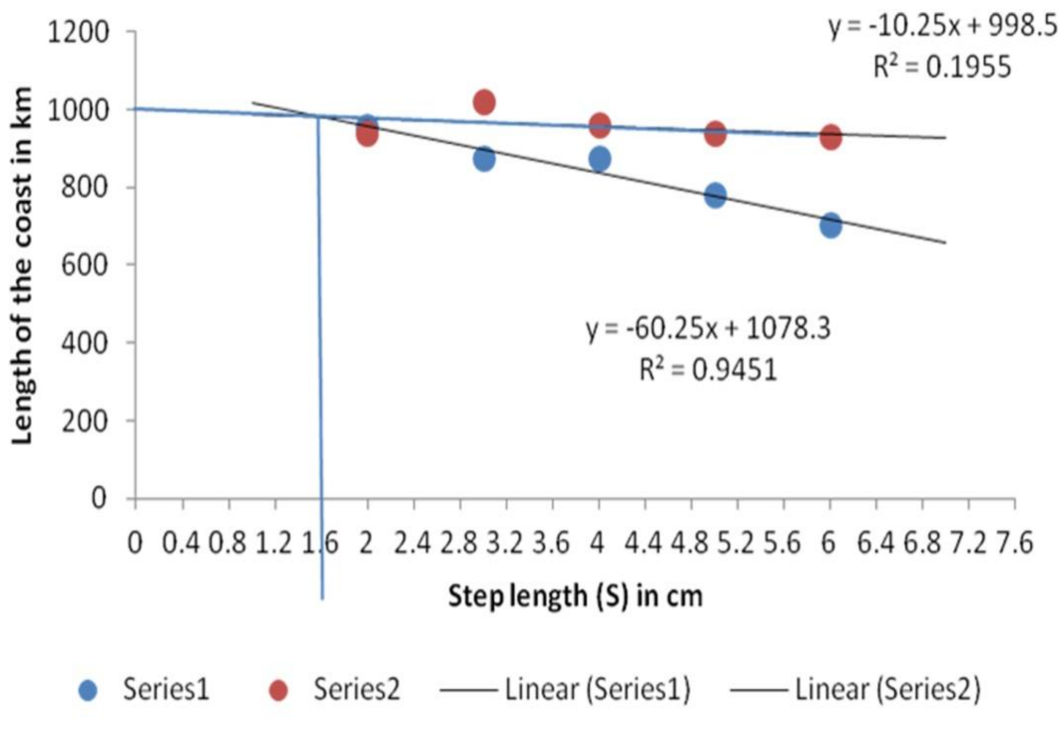

Figure 3: Determination of average step length and average coast length using fractal data of maps of different scales.

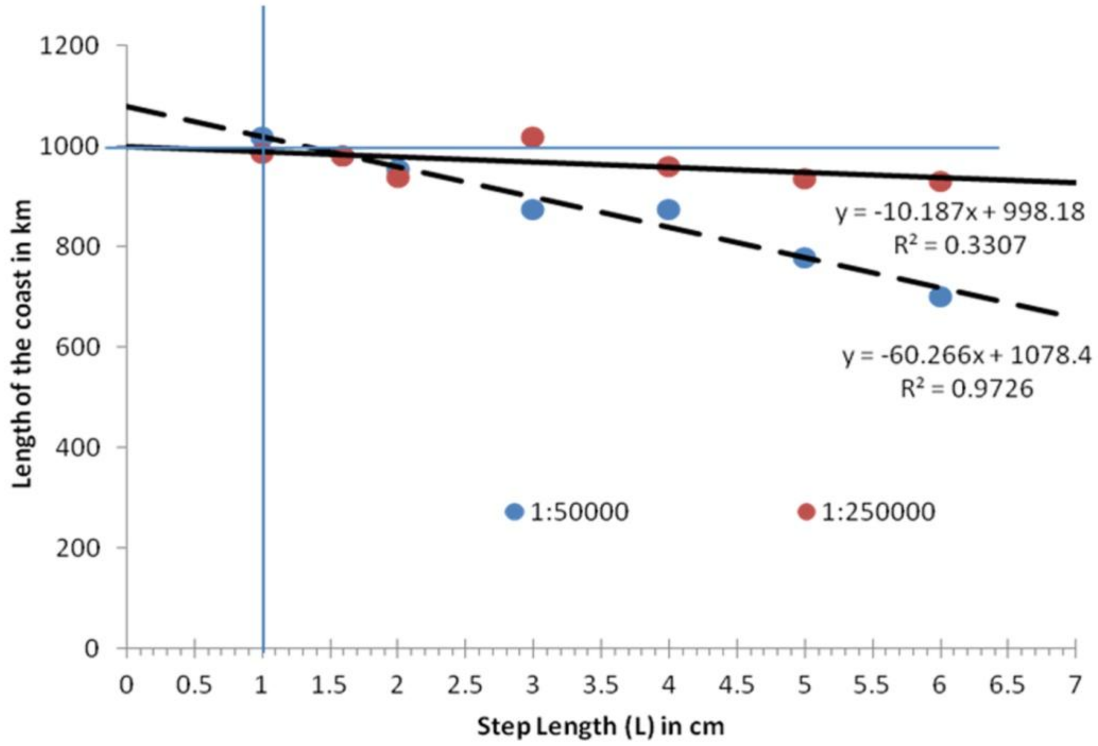

Figure 4: Determination of total coastal length of Tamil Nadu by interpolated for step length 1 using data on maps of different scales

Conclusion

Fractal dimensions of Tamil Nadu coast at two scales are as follow: FD_{50000} = 1.29 and FD_{250000} =1.04. By deploying data into a computer program the average fractal dimension is calculated as FD = 1.09. From the graphical plots of step lengths versus coastal length for two different scales of maps the attempt made to arrive an average step length that could reveal a more or less close approximation to the actual coastal length has been met through the interpolation of intersecting point of those two trend lines, i.e., average step length S = 1.6 cm and the corresponding coast length is 1078.05 Km for 1:50000 and 998.5 Km for 1:250000. Further interpolation was exercised to know coast lengths for supposed step length S = 1cm and are found to be 1078.4 Km for 1:50000 and 998.18 Km for 1:250000 respectively for different scale of maps considered.

t-test also reinforced the scale invariability and self-similarity properties of coasts, in general by the acceptance of null hypothesis.

It is found that Tamil Nadu coast is not any exception with regard to fractal concept.

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Reference

[1] Aschwanden, M., 2011. Self-organized criticality in Astrophysics: The statistics of nonlinear processes in the universe. Praxis books in Astronomy and planetary science.

[2] Chattopadhyay, S., and Kumar, S., 2007. Fractal dimension of selected coastal water bodies in Kerala, SW coast of India-A case study, Indian Journal of Marine Sciences, 36(2):162-166

[3] Falconer, K., 2013. Fractals-a very short introduction. Oxford Book.

[4] Kamalakannan, M., 2013. Fractal analysis of Nagapattinam coastline, India. Unpublished M.Sc. dissertation submitted to Bharathidasan University, Tiruchirappalli. 36p.

[5] Klinkenberg,B., and Goodchild, M., 1992.The fractal properties of Topography: a comparison of methods, Earth processes and landforms, 19:217-34.

[6] Mandelbrot,B., 1967. How long is the coast of Britain? Statistical self-similarity and fractional dimension Science, 156:636–638

[7] Menger, K., 1928. Dimensions theories, *B.G* Teubner Publishers

[8] Paszto, V., Marek, L., and Tucek, P., 2011. Fractal dimension calculation for chorine land-cover evolution in GIS – *A case study*, 196-205

[9] Peter, S., 2003. Fractals and Fractal Dimension; http://artemis.wszib.edu.pl/~sloot/2_2.html

[10] Radhakrishnan, V., 2002. A study of sediments of Tamirabarani River system, Thirunelveli district, Tamil Nadu, *Ph. D.*, Unpublished Thesis.

[11] Salingaros N.A., 2008. Connecting the Fractal Coast. Plenary Talk at the Laboratorio Internazionale d'Architettura. Università degli studi Mediterranea di Raggio Calebria, Italy.

[12] Shamsgovara, A., 2012. Analytic and numerical calculations of fractal dimension, Department of Mathematics, Royal Institute of Technology, KTH, Published Research Thesis.

[13] Southgate, H.N., Moller, I., 2000. Fractal properties of coastal profile evolution at Duck, North Carolina, Journal of Geophysical Research, 11:489-11,507

[14] Tanner, B.R., Perfect, E., and Kelley, J.T., 2006. Fractal analysis of Maine's glaciated shoreline tests established coastal classification scheme. Journal of Coastal Research, 22(5):1300-1304.