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Magnetic Helix - Relativistic Ampère-Maxwell Law from Geometric Viewpoint

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We think of relativistic Ampére-Maxwell equation from geometric viewpoint: this is an analogic discussion from [1].
As shown in Fig. 1, magnetic field in the static system κ is around the observer on the x-axis. However, as shown in Fig. 2, it will converge onto line in the x-axis drawing helix in the inertia system κ' (, it oddly suggests that photon has no spin though.)

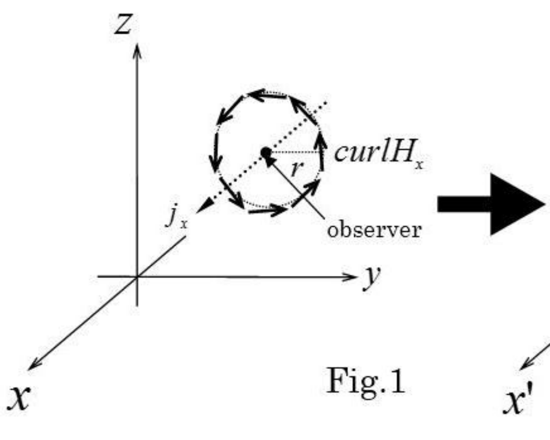

Fig.1

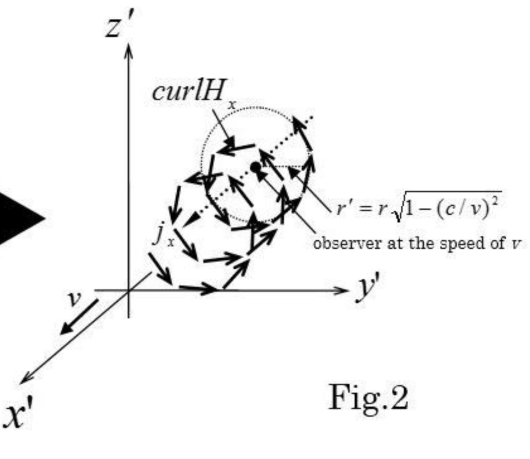

Fig.2

Let us think of the magnetic flux H around the observer in κ' as shown in Fig. 2. The original magnetic flux H in y-z plane converges onto x-axis as the observer's speed closes to the light one. Since the magnetic flux in κ' consists of two components of vector in the y-z plane and parallel to x-axis as shown in Fig. 3, $curlH$ in κ' is $curlH_x = \sqrt{1-(v^2/c^2)}curlH'_x$ - $(v/c)curlH'_x$.

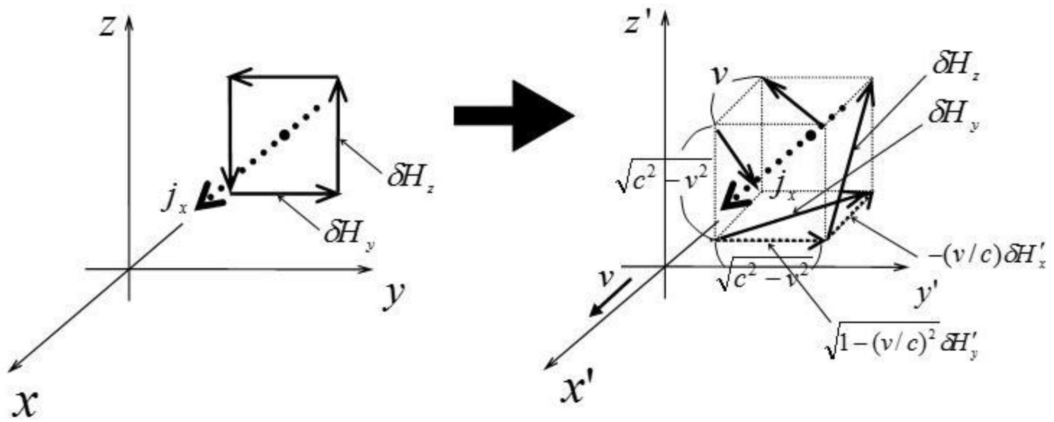

Fig. 3 (intuitive images for Fig. 1 and 2)

Since $(v/c)curlH'_x$ is vertical to $\sqrt{1-(v/c)^2}curlH'_x$, it could be considered as *D'* in the manner of [1]. Then, from $D = \frac{1}{c}H$ (: from $B = \frac{1}{c}E$, $B = \mu_0 H$, and $D = \varepsilon_0 E$),

$$curlH_x = \sqrt{1-(v/c)^2}curlH'_x - v\frac{\partial D'_x}{\partial x} = \sqrt{1-(v/c)^2}curlH'_x - \frac{\partial x}{\partial t}\frac{\partial D'_x}{\partial x} = \sqrt{1-(v/c)^2}curlH'_x - \frac{\partial D'_x}{\partial t}.$$

Since D'_x is equivalent to D_x by the consistency of light speed,

$$curlH_x = \sqrt{1-(v/c)^2}curlH'_x - \frac{\partial D_x}{\partial t}.$$

Therefore, the relativistic Ampére-Maxwell law is,

$$j_x = curlH_x = \sqrt{1-(v/c)^2}curlH'_x - \frac{\partial D_x}{\partial t}.$$

Likewise, $j_y = \sqrt{1-(v/c)^2}\, curlH'_y - \frac{\partial D_y}{\partial t}$ and $j_z = \sqrt{1-(v/c)^2}\, curlH'_z - \frac{\partial D_z}{\partial t}$ hold. Therefore,

$$j = \sqrt{1-(v/c)^2}\, curlH' - \frac{\partial D}{\partial t}.$$

Furthermore, since the propagation rate of electro-magnetic wave is always *c,* the equation of helix in κ' is

$$x' = -vt\ ,\quad y' = r\sqrt{1-(c^2/v^2)}\cos(c/r)t\ ,\quad z' = r\sqrt{1-(c^2/v^2)}\sin(c/r)t\ .$$

REFERENCES

[1] E. Miztani, Special Relativity from Geometric Viewpoint, Communications in Applied Geometry, Vol.1, No.1, 17-25, (2011)