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The Hexagon

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Abstract

We provide coordinates of a regular 6-gon

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There are some general facts about a regular n-gon, see [1], but we have found only one website about a *regular 6-gon* or *hexagon*.
There are approximate coordinates of a hexagon in the German wikipedia, see [2]. The exact values of a hexagon appear as a mystery. To our great surprise, we have found only one more information. It seems that the first who calculated the coordinates of a hexagon was the Indian Gopal Menon. See [3]. He yields formulas for a hexagon depending on a number a. Here are exact values if $a = 2$. It holds that

$$(-2,0);\left(-1,+\sqrt{3}\right);\left(+1,+\sqrt{3}\right);(+2,0);\left(+1,-\sqrt{3}\right);\left(-1,-\sqrt{3}\right)$$

are coordinates for a horizontal hexagon, and

$$(0,+2);\left(+\sqrt{3},+1\right);\left(+\sqrt{3},-1\right);(0,-2);\left(-\sqrt{3},-1\right);\left(-\sqrt{3},+1\right)$$

are coordinates of an upright hexagon.

Proof. An easy calculation shows that in the horizontal 6-gon the six interior angles have 120 degrees. All edgelengths are 2. The proof is done. □

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We have made the coodinates as simple as possible. The calculation of the above coordinates without [3] would be easy, if one knows that a hexagon consists of six equilateral triangles.

Now we make a conjecture about regular polygons.

Definition
A regular n-gon is a polygon with n vertices such that the edges have the same length, and all inner angles are equal.

Conjecture
Let $n > 2$ be a natural number, where n is not 4. It holds:
An n-gon is regular if and only if all inner angles are equal.

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REFERENCES

[1] https://math.stackexchange.com/q/117164

[2] https://de.wikipedia.org/wiki/Sechseck

[3] https://www.quora.com/How-can-you-find-the-coordinates-in-a-hexagon