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Sudoku

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Abstract

The foundation of this paper is the well-known Sudoko riddle. We ask how many fields at most can have a default entry such that there is only a single possibility to complete the board. As well as we ask how many fields can be left blank at most such that there is only a single possibility to complete the board. Further, we ask how many possibilities there are all in all to fill the board. We introduce some sequences.

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Nearly everybody knows the Sudoku game, which is the foundation of this paper. This riddle comes in millions of variations. There are easy and difficult ones, and they offer much entertainment for many. Besides from that, we can define some sequences.

In the following let n be a natural number, i.e. $\mathsf{n} \in \mathbb{N} := \{1, 2, 3, \ldots\}$.

We define a *Sudoku board* as a quadratic board of n^2 fields (n fields in each row). Every field has to be filled with one symbol out of a set of n different symbols, where somes rules have to be respected. The Sudoku board has n columns, which are numbered by 1,2,3 $\ldots$, $\mathsf{n} - 1$, n; as well as it has n rows, which are also numbered by 1,2,3 $\ldots$, $\mathsf{n} - 1, \mathsf{n}$.

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If n is a square number, i.e. $\mathsf{n} = \mathsf{k}^2$ for a natural number k, we define a *block* as a quadratic area of n fields, such that k fields are in every row of the block, and both the number of the first column - 1 and the number of first row - 1 of each block is devisible by k. If n is a square number, the Sudoku board consists of n disjunct blocks, each of them have n fields.

In the usual Sudoku game is $\mathsf{n} = 9$.

There are some rules to fill the Sudoku board:
- (1) In every column occurs all n symbols.
- (2) In every row occurs all n symbols.
- (3) If n is a square number, in every block occurs all n symbols.

We say that a Sodoku board is *filled* if and only if all n^2 fields have one symbol out of the set of the n symbols, where the rules (1) and (2) or, alternatively to another demand, (1), (2), and (3) have to be respected.

A filled Sudoku board is called a *solution*.

We name the fields of a Sodoku board by two indices (i, j), where i is the number of the column, j is the number of the row, i.e. $1 \leq \mathsf{i}, \mathsf{j} \leq \mathsf{n}$.

We say that two solutions are *different* if and only if there is a field such that the entry of one solution on this field differs from the other.

Now we introduce for every sidelength n of a Sudoku board 6 sequences. The elements are natural numbers or zero.

We define $2\ Rules(\mathsf{k})$, for all natural numbers k, as the number of possibilities to fill the board with different solutions, where we use a Sudoku board of sidelength n, and we respect the rules (1) and (2).

We define $3\ Rules(\mathsf{k})$, for all natural numbers k, as the number of possibilities to fill the board with different solutions, where we use a Sudoku board of sidelength n, and we respect the rules (1), (2), and (3).

We define $Default, 2\ Rules(\mathsf{k})$, for all natural numbers k, as the maximum natural number of the fields with a default entry, using a Sudoku board of sidelength n , such that there is exactly one possible way to complete the board, and we respect the rules (1) and (2).

We define $Default, 3\ Rules(\mathsf{k})$, for all natural numbers k, as the maximum natural number of the fields with a default entry, where we use a Sudoku board of sidelength n, such that there is exactly one possible way to complete the board, and we respect the rules (1), (2), and (3).

We define $Blank, 2\ Rules(\mathsf{k})$, for all natural numbers k, as the maximum natural number of fields that can be left blank such that the Sudoku board of sidelength n still has exactly one possible completion while respecting the Sudoku rules (1) and (2).

We define $Blank, 3\ Rules(\mathsf{k})$, for all natural numbers k, as the maximum natural number of fields that can be left blank such that the Sudoku board of sidelength n still has exactly one possible completion while respecting the Sudoku rules (1), (2), and (3).

Remark 0.1. The searched numbers in the last 4 sequences are less than n^2.

Remark 0.2. If n is not a square number, some elements of distinct sequences are equal because of (3).

Remark 0.3. The concept can be extended into higher dimensions.

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