Communications in Applied Geometry.
Volume 1, Number 1 (2011) pp. 17-25

https://dx.doi.org/10.37622/CAG/1.1.2011.17-25

Special Relativity from Geometric Viewpoint

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Abstract

Theory of special relativity mainly consists of the principle of relativity and derivation of relativistic electromagnetics by the principle. The principle of relativity is perfectly described by simple and visible geometry (, the original paper does not include graphs to explain it though.) However, the relativistic electric field and magnetic flux are derived by algebraic transformation based on the principle. Although it has no problem, it will be intuitionally understandable to derive also the electromagnatics such by geometry as well as the principle of relativity. In this paper, we discuss it from geometric viewpoint.

Keywords: relativistic aberration, relativistic Coulomb force, relativistic electromagnetic field, Lorenz Force, Fleming's left hand rule.

Relativistic Aberration

First of all, let us think of an observer when in the stationary system κ and inertia one κ' as shown in Figure1 and 2. A ray of light to the observer declines to the x' axis by aberration and *principle of the constancy of the light speed* as velocity of the observer *v* approaches to light velocity.

Furthermore, formula of the relativistic aberration is

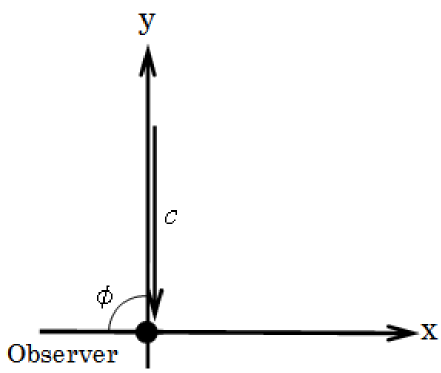
$$\cos\phi' = \frac{\cos\phi - v/c}{1 - \cos\phi \cdot v/c} \quad (1)$$

where ϕ and ϕ' are angles between the ray of light and x axis in κ, κ', and *c* is light velocity. Since $\phi = \pi/2$ in this case as shown in Figure 1, it results in

$$\cos\phi' = -v/c \quad (2),$$

$$\cos(\pi - \phi') = \cos\theta = v/c \quad (3)$$

It corresponds to what Figure 2 shows. Discussing special relativity from geometrical viewpoint, the relativistic aberration has a key role.



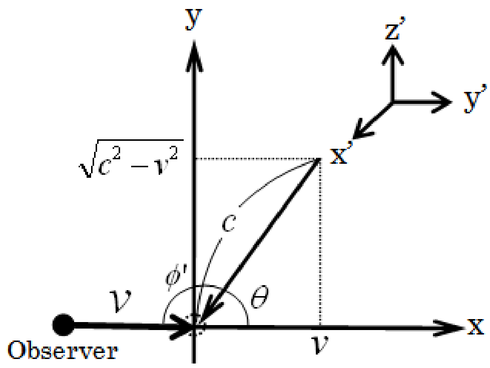


Figure 1 **Figure 2**

Relativistic Coulomb Force

We know that Coulomb force between electron and positron moving with relativistic speed is considerably reduced. It will be explained by the aberration we discussed above. As shown in Figure 3, when a pair of positron and electron travels horizontally at relativistic speed, an observer detects the electric field E (or $E'_{\perp}$ if expressing it by the coordinates of inertia system viewed from the observer in the stationary system) declined by the relativistic aberration.

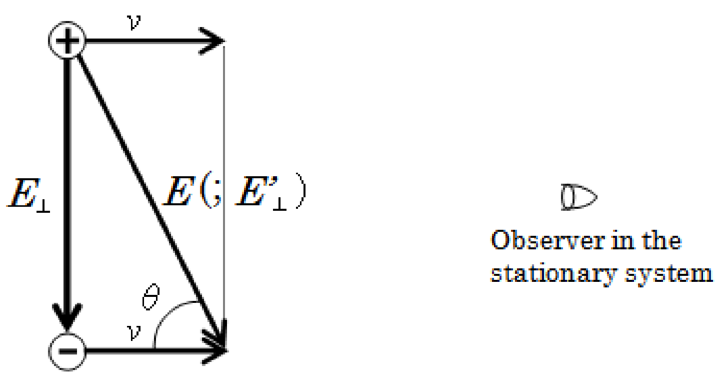


Figure 3

Since only the vertical component of the electric field is active for the electron, relativistic Coulomb force *F'* from the observer's viewpoint is

$$F' = qE_{\perp} \tag{4}$$

where q is quantity of electric charge, $E_\perp$ is vertical component of E expressed by the stationary system. Since $E_\perp = E\sin\theta$, Eq. (4) is

$$F' = qE\sin\theta \tag{5}$$

From Eq. (3), Eq. (5) is

$$F' = qE\sqrt{1-(v/c)^2} \tag{6}$$

It corresponds to the general expression.

Relativistic Electromagnetic Field

Relativistic magnetic flux density B'_x, B'_y, B'_z and electric field E'_x, E'_y, E'_z are expressed by

$$B'_x = B_x \tag{7}$$

$$B'_y = \beta\left(B_y + \frac{v}{c^2}E_z\right) \tag{8}$$

$$B'_z = \beta\left(B_z - \frac{v}{c^2}E_y\right) \tag{9}$$

$$E'_x = E_x \tag{10}$$

$$E'_y = \beta\left(E_y - vB_z\right) \tag{11}$$

$$E'_z = \beta\left(E_z + vB_y\right) \tag{12}$$

where $\beta = 1/\sqrt{1-v^2/c^2}$. Let us discuss those equations from geometric viewpoint. First of all, let us think of the magnetic flux. As shown in Figure 4, an observer moves at a speed v along the $+y$-axis and detects declined magnetic flux of the stationary system.

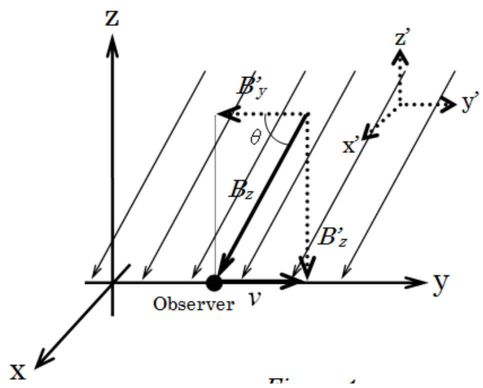


Figure 4

Expressing the magnetic flux by the coordinates of the inertia system,

$$B_z = B'_z \sin\theta + B'_y \cos\theta \tag{13.}$$

From Eq. (3),

$$B_z = \sqrt{1-\left(\frac{v}{c}\right)^2} B'_z + \frac{v}{c} B'_y = \frac{1}{\beta} B'_z + \frac{v}{c} B'_y \tag{14}$$

B_z denotes vector of declined magnetic flux density of the stationary system, B'_z the vertical component and B'_z horizontal one by the inertia system. Since B'_y is horizontal to direction of the observer moving with *v*, $B'_y = B_y$ by the constancy of light speed. So that Eq. (13) is

$$B_z = \frac{1}{\beta} B'_z + \frac{v}{c} B_y \tag{15.}$$

By the way, since B'_y (or B_y) is perpendicular to B'_z , it makes the observer look electric field. Considering it as if another field, then substituting Maxwell's equation $B = E/c$ to correct the magnitude gap between *B* and *E* , Eq. (15) is

$$B_z = \frac{1}{\beta} B'_z + \frac{v}{c^2} E_y \tag{16.}$$

Expanding B'_z ,

$$B'_z = \beta\left(B_z - \frac{v}{c^2} E_y \right) \tag{17.}$$

It corresponds to the original equation (9).

Let us think of B'_y in the same way. As shown in Figure 5, B_y is

$$B_y = B'_y \sin\theta + B'_z \cos\theta \tag{18.}$$

From Eq. (3),

$$B_y = \frac{1}{\beta} B'_y + \frac{v}{c} B'_z \tag{19}$$

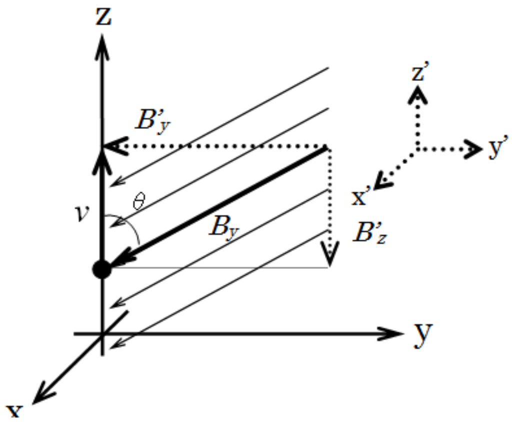


Figure 5

Since B'_z is parallel to direction of the moving observer, $B'_z = B_z$ by constancy of light speed. So that Eq. (19) is

$$B_y = \frac{1}{\beta} B'_y + \frac{v}{c} B_z \quad (20).$$

Since B'_z (or B_z) is perpendicular to B'_y , observer look electric field. Substituting Maxwell's equation $B = E/c$ to correct the magnitude gap between B and E , Eq. (20) is

$$B_y = \frac{1}{\beta} B'_y + \frac{v}{c^2} E_z \quad (21).$$

Expanding B'_y ,

$$B'_y = \beta (B_y - \frac{v}{c^2} E_z) \quad (22).$$

However, it does not correspond to the original equation (8). Now, to solve the problem, let us introduce Fleming's left hand rule applied to Lorenz force(; $F = q(E + v \times B)$) by assuming that the moving observer were electron and then pay attention to each direction of vectors B_y , B'_y , B'_z for the coordinates. First of all, let us set the rule in the coordinates as shown in Figure 6. Then, let us see the coordinates from different viewpoint as shown in Figure 7. Although the Lorenz force is negative(; $-F'_x$) in the coordinates of Figure 6, it is positive (; $+F'_x$) from different viewpoint in the coordinates of Figure 7. However, they are equivalent with each other. So that, let us unify all the settings by positive direction(; $+F'_x$) from now on.

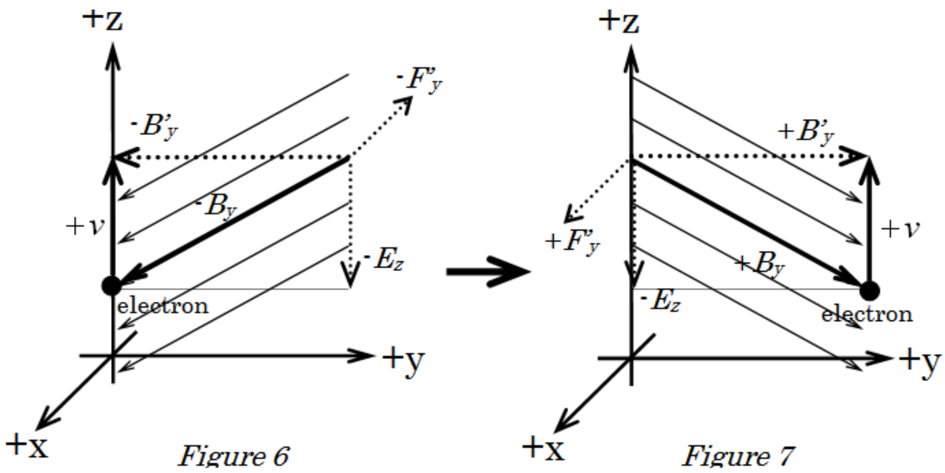


Figure 6 **Figure 7**

Again, let us reconsider Eq. (21) and (22) from the latter viewpoint. The revised equation is

$$B_y = \frac{1}{\beta} B'_y + \frac{v}{c^2}(-E_z) \quad (23).$$

Expanding B'_y ,

$$B'_y = \beta\left(B_y + \frac{v}{c^2} E_z\right) \quad (24).$$

It eventually corresponds to the original equation (8). Furthermore, let us reconsider a series of Eq. (13) to (17) similarly. Since Figure 4 is redrawn as shown in Figure 8, those therefore need to be corrected as follows: Eq. (13) is revised as

$$-B_z = -B'_z \sin\theta + \left(-B'_y\right)\cos\theta \quad (25).$$

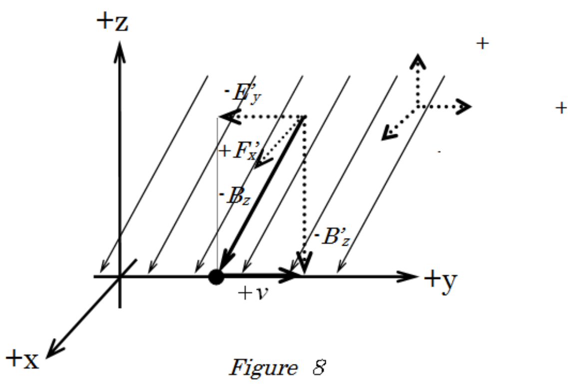


Figure 8

However, multiplying both sides of each equation by -1, it corresponds to Eq. (13).

Secondarily, let us think of relativistic electric field. As shown in Figure 9, the equation of electric field of the stationary system E_y is

$$-E_y = -E'_y \sin\theta + (-E'_z)\cos\theta \qquad (26).$$

Multiplying both sides of each equation by -1,

$$E_y = E'_y \sin\theta + E'_z \cos\theta \qquad (27).$$

From Eq. (3),

$$E_y = \frac{1}{\beta}E'_y + \frac{v}{c}E'_z \qquad (28).$$

E'_y denotes horizontal component and E'_z vertical one by the inertia system. Since E'_z is parallel to direction of moving observer, $E'_z = E_z$ by the constancy of light speed. So that Eq. (28) is

$$E_y = \frac{1}{\beta}E'_y + \frac{v}{c}E_z \qquad (29).$$

Since E'_z (or E_z) is perpendicular to E'_y , it makes observer look magnetic flux. Substituting Maxwell's equation $E = cB$ to correct the magnitude gap between B and E , Eq. (29) is

$$E_y = \frac{1}{\beta}E'_y + vB_z \qquad (30).$$

Expanding E'_y ,

$$E'_y = \beta(E_y - vB_z) \qquad (31).$$

It corresponds to the original equation (11).

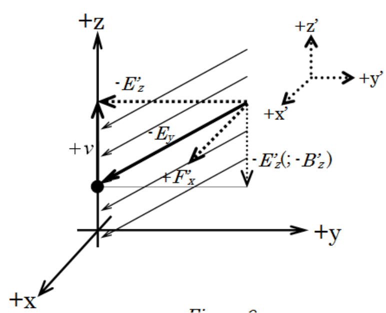


Figure 9

Let us think of E'_z in the same way. As shown in Figure 10, the equation of E_z is

$$E_z = E'_z \sin\theta + \left(-E'_y\right)\cos\theta \qquad (32).$$

From Eq. (3),

$$E_z = \frac{1}{\beta}E'_z - \frac{v}{c}E'_y \qquad (33)$$

Since E'_y is horizontal to direction of moving observer, $E'_y = E_y$ by constancy of light speed. Therefore, Eq. (33) is

$$E_z = \frac{1}{\beta}E'_z - \frac{v}{c}E_y \qquad (34).$$

Since E'_y , (or E_y) is perpendicular to E'_z , it makes observer magnetic flux. Substituting Maxwell's equation $E = cB$ to correct the magnitude gap between B and E , Eq. (34) is

$$E_z = \frac{1}{\beta}E'_z - vB_y \qquad (35)$$

Expanding E'_z ,

$$E'_z = \beta(E_z + vB_y) \qquad (36).$$

It corresponds to the original equation (12).

At last, original equations (7) and (10) are easily verified by constancy of light velocity as shown in Fig.11.

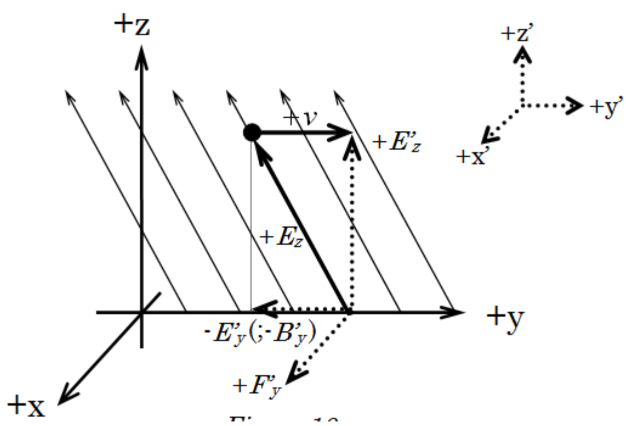


Figure 10

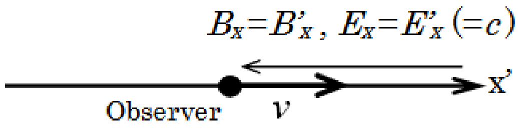


Figure 11

Acknowledgment

I appreciate Sibbett Frank Sapp with his support.

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