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Geographically Weighted Regression Modeling For Corn Production In Central Java

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ABSTRACT

The main issue in enhancing the corn production lies on the dependency on certain areas as the main producers of corn in Central Java. The difference in production most frequently not only makes the needs for corn in many areas unfulfilled but also leads the price for this crop unequal. To fulfill the needs for corn in Central Java, the mapping of the corn producer areas aimed at enhancing the potential of the areas in corn production is deemed essential. Meanwhile, a special concern will be given for the areas that have insufficient number of corn production. In this case, considering the difference of production in some areas in Central Java that are still relied highly on the climate and soil condition suitable for corn plant, we propose the model of corn production by using Geographically Weighted Regression (GWR) method.Based on the GWR model for each regency and city, it was found that the highest production of corn was from Grobogan, Blora and Wonogiri Regencies with R^2near 1.

Keywords: Geographically weighted regression, kernel function, bandwidth.

Mathematics Subject Classification: 62J05, 62-07

1. Introduction

Corn refers to the second-ranking crop commodity in Central Java after paddy that is widely consumed by most of society. In addition to be crop commodity, corn is mostly used as feedstuff, pharmaceutical industry, or food industry. This commodity has a strategic and political characteristic in considering that the stability in availability and price of commodity come to the indicators of government achievement. Hence, the existence and sufficiency of such commodity must always be a concern for the increasing number of demand coincided with the increasing number of population in Indonesia, particularly in Central Java.

An attempt to fulfill the food needs through the achievement of food self-sufficient, in this case corn, has not shown any optimal result. This can be reflected from the level of the availability of domestic food commodity that is still in high reliance on import of corn that accounts for 11 percent. However, through the Development Program of National Food Securityin 2012, the production of corn commodity in Central Java in comparison to the production in 2011 has been increasing by 9.7 percent amounting for 3.04 million tons of dried shelled corn and has been increasing in production by 269.06 thousand tons.

The policy in the price of staple food is one of essential instruments in prompting the national food security. Since the food price is highly determined by the availibility or production of food, an attempt to predict the corn production in future is deemed necessary. There are some methods that can be used to predict the production of the staple food and to observe its influencing factors, one of which is by regression analysis.

The main issue in enhancing the corn production lies on the dependency on certain areas as the main producers of corn in Central Java. The land mass with different natural resources has led to the variance of productions in any areas. Most of areas in Central Java possess good potentials for the development of corn plant. However, since it does not provide any significant profit compared to other plant commodities, the corn is planted only in certain areas as their geographic condition is not suitable for other plant commodities. As a consequence, the difference in production most frequently not only makes the needs for corn in many areas unfulfilled but also leads the price for this crop unequal. To fulfill the needs for corn in Central Java, the mapping of the corn producer areas aimed at enhancing the potential of the areas in corn production is deemed essential. Meanwhile, a special concern will be given for the areas that have insufficient number of corn production. In this case, considering the difference of production in some areas in Central Java that are still relied highly on the climate and soil condition suitable for corn plant, the method of Geographically Weighted Regression (GWR) is used purposely to arrange the model of corn production.

2. Model of Geographically Weighted Regression (GWR)

This model is a development of a regression model in which each parameter is measured in each of observation site. In this way, each of the observation sites will have a different value of regression parameter. Model of GWR is a development of global regression model in which its basic concept has been taken from the non-parametric regression (Mei, *et al.*, 2006). The variable of *Y* response in GWR model is predicted by means of predictor variables, the regression coefficient of which depends on the location where the data is observed. The model of GWR can be written as follows (Fotheringham, *et al.*, 2002):

$$y_i = \beta_0(u_i, v_i) + \sum_{k=1}^{p} \beta_k(u_i, v_i)\, x_{ik} + \varepsilon_i \qquad (2.1)$$

where

y_i : The observation value of the response variable for location *i*

(u_i, v_i) : Longitude and latitude coordinate from the observation location *i*

$\beta_k(u_i, v_i)$: Regression coefficient for the variable of predictors *k*in the observation location

x_{ik} : The observation value for the variable of predictor *i* assumed as identical, independent, and normally distributed with the zero mean and constant variance of σ^2.

2.1.Estimation of Parameter of GWR Model

Estimating the parameter of GWR model was performed by means of Weighted Least Squares(WLS) method by giving a different weight for each location where the data was observed. The weighting was in accordance with the Law I of Tobler: “Everything is related to everything else, but near thing are more related than distant things”.Hence, in GWR model, it is assumed that the area near the observation site *i*had a more significant impact on its parameter estimation compared to the distant ones. To illustrate this, the weight for each location (u_i, v_i) is $w_j(u_i, v_i)$, *j* = 1, 2, …,*n*, then the parameter in the observation site (u_i, v_i) is estimated by adding the weight elements of $w_j(u_i, v_i)$ in the equation (2.1) prior to minimum the number of the following residual square:

$$\sum_{j=1}^{n} w_j(u_i, v_i)\, \varepsilon_j^2 = \sum_{j=1}^{n} w_j(u_i, v_i) \left[y_j - \beta_0(u_i, v_i) - \sum_{k=1}^{p} \beta_k(u_i, v_i)\, x_{jk} \right]^2$$

In the form of matrix, the number of residual square is written as:

$$\begin{aligned} \boldsymbol{\varepsilon}^T \mathbf{W}(u_i, v_i)\, \boldsymbol{\varepsilon} = {} & \mathbf{y}^T \mathbf{W}(u_i, v_i)\, \mathbf{y} - 2\boldsymbol{\beta}^T(u_i, v_i)\, \mathbf{X}^T \mathbf{W}(u_i, v_i)\, \mathbf{y} \\ & + \boldsymbol{\beta}^T(u_i, v_i)\, \mathbf{X}^T \mathbf{W}(u_i, v_i)\, \mathbf{X}\boldsymbol{\beta}(u_i, v_i) \end{aligned} \qquad (2.2)$$

where

$$\boldsymbol{\beta}(u_i, v_i) = \begin{pmatrix} \beta_0(u_i, v_i) \\ \beta_1(u_i, v_i) \\ \vdots \\ \beta_p(u_i, v_i) \end{pmatrix} \text{ and } \mathbf{W}(u_i, v_i) = \text{diag}\left[w_1(u_i, v_i), w_2(u_i, v_i), \cdots, w_n(u_i, v_i) \right].$$

If the equation (2.2) is generated towards $\boldsymbol{\beta}^T(u_i, v_i)$ and the result is equalized with zero, then the estimator of parameter of GWR model is obtained.

$$\frac{\partial \boldsymbol{\varepsilon}^T \mathbf{W}(u_i,v_i)\boldsymbol{\varepsilon}}{\partial \boldsymbol{\beta}^T(u_i,v_i)} = 0 - 2\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{y} + 2\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\boldsymbol{\beta}(u_i,v_i) = 0$$

$$-2\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{y} + 2\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\boldsymbol{\beta}(u_i,v_i) = 0$$

$$\left[\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\right]^{-1}\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\boldsymbol{\beta}(u_i,v_i) = \left[\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\right]^{-1}\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{y}$$

$$\hat{\boldsymbol{\beta}}(u_i,v_i) = \left[\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\right]^{-1}\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{y}\ . \qquad (2.3)$$

For example, $\mathbf{x}_i^T = (1, x_{i1}, x_{i2}, \cdots, x_{ip})$ is the element of *i*line from the Matrix X. So, the value of prediction for *y*in the location of observation (u_i,v_i) can be obtained through the following method:

$$\hat{y}_i = \mathbf{x}_i^T\hat{\boldsymbol{\beta}}(u_i,v_i) = \mathbf{x}_i^T\left(\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{y}$$

Thus, for all observations, it can be written as follows:

$$\hat{\mathbf{y}} = (\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_n)^T = \mathbf{L}\mathbf{y} \text{ and}$$

$$\hat{\boldsymbol{\varepsilon}} = (\hat{\varepsilon}_1, \hat{\varepsilon}_2, \cdots, \hat{\varepsilon}_n)^T = (\mathbf{I} - \mathbf{L})\mathbf{y}$$

with **I**is the matrix of identity in the size of *nxn* and

$$\mathbf{L} = \begin{pmatrix} \mathbf{x}_1^T\left(\mathbf{X}^T\mathbf{W}(u_1,v_1)\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}(u_1,v_1) \\ \mathbf{x}_2^T\left(\mathbf{X}^T\mathbf{W}(u_2,v_2)\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}(u_2,v_2) \\ \vdots \\ \mathbf{x}_n^T\left(\mathbf{X}^T\mathbf{W}(u_n,v_n)\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}(u_n,v_n) \end{pmatrix} \qquad (2.4)$$

The estimator of $\hat{\boldsymbol{\beta}}(u_i,v_i)$ in equation (2.2) is the estimator that is unbiased and consistent for $\boldsymbol{\beta}(u_i,v_i)$ (Nurdim, 2008).

2.2. Weighting of GWR Model

The role of weighting in GWR model is very crucial since the value of weighting represents the position of observation data one to another. The scheme of weighting in GWR can be applied in some various methods. There are some literatures that can be used to determine the level of weighting for each different location to GWR model, one of which by usingkernel functions.

Kernel function is used to estimate the parameter in GWR function if the function of the distance (w_j) refers to a continuous and monotonous function turun (Chasco,*et al*, 2007). The weightings formed using this kernel function include Gaussian distance function,exponentialfunction, bisquarefunction, and kernel function of tricube. The followingweighting function of Gaussian (Lesage, 2001)was selected in the analysis performed here:

$$w_j(u_i,v_i) = \phi\left(d_{ij}/\sigma h\right)$$

with ϕ as a standard normal density, and σ representing the standard deviation of the distance vector d_{ij}, $d_{ij}=\sqrt{(u_i-u_j)^2+(v_i-v_j)^2}$ as the euclideandistance betweeen the locations of (u_i,v_i) and the location of (u_j,v_j) and h as the known non-negative parameter commonly called as bandwidth.

There are some methods used to select the optimum bandwidth such as the method of cross validation(*CV*) that is defined as follows:

$$CV(h)=\sum_{i=1}^{n}\left(y_i-\hat{y}_{\neq i}(h)\right)^2$$

with $\hat{y}_{\neq i}(h)$ as the estimator value of y_i and with the observation in the location (u_i,v_i) eliminated from the estimating process. The optimal value of h was obtained from h resulting in a minimum of *CV*value.

Some methods were used to select the best model. Akaike Information Criterion(*AIC*) is one of them and is defined as follows:

AIC = D(G) + 2K(G)

with $D(G)=\sum_{i=1}^{n}\left\{y_i\ln\left(\hat{y}_i(\beta(u_i,v_i),G)/y_i\right)+\left(y_i-\hat{y}_i(\beta(u_i,v_i),G)\right)\right\}$

*D(G)*is a devians value of model with the bandwidth *G*and *K*is the number of parameters in the model with bandwidth *G*. The best model is the one with the least *AIC* value.

2.3. Hypothesis Test on GWR Model

The test of significance of parameter of GWR model in each location was conducted by partially testing the parameter. Such testing was done to observe which parameter is significant in impacting their response variable. The form of hypothesis is presented as follows:

$$\mathrm{H}_0:\beta_k(u_i,v_i)=0$$

$$\mathrm{H}_1:\beta_k(u_i,v_i)\neq 0 \text{ with } k=1,2,\cdots,p$$

The estimator of parameter $\hat{\boldsymbol{\beta}}(u_i,v_i)$ will follow the multivariat normal distribution on the average of $\boldsymbol{\beta}(u_i,v_i)$ and covariant-variant matrix of $\mathbf{C}_i\mathbf{C}_i^T\sigma^2$ with $\mathbf{C}_i=\left(\mathbf{X}^T\mathbf{W}(u_i,v_i)\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}(u_i,v_i)$, thus, it is obtained $\frac{\hat{\beta}_k(u_i,v_i)-\beta_k(u_i,v_i)}{\sigma\sqrt{c_{kk}}}$ ~N(0,1)with c_{kk} as the diagonal element of $k$from the matrix of $\mathbf{C}_i\mathbf{C}_i^T$. Thus, the test statistic used is

$$T_{hit}=\frac{\hat{\beta}_k(u_i,v_i)}{\hat{\sigma}\sqrt{c_{kk}}}.$$

Under H_0, Twould follow the distribution of t with the free level of $\left(\frac{\delta_1^2}{\delta_2}\right)$. Meanwhile, $\hat{\sigma}$ was obtained by rooting $\hat{\sigma}^2 = \frac{\text{RSS}(\text{H}_1)}{\delta_1}$. If the significance level given was α then H_0would be rejected or in other words, the parameter $\beta_k(u_i, v_i)$was significant to the model if $|T_{hit}| > t_{\alpha/2, df}$, where $df = \left(\frac{\delta_1^2}{\delta_2}\right)$.

The method of Breusch-Pagan (*BP*) test can be used to test the spatial heterogeneity(Anselin 1988). The hypothesis is as follows:

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_n^2 = \sigma^2$ (homoscedasticity)

H_1: minimum one $\sigma_i^2 \neq \sigma^2$ (heteroscedasticity)

The value of *BP* test is $BP = (1/2)\,\mathbf{f}^T\mathbf{Z}\left(\mathbf{Z}^T\mathbf{Z}\right)^{-1}\mathbf{Z}^T\mathbf{f} \sim \chi^2_{(p)}$

with

$e_i = y_i - \hat{y}_i$: The least square if residual for the observation$i$

$\mathbf{f} = (f_1, f_2, \cdots, f_n)^T$ with $f_i = \left(\frac{e_i^2}{\sigma^2} - 1\right)$

$\mathbf{Z}$: A matrix in the size of $n \times (p+1)$ containing the vector that has been at normal standard (Z) for each observation.

$H_0$is rejected if $BP > \chi^2_p$ with p representing the number of predictors.

3. Research Method

Data used in this research was the secondary data obtained from the relevant government institutions. The data is related to the data of corn in 2012 from the Indonesian Ministry of Agriculture and StatisticsIndonesia.

In this research, the data was presented in each of regional or regency/city unit. Thus, the analysis based on the location is very important since the ignorance on it can make the estimation unefficinet and the conclusion inaccurate. The steps taken in this research included:

1. Determining the model of linear regression using OLS to obtain an independent that can have an influence for corn production.
2. Testing the assumption of normality, homoscedasticity, multicollinearityfrom the model of linear regression
3. Testing the spatial heterogeneity
4. Determining the weighting used in GWR model
5. Estimating the parameter of GWR model
6. Testing the hypothesis of GWR model
7. Conducting the best model selection using *AIC* and R^2.

4. Results and Discussion

4.1 Regression Model of Corn in Central Java in 2012

The regression analysis was initiated by determining the dependent and independent variable. Based on the case of the data about the corn in 2012, production (Y) was used as the dependent variable; on the other hand, the independent variable was taken from the crop area (X_1), rainfall (X_2), temperature (X_3), humidity (X_4), and labor (X_5). Once given the analysis, the model of regression can be written as follows:

$$Y = 173663.2 + 5.123494X_1 + 45.61485X_2 - 159.7547X_3 - 3033.016\,X_4 - 0.0528042$$

The regression model had the value of R^2-adjusted at 0.649280 or 64.928%; thus, it can be interpreted that 64.928% of variable Y (corn production)was explained by X variables. Meanwhile, the remains at 35.072% were explained by other variables. Having obtained the regression model, it was followed by searching the best regression model by looking at the most significant independent variable towards the dependent variable. By using the significance test $\alpha = 0.05$, a result was obtained that X variable significantly had an influence towards the Y variable (production).

From the significance test of parameter, it was found that the crop area (X_1) was significantly influential. Thus, the data for the corn in Central Java in 2012 had the best regression model:

$$Y = 21151.51 + 5.016767X_1$$

The regression model has the value of R-squared adjusted at 0.60860 or 60.86%. Thus, it can be said that 60.68% of Y variable (production) was explained by X_1 variable, while the rest at 39.32% was explained by other variable. Having obtained the best regression model for the data of corn in Central Java in 2012, the test of normality assumption was performed with $\alpha = 0.05$and it can be concluded that the residual was not normally distributed and the assumption of normality was not fulfilled. The test of non-multicollinearitywas performed to observe the relationship between the independent variable in regression model. Through the calculation, it was found that the value of multicollinearitywas at 2.030403;thus, it can be concluded that the assumption of non-homoscedasticity with Breusch-Pagan test,and with the probability value of 0.2479630and $\alpha = 0.05$, it can be concluded that there was no any heteroscedasticity. In previous research, according to Rousseeuwand Yohai (1984)in order to cope with the unfulfilled assumption of normality, robust regression model can be used (SusantiandPratiwi, 2011; Susanti, *et al.*, 2009; Susantiand Pratiwi, 2012). However, in this research, by weighting in GWR model, it is expected that it can result a good model.

4.2. Geographically Weighted Regression Model

Once checking was conducted in terms of spatial effect,it was found that the data of corn in Central Java Province with the data in each city/regency had no any spatial effect both in lag and in error.Hence, it was then attempted to use the method of spot approach using GWR model with the model obtained in the previous best regression model.

The step to model with GWR was by selecting the best bandwidth, thus, it was possible to determine the weighting matrix. Following this, the parameter was estimated and the local determination coefficient was determined in order to create the model for each observation spot.

For the data of corn in 2012 in Central Java province, it was found that the best bandwidthwas at 1.319with *CV*of 525840411.5.The function of weighting used was kernel Gaussian, with the longitude and the latitude for each observation spot with 31 data of observation spots and 35 observation spots. The GWR model could not be done if the data was valued 0. Thus, this data would be deleted. In this way, GWR model for 31 regencies/cities after obtained the bandwidth, longitudeand latitude,and estimation of parameter was obtained as in the result presented in Table 1.

Table1.The Estimation and the Test of Parameter from GWR model to Regency/City in Central Java

Regency/ City	β_0	*t*	β_1	*t*	Local R^2
Cilacap	5105.642509	0.972850	5.264407	27.369733	0.953035
Purworejo	4349.343979	0.892310	5.265329	29.682001	0.967388
Magelang	4249.805779	0.873457	5.269813	29.855530	0.969870
Klaten	4174.700726	0.845847	5.261021	29.745685	0.972564
Sukoharjo	4147.725203	0.830188	5.257583	29.632278	0.973832
Wonogiri	4187.803911	0.824852	5.248788	29.390371	0.974887
Karanganyar	4117.499283	0.814979	5.255689	29.530463	0.974658
Temanggung	4300.922656	0.886326	5.277161	29.853183	0.968833
Kendal	4310.667371	0.887879	5.282646	29.836803	0.968928
Pemalang	4787.689640	0.962639	5.286749	28.999232	0.960634
Tegal	4992.407987	0.983502	5.287144	28.401738	0.957191
Brebes	5208.954726	1.000000	5.286207	27.611971	0.953307
Kt Salatiga	4177.375751	0.853466	5.271308	29.852515	0.971586
Kt Semarang	4219.851144	0.864936	5.280420	29.839695	0.970652
Jepara	4122.839805	0.819963	5.285651	29.420539	0.972416
Kudus	4051.505227	0.807199	5.274634	29.551094	0.973440
Pati	3991.047571	0.784176	5.270895	29.395657	0.974153
Rembang	3859.307911	0.733644	5.258695	29.036330	0.975442
Demak	4134.160363	0.837666	5.278036	29.737264	0.972220
Semarang	4173.665950	0.852440	5.272262	29.849623	0.971630
Boyolali	4144.120704	0.840454	5.266997	29.792384	0.972613
Sragen	4081.475288	0.811125	5.261100	29.592627	0.974334
Blora	3940.334245	0.756709	5.255634	29.198170	0.975609
Grobogan	4073.712536	0.815904	5.269258	29.658514	0.973569
Banyumas	4887.987260	0.961148	5.270810	28.347334	0.957400
Wonosobo	4393.483552	0.903470	5.274017	29.716541	0.966723
Pekalongan	4605.789654	0.939554	5.285513	29.422155	0.963644
Batang	4459.319452	0.917141	5.285678	29.689167	0.966303

Kebumen	4531.031211	0.919467	5.265962	29.296252	0.963525
Purbalingga	4720.594233	0.918832	5.276681	28.996900	0.960812
Banjarnegara	4540.129803	0.926544	5.276298	29.450413	0.964014

From the analysis, it was found that of 31 regencies/cities, the variable of crop area was significantly influential for the number of corn production in which the best model in the perspective of R^2value was Bloraregency with R^2= 0.9756and AIC=707.309136. GWR model for this regency was $Y_1 = 3940.334245 + 5.255634\ X_1$.This indicates that 97.56% of production can be explained by the crop area while 2.34 % of the restcan be explained by other variables. From the test of significance of model parameter that has been performed, it was found t_{value}=29.198 with α=0.05. Thus, it can be concluded thatparameter $\beta_1$is influential for the model meaning that the variable crop area was significantly influential for the number of crop production in which in each increase of 1 hectare of crop area of corn it can increase the number of corn production at 3945.6 tons.Besides, from the value of local R^2near 1 showed that the model is quite suitable. This can be seen from other regency/city that also produced the good model of GWR regression.

From the GWR model obtained, it was then continued by predicting the corn production. This later has created the following mapping. Figure 1 shows that the prediction areas of the highest corn production included Grobogan, Blora and Wonogiri Regencies.

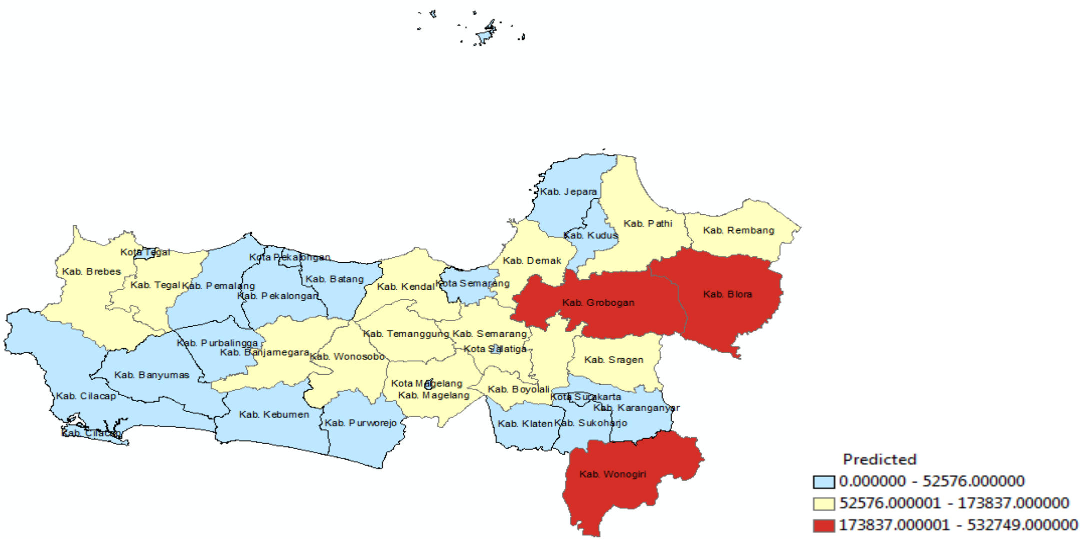

Figure 1. The Mapping of Corn Production Prediction in Central Java in 2012

5. Conclusion

Based on the GWR model for each regency/city, it was found that the highest production of corn was from Grobogan, Blora and Wonogiri with the GWR model of

$Y_1 = 4073.712536 + 5.269258\,X_1$ with R^2= 97.36%, $Y_1 = 3940.334245 + 5.255634\,X_1$ with R^2= 97.56%,and $Y_1 = 4187.803911 + 5.248788\,X_1$ with R^2= 97.49%, respectively.

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