Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 1 (2015), pp. 205-213

http://www.ripublication.com

# Implementation Of Fuzzy Analytic Hierarchy Process For Selection Of Location For Constructing A Firm

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**Abstract**

Manufacturing companies play a vital role in selection of location for constructing a firm. Selecting a site is important because they are costly and difficult to reverse, and they entail a long-term commitment. It also had an impact on operating costs and revenues. For instance, a poor choice of location might result in excessive transportation costs, a shortage of qualified labour, less competitive advantage, inadequate supplies of raw materials, or some similar condition that would be detrimental to operations. Fuzzy Multi Criteria Decision Making Method [FMCDM] is used for dealing the imprecise (or) vague nature of linguistic assessment. Fuzzy linguistic term approach is applied to capture the fuzziness and subjectiveness of decision makers' judgments. In Fuzzy Analytic Hierarchy Process [FAHP], α-cut based method has been utilized to prevent the controversial of fuzzy number ranking process. Confidence level and the optimistic level of decision maker are captured by using α-cut based fuzzy number results.

**Keywords:** Selection of Location, Fuzzy Multi Criteria Decision Making, Fuzzy Analytic Hierarchy Process.

## 1. Introduction

Methodological tool for dealing real world complex engineering problem is Multiple Criteria Decision Making (MCDM).Decision makers face many problems with incomplete and vague information in MCDM problems. Inability of normal decision making methods, address the imprecision and uncertainty paved the path for fuzzy decision making techniques. Goals, Constraints and consequences are known imprecisely in much of the real world decision-making processes and in such a situation fuzzy set theory becomes functional [1].

In FMCDM [3], performance ratings and weights are usually represented by fuzzy numbers. An alternative is calculated by aggregating all criteria weights and alternatives ratings, where alternatives with a higher utility are preferred. Since location selection is basically determined by subjective perceptions and feelings towards each of the evaluated criterion, FMCDM approach can be more suitable to explain how customers make decisions to select the best location for constructing a firm.

Fuzzy set theory has proven advantages within vague, imprecise and uncertain contexts. In [8] one basic application of fuzzy set theory is Fuzzy Synthetic Evaluation (FSE), which is a decision-making approach within a fuzzy environment. AHP can give a comprehensive and consistent analysis on the weights of all factors; therefore, many works on the integration of FSE and AHP have been performed to obtain the benefits from both. In [2] researchers use a membership function to describe how the alternatives satisfy the criteria (factors) and then use an AHP to make the decision. Some works use the AHP to get the weights of the factor only, fuzzify the weights, and then use FSE or ranking as the decision-making strategy. [7] Employed triangular fuzzy to represent a pairwise comparison ratio in AHP instead of exact numbers and a new decision-making methodology called FAHP came to exist. In [4] Fuzzy VIKOR method is used for assessing the qualitative factors and mathematical model such as Fuzzy Mixed Integer Linear programming is used for assessing the quantitative factors.

## 2. Fuzzy Analytic Hierarchy Process

FAHP provides a systematic method for comparison and weighting of the multiple criteria, alternatives to decision makers in the case of incomplete information. Instead of single crisp value, FAHP used a range of value to incorporate decision maker's uncertainty [6].From this range, decision maker can select the value that reflects his confidence and also he can specify his attitude like optimistic, pessimistic or moderate [5]. Optimistic attitude is represented by the highest value of range, moderate attitude is represented by the middle value of the range and pessimistic attitude is represented by the lowest value of the range.

In FAHP approach, we use triangular fuzzy numbers for the simplicity, effectiveness, fuzzification of the crisp Pairwise Comparison Matrices (PCM). It is fuzzified by the triangular fuzzy number ( $l, m, u$ ) where $l$ and $u$ are the lower and upper bound which represent the vagueness in scores. Basic concept of fuzzy extent analysis is to obtain the criteria importance and alternative performances by solving these fuzzified reciprocal PCMs. After obtaining the fuzzy performances, the ultimate aim will be to get the final results in crisp form. Therefore, the fuzzy performance matrices are transformed into interval performance matrices using the $\alpha$-cut concept. Then, to obtain the crisp output, the concept of optimism index is introduced $\lambda$. $\alpha$-cut analysis will avoid complex comparison of fuzzy utilities. $\alpha$-cut analysis allows to incorporate ambiguity in expert knowledge and the optimism index $\lambda$ to address the decision makers attitude.

**2.1 Methodology of FAHP**

Step 1: Acquisition of Crisp PCM and Fuzzyfying the Crisp PCM to Fuzzy PCM

Crisp PCM, *A*,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \tag{1}$$

The fuzzy PCM, $\tilde{A}$ will be as follows,

$$\tilde{A} = \begin{bmatrix} (a_{11l}\, a_{11m}\, a_{11u}) & (a_{12l}\, a_{12m}\, a_{12u}) & \cdots\cdots & (a_{1nl}\, a_{1nm}\, a_{1nu}) \\ (a_{21l}\, a_{21m}\, a_{21u}) & (a_{22l}\, a_{22m}\, a_{22u}) & \cdots\cdots & (a_{2nl}\, a_{2nm}\, a_{2nu}) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ (a_{m1l}\, a_{m1m}\, a_{m1u}) & (a_{m2l}\, a_{m2m}\, a_{m2u}) & \cdots\cdots & (a_{mnl}\, a_{mnm}\, a_{mnu}) \end{bmatrix} \tag{2}$$

Step 2: Fuzzy Extent Analysis for Calculation of Performance Ratings, Weight Multiplication and Summation

Then, the fuzzy extent analysis is applied on the above fuzzy PCM to obtain the fuzzy performance matrix. The purpose of fuzzy extent analysis is to obtain the criteria importance and alternative performance by solving these fuzzified reciprocal PCMs.

$$\tilde{X}_i \text{ or } \tilde{W}_j = \frac{\sum_{j=1}^{k} \tilde{a}_j}{\sum_{i=1}^{k} \sum_{j=1}^{k} \tilde{a}_{ij}} \tag{3}$$

*where i= 1,2,3...... p , j= 1,2,3......qand k=p, or k=q,* depending upon the element under operation, whether it is an alternative or criteria (the number of rows and columns in the PCM)

$$\tilde{X}_i = \begin{pmatrix} (x_{11l}\, x_{11m}\, x_{11u}) \\ (x_{21l}\, x_{21m}\, x_{21u}) \\ \cdots\cdots \\ \cdots\cdots \\ (x_{ijl}\, x_{ijm}\, x_{iju}) \end{pmatrix} \tag{4}$$

Fuzzy extent analysis is applied to get the fuzzy decision or performance matrix $(\tilde{X}_i)$ and fuzzy weights $(\tilde{W})$. After that, a fuzzy weighted performance matrix $(\tilde{P})$ can thus be obtained by multiplying the weight vector with the decision matrix.

$$\tilde{p} = \tilde{x}_i * \tilde{w} = \begin{pmatrix} w_l x_{11l} & w_m x_{11m} & w_u x_{11u} \\ w_l x_{21l} & w_m x_{21m} & w_u x_{21u} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ w_l x_{ijl} & w_m x_{ijm} & w_u x_{iju} \end{pmatrix} = \begin{pmatrix} p_{1l} & p_{1m} & p_{1u} \\ p_{2l} & p_{2m} & p_{2u} \\ & & \\ p_{il} & p_{im} & p_{iu} \end{pmatrix} \quad (5)$$

The next step will be weight summation where the weighted performance matrix $\tilde{p}$ for each alternative under each criteria context is sum up to obtain a total weighted performance matrix for each alternative.

Step 3: Check Fuzzy Ranking with Alpha-Cuts-Based Method

According to Wang (1997), in order to make a crisp choice among the alternatives, alpha-cuts-based method is needed for checking and comparing fuzzy number. The alpha-cuts-based method stated that if let A and B be fuzzy numbers with α-cuts, $A_\alpha = [a_\alpha^-, a_\alpha^+]$ and $B_\alpha = [b_\alpha^-, b_\alpha^+]$.It say A is smaller than B, denoted by A ≤ B, if $a_\alpha^- < b_\alpha^-$ and $a_\alpha^+ < b_\alpha^+$ for all $\alpha \in (0,1]$. The advantage of this method is the conclusion is less controversial. Here, apply the alpha cut analysis to the total weighted performance matrices for each alternative, and checking for the ranking consistency for each alternative under different alpha level circumstances.

Step 4: Alpha Cut Analysis for Confidence Level Representation

The alpha cut analysis is applied to transform the total weighted performance matrices into interval performance matrices. The alpha cut is to account for the uncertainty in the fuzzy range chosen. In this case, the decision maker expressed personal confidence about this range. The confidence value ranges between 0 and 1, from the least confidence to the most confidence. Alpha Cuts Analysis

$$\alpha_{left} = [\alpha * \ Middle\_fuzzy - Left\_fuzzy\ ] + Left\_fuzzy$$

$$\alpha_{Right} = Right\_fuzzy - [\alpha * \ Right\_fuzzy - Middle\_fuzzy\ ]$$

$$\tilde{P}_\alpha = \begin{pmatrix} [P_{1l\alpha}, P_{1r\alpha}] \\ [P_{2l\alpha}, P_{2r\alpha}] \\ \\ [P_{il\alpha}, P_{ir\alpha}] \end{pmatrix} \quad (6)$$

where *l* and *r* represent the left and right value of the interval set.

Step 5: Lambda Function and Crisp Values Normalization

Through the alpha cut analysis, it will get two values namely Alpha_Right (maximum range) and Alpha_Left (minimum range) which need to been converted into a crisp value. It is done by applying the Lambda function which represents the attitude of the decision maker. The attitude of the decision maker is maybe optimistic, moderate or pessimistic. Here, the concept of optimism index, λ, is introduced to obtain the crisp output.

$$crisp\_value = \lambda * \alpha_{right} + \left[ 1-\lambda * \alpha_{left} \right]$$

$$c_{\lambda} = \lambda * P_{r\alpha} + 1-\lambda * P_{l\alpha} \text{ where } \lambda = 0, 1 \quad \Rightarrow c_{\lambda} = \begin{pmatrix} c_{1\lambda} \\ c_{2\lambda} \\ \cdots \\ c_{i\lambda} \end{pmatrix} \tag{7}$$

Finally, the crisp values need to be normalized, because the elements of the PCM do not have the same scale. It is important to note that elements can be compared if they have the uniform scale.

$$c_{i\lambda} = \frac{c_{i\lambda}}{\sum c_{i\lambda}} \tag{8}$$

## 3. Numerical Calculations And Graphical Representations For Multi Criteria Decision Making

In order to illustrate the model, a hypothetical case study is done for a Manufacturing Company to find a new location for constructing a firm and it has three alternatives ($A_1$, $A_2$, $A_3$). First of all, a committee of decision-makers is formed. There are three decision-makers ($D_1$, $D_2$, $D_3$) in the committee. Then evaluation criteria are determined as Plant construction cost ($C_1$), Materials and equipment ($C_2$), Electricity and water ($C_3$), Transportation ($C_4$), Taxes ($C_5$).It is shown in Figure 1.1.

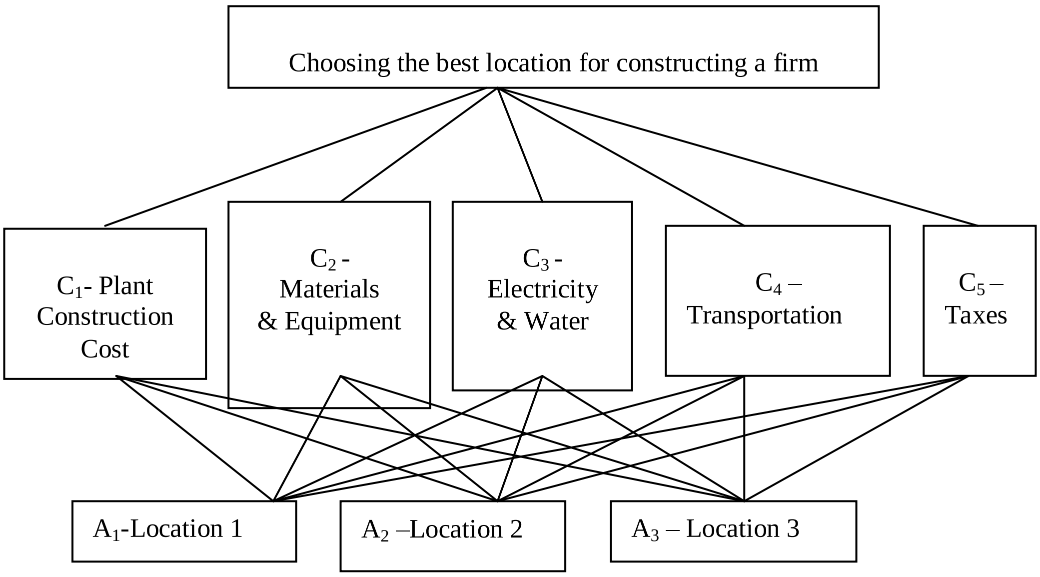


**Figure 1.1** Hierarchical structure of location selection process

## 4. Application of Location selection with FAHP Method

FAHP method is used for the same case study. First each decision-maker individually carries out pair-wise comparison by using triangular fuzzy number ranges from 1/9 to 9. Then crisp PCM is fuzzified using triangular fuzzy number which fuzzified the original PCM. Table 1.1 represents the fuzzy evaluation matrix with respect to goal.

**Table 1.1** Fuzzy evaluation matrix with respect to goal

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $C_1$ | (1,1,1) | (1, 1.67, 3) | (1, 2.33, 3) | (3, 4.33, 5) | (3, 3, 3) |
| $C_2$ | (0.33, 0.77, 1) | (1,1,1 | (0.33, 2.11, 3) | (3, 3.67, 5) | (1, 2.33, 3) |
| $C_3$ | (0.33, 0.77, 1) | (0.33, 1.22, 3) | (1,1,1) | (1,3,5) | (1, 1.67, 3) |
| $C_4$ | (0.20, 0.24, 0.33) | (0.20,0.28, 0.33) | (0.20, 0.51, 1) | (1,1,1) | (0.33,0.55, 1) |
| $C_5$ | (0.33, 0.33, 0.33) | (0.33, 0.55, 1) | (0.33, 0.77, 1) | (1, 2.33, 3) | (1,1,1) |

**Table 1.2** Fuzzy evaluation matrix with respect to $C_1$

| $C_1$ | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $A_1$ | (1,1,1) | (0.33, 2.11, 3) | (3, 3.67, 5) |
| $A_2$ | (0.33, 1.22, 3) | (1,1,1) | (1, 3, 5) |
| $A_3$ | (0.2, 0.29, 0.33) | (0.33, 0.51, 1) | (1,1,1) |

**Table 1.3** Fuzzy evaluation matrix with respect to $C_2$

| $C_2$ | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $A_1$ | (1,1,1) | (3, 5.67, 7) | (3, 5, 7) |
| $A_2$ | (0.14, 0.21, 0.33) | (1,1,1) | (0.33, 0.77, 1) |
| $A_3$ | (0.14, 0.23, 0.33) | (1, 1.67, 3) | (1,1,1) |

**Table 1.4** Fuzzy evaluation matrix with respect to $C_3$

| $C_3$ | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $A_1$ | (1,1,1) | (0.2, 0.2, 0.2) | (0.14, 0.23, 0.33) |
| $A_2$ | (5, 5, 5) | (1,1,1) | (0.33, 1.44, 3) |
| $A_3$ | (3, 5, 7) | (0.33, 1.44, 3) | (1,1,1) |

**Table 1.5** Fuzzy evaluation matrix with respect to $C_4$

| $C_4$ | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $A_1$ | (1,1,1) | (3, 3, 3) | (0.33, 2.78, 5) |
| $A_2$ | (0.33, 0.33, 0.33) | (1,1,1) | (0.2, 1.4, 3) |
| $A_3$ | (0.2, 1.17, 3) | (0.33, 2.11, 5) | (1,1,1) |

**Table 1.6** Fuzzy evaluation matrix with respect to $C_5$

| $C_5$ | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $A_1$ | (1,1,1) | (1, 2.33, 5) | (3, 3, 3) |
| $A_2$ | (0.2, 0.73, 1) | (1,1,1) | (0.33, 2.11, 3) |
| $A_3$ | (0.33,0.33, 0.33) | (0.33, 1.22, 3) | (1,1,1) |

After obtaining the fuzzified Pairwise comparison matrices, the fuzzy extent analysis are applied using equation (3).Table 1.2, 1.3, 1.4, 1.5, 1.6 represents the fuzzy evaluation matrix with respect to each criterias.

**Table 1.7** Overall Weight of each Criterion (after Fuzzy Extent Analysis)

| Criteria | Overall Weight | | |
|---|---|---|---|
| | Left | Middle | Right |
| $C_1$ | 0.1765 | 0.3313 | 0.64544 |
| $C_2$ | 0.1110 | 0.26545 | 0.55938 |
| $C_3$ | 0.07178 | 0.20016 | 0.5594 |
| $C_4$ | 0.0379 | 0.06932 | 0.15749 |
| $C_5$ | 0.05864 | 0.1338 | 0.2724 |

Table 1.7 represent the overall weight of each criterion. After obtaining the overall weight of each criterion, calculate performance of each alternative by fuzzy extent analysis using equation (3). Table 1.8 and 1.9 represent the weighted performance and total performance of the alternatives.

**Table 1.8** Weighted Performance of each Alternative

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $A_1$ | (0.03758, 0.16277, 0.7093) | (0.03587, 0.18718, 0.79083) | (0.00446, 0.0175, 0.07132) | (0.00735, 0.0341, 0.19180) | (0.0159, 0.0666, 0.2993) |
| $A_2$ | (0.02023, 0.12533, 0.7093) | (0.0075, 0.03176, 0.1229) | (0.02110, 0.09131, 0.41955) | (0.00259, 0.01372, 0.09228) | (0.0049, 0.04039, 0.16630) |
| $A_3$ | (0.01328, 0.04321, 0.18363) | (0.01097, 0.0465 0.2283) | (0.01444, 0.09131, 0.51278) | (0.00259, 0.02152, 0.19180) | (0.0053, 0.0268, 0.14402) |

**Table 1.9** Total Weighted Performances

| | Total Weighted Performance | | |
|---|---|---|---|
| | Left | Middle | Right |
| $A_1$ | 0.101257 | 0.46815 | 2.06259 |
| $A_2$ | 0.056354 | 0.302513 | 1.51028 |
| $A_3$ | 0.034639 | 0.22933 | 1.26053 |

Then calculate fuzzy ranking check through alpha-cut based method using equation (6). From the alpha-cut based method, the result is consistent enough to show Alternative 1 has the highest fuzzy ranking at all alpha level following by Alteration 2 and 3 respectively. Results of FAHP through alpha cut and lambda function are represented in Table 1.10 and 1.11.

**Table 1.10** Result of FAHP through Alpha Cut and Lambda Function

| | **Alpha Cut(α=0.5)** | | **Crisp Value** | | **Crisp Value (after normalization)** | |
|---|---|---|---|---|---|---|
| | $\alpha_{Left}$ | $\alpha_{Right}$ | **λ=0.5** | **λ=0.7** | **λ=0.5** | **λ=0.7** |
| **$A_1$** | 0.28470 | 1.2654 | 0.775036 | 0.97117 | 0.44126 | 0.43736 |
| **$A_2$** | 0.17943 | 0.9064 | 0.542915 | 0.688307 | 0.309105 | 0.30997 |
| **$A_3$** | 0.13198 | 0.7449 | 0.438457 | 0.561046 | 0.24963 | 0.25266 |

**Table 1.11** FAHP value for the selection of Location

| | **FAHP(α=0.5)** | |
|---|---|---|
| | **λ = 0.5** | **λ =0.7** |
| **$A_1$** | 0.44126 | 0.43736 |
| **$A_2$** | 0.309105 | 0.30997 |
| **$A_3$** | 0.24963 | 0.25266 |

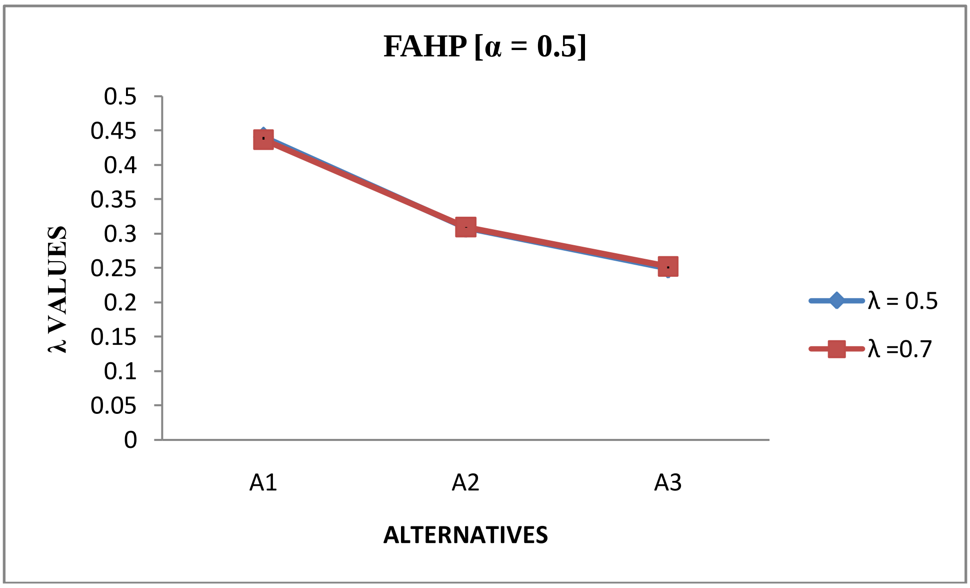


**Figure 1.2** FAHP value for the Selection of Location

The category defined as Alternative $A_1$ has the highest priority weight, is selected as the best facility location for constructing a firm. The ranking order of the alternatives with FAHP method is $A_1>A_2>A_3$ and it is shown in Figure 1.2. In FAHP, α-cut based method has been utilized to prevent the controversial of fuzzy number ranking process. Confidence level and the optimistic level of decision maker are captured by using α-cut based fuzzy number.

## 5. Conclusion

In FAHP linguistic variables are converted into triangular numbers. Here decision-makers made pair-wise comparisons for the criteria and alternatives under each criterion. Then these comparisons integrated and decision-makers' pair-wise comparison values are transformed into triangular fuzzy numbers. The priority weights of criteria and alternatives are determined by extent analysis. According to the combination of the priority weights of criteria and alternatives, the best alternative is determined. According to the FAHP, the best alternative is $A_1$ (Location 1) for constructing a firm.

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