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Fuzzy Weakly Completely Continuous Functions

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1. Introduction

Throughout this section spaces will always mean fuzzy topological spaces and f: X →Y denotes a function of a space X into a space Y . A fuzzy subset S of a space X is said to be fuzzy regular open (resp fuzzy regular closed) if Fint (Fcl(S))= S (resp Fcl (Fint (S)) = S) . Where Fcl(S) and Fint (S) denote the fuzzy closure of S and the fuzzy interior of S respectively . A fuzzy subset S of a space X is called θ-fuzzy open (resp δ-fuzzy open) if for each x∈S ,there exists an fuzzy open set U such that x∈U ⊂ Fint (Fcl(U)) ⊂ S). The complement of a θ-fuzzy open (resp δ-fuzzy open) set is called θ-fuzzy closed (resp δ-fuzzy closed).

2. Fuzzy Weakly Completely Continuous Functions

Definition: 2.1

A function f : X →Y is said to be fuzzy completely continuous if for each fuzzy open set V of Y , $f^{-1}(V)$ is fuzzy regular open in X .

A function f: X →Y is said to be fuzzy strongly continuous if for each fuzzy subset A of X , f (Fcl (A)) ⊂ f (A). It was shown that fuzzy strong continuity implies fuzzy complete continuity and fuzzy complete continuity implies fuzzy continuity and also the converse are not true in general.

Definition: 2.2

A function f: X →Y is said to be fuzzy weakly completely continuous (briefly FWCC) if for each θ-fuzzy open set V of Y, $f^{-1}(V)$ is fuzzy regular open in X.

Theorem: 2.3

A function f: X →Y is FWCC if and only if for each θ-fuzzy closed set F of Y , $f^{-1}(F)$ is fuzzy regular closed in X.

Theorem: 2.4

If f: X →Y is FWCC ,then the following equivalent properties can hold:

- **(a)** For each x∈ X and each θ-fuzzy open set V containing f(x) ,there exists a fuzzy regular open set U containing x such that f(U) ⊂ V.
- **(b)** For each x∈ X and each θ-fuzzy open set V containing f(x) ,there exists a δ-fuzzy open set U containing x such that f(U) ⊂ V.
- **(c)** For any θ-fuzzy open set V of Y , f^{-1} (V) is δ-fuzzy open in X.
- **(d)** For any θ-fuzzy closed set F of Y , f^{-1}(F) is δ-fuzzy closed in X .

Proof: It is obvious that fuzzy weak complete continuity implies (a). Since a δ-fuzzy open set is the union of fuzzy regular open sets it follows that (a) implies (b) and (c) implies (a). Since the union of δ-fuzzy open sets is δ-fuzzy open ,We observe that (b) implies (c) . It is obvious that (c) and (d) are equivalent.

Remark: 2.5

In theorem 2.4(a) does not necessarily imply FWCC . In example 3.5 since X is fuzzy regular , fuzzy open , θ-fuzzy open and δ-fuzzy open are equivalent and hence f satisfies (a) but it is not FWCC.

Lemma : 2.6

Let A be either fuzzy dense or fuzzy open in a space X .If U is a fuzzy regular open set of X then A∩U is fuzzy regular open in the fuzzy subspace A.

Theorem: 2.7

If f: X →Y is FWCC and A is either fuzzy open or fuzzy dense in a space X ,then the restriction f/A: A→Y is FWCC.

Proof: Let V be a θ-fuzzy open set of Y. Then f^{-1}(V) is fuzzy regular open in X. It follows from lemma 2.6 that $(f/A)^{-1}$(V)=f^{-1}(V)∩A is fuzzy regular open in the fuzzy subspace A . Therefore f/A is FWCC

Remark: 2.8

Let f : X → Y be FWCC and A, B fuzzy subsets of X. Then ,the restriction f/A: A→ f(A) need not be FWCC. More over f/A□ B: A□ B→ f(A□ B) is not always FWCC even if f/A: A→ f(A) f/B: B→ f(B) and f are all FWCC.

Example:2.9

Let X={ a, b, c, d} , τ = { 0, {a}, {b}, {a, b}, X } and σ = {0 ,{a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b,c}, X }

Let A = {a, b} , and B = {c} , Then the identity function f: (X, τ) →(X, σ) , f/A : A→ f(A) and f/B : B→ f(B) are FWCC . However f/A□ B : A□ B → f(A□ B) is not FWCC.

Remark: 2.10

There exists a FWCC function under which the inverse image of a fuzzy regular open set is not always fuzzy regular open .

Definition:2.11

A function $f : X \rightarrow Y$ is said to be θ-fuzzy continuous if for each $x \epsilon X$ and each fuzzy neighborhood V of f(x), there exists a fuzzy neighborhood U of x such that f(Fcl(U) $\subset$ Fcl(V) .

Lemma: 2.12

If $f : X \rightarrow Y$ is θ-fuzzy continuous and V is θ-fuzzy closed in Y then $f^{-1}(V)$ is θ-fuzzy closed in X .

Theorem :2.13

If $f :X \rightarrow Y$ is FWCC and $g :Y \rightarrow Z$ is θ-fuzzy continuous , then $g\bullet f :X\rightarrow Z$ is FWCC .

Proof: Let W be a θ-fuzzy closed set of Z. By lemma 2.12 , $g^{-1}(W)$ is θ-fuzzy closed in Y. Since f is FWCC ,By theorem 2.3 $f^{-1}(g^{-1}(W)) = (g\bullet f)^{-1}(W)$ is fuzzy regular closed in X . It follows from theorem 2.3 that g•f is FWCC.

Definition: 2.14

A function $f: X \rightarrow Y$ is called an R-fuzzy map if for each fuzzy regular open set V of Y, $f^{-1}(V)$ is fuzzy regular open in X .

Theorem:2.15

If $f :X \rightarrow Y$ is an R-fuzzy map and $g :Y \rightarrow Z$ is FWCC then $g\bullet f : X \rightarrow Z$ is FWCC .

Proof: Let W be a θ-fuzzy open set of Z , Then $g^{-1}(w)$ is fuzzy regular open in Y and hence $(g\bullet f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is fuzzy regular open in X .

A space X is said to be fuzzy extremely disconnected if the fuzzy closure of each fuzzy open set of X is fuzzy open in X . A space X is said to be fuzzy almost regular , if for each fuzzy regular closed set F and each point $x \epsilon X-F$,there exist disjoint fuzzy open sets U and V such that $x\epsilon U$ and $F\subset V$. A fuzzy almost – regularity is implied by fuzzy regularity and fuzzy independent of fuzzy semi – regularity , and also an fuzzy almost-regular and fuzzy semi –regular space is fuzzy regular . Every fuzzy open set of a fuzzy regular space is θ-fuzzy open , However a θ-fuzzy open set in a fuzzy regular space is not necessarily fuzzy regular open .

Example :2.16

Let X be the real numbers with the usual fuzzy topology and $A = (0,1) \cup (1,2)$. Then A is fuzzy open and hence θ-fuzzy open in X , but it is not fuzzy regular open in X .

Example :2.17

Let X ={ a, b, c } and τ ={ 0, {a}, {b}, {a,b}, X } . Then (X , τ) is fuzzy semi-regular and {a} is fuzzy regular open in (X , τ), but it is not θ-fuzzy open in (X ,τ) .

Lemma :2.18

If a space X is extremely disconnected , every fuzzy regular open set of X is θ-fuzzy open .

Proof: Let X be extremely disconnected and V fuzzy regular open in X . Then we have V = Fint (Fcl(V)) = Fcl (V) and hence V is fuzzy open closed in X . Therefore V is θ-fuzzy open.

Theorem: 2.19

Let Y be either extremely disconnected or fuzzy almost regular . If f : X → Y and g : Y →Z are FWCC , then g•f is FWCC.

Proof: Let W be a θ-fuzzy open set of Z . Then $g^{-1}(W)$ is fuzzy regular open in Y and hence $g^{-1}(W)$ is θ-fuzzy open in Y by lemma 2.18 $(g\bullet f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is fuzzy regular open in X. This shows that g•f is FWCC.A space X is said to be fuzzy weakly Hausdorff if each point of X is the intersection of fuzzy regular closed set of X.

Theorem : 2.20

Let f: X →Y be a FWCC injection. If Y is Hausdorff then X is fuzzy weakly-Haudorff

Proof: Let x∊X . Then { f(x) } is a fuzzy compact set of a Hausdroff space Y and hence { f(x) } is θ-fuzzy closed in Y since f is FWCC and injective , $\{x\}= f^{-1}(f(x))$ is fuzzy regular closed in X by theorem 2.3. Therefore X is fuzzy weakly – Hausdroff .

Theorem :2.21

Let f :X→Y be a function and g :X→Y given by g(x) = (x , f(x)) for each x∊X , be the graph function. If g is FWCC , then f is FWCC .

Proof : Let V be a θ-fuzzy open set of Y. If X×V is a θ=fuzzy open set of X×Y . Since g is FWCC , $g^{-1}(X\times V)$ is fuzzy regular open in X . However , by a simple calculation we obtain $g^{-1}(X\times V) = f^{-1}(V)$ Therefore f is FWCC .

3. Comparisons

Definition :3.1

A function f : X→Y is said to be fuzzy faintly continuous if for each x∊ X and each θ-fuzzy open set V containing f(x) , there exists an fuzzy open set U containing x such that f(U) ⊂V.

Theorem :3.2

For a function the following implications hold : fuzzy compete continuity → fuzzy weak complete continuity → fuzzy faint- continuity .

Proof : Since every θ-fuzzy open set is fuzzy open the first implication holds . The second implication follows from the result that a function is fuzzy faintly – continuous if and only if the inverse image of each θ-fuzzy open set is an fuzzy open set .

Definition :3.3

A function f : X→Y is said to be δ-fuzzy continuous (res . fuzzy almost continuous , fuzzy weakly continuous) if for each x∊X and each fuzzy open neighborhood V of f(x) , there exists an fuzzy open neighborhood U of x such that f(Fint (Fcl (U)) ⊂ Fint (Fcl (V)) (res . f(U) ⊂ Fint (Fcl (V)) , f(U) ⊂ Fcl (V) .

Remark : 3.4
For a function the following implications hold but none of these implications is reversible .

(1) fuzzy completely continuous → R-fuzzy map → δ-fuzzy continuous →fuzzy almost continuous .

(2) fuzzy continuous → fuzzy almost continuous → θ-fuzzy continuous → fuzzy weakly continuous →fuzzy faintly continuous .

We shall show that fuzzy weak complete continuity is independent of each one of R-fuzzy map , δ-fuzzy continuity , fuzzy continuity ,fuzzy almost-continuity, θ-fuzzy continuity and fuzzy weak – continuity .

Example :3.5
Let X be the real numbers with the usual fuzzy topology and f :X→Y the fuzzy identity function .

Then f is fuzzy continuous and also an R-fuzzy map but is not FWCC . Let A = (0,1) □ (1,2) , then A is θ- fuzzy open but it is not fuzzy regular open in X .

Example :3.6
Let X = { a, b, c, d } , τ = { φ, {c}, {a,d}, {a,c,d}, X} and σ = { φ, {a}, {b,c}, {a,b,c}, X } . Define a function f : (X, τ) →(X, σ) as follows f(a) = f(b) = f(c) = a and f(d) = b , then f is FWCC but it is not fuzzy weakly continuous .

Theorem :3.7
Let Y be either fuzzy almost – regular or fuzzy extremely disconnected . If f :X→Y is FWCC then it is an R-fuzzy map .

Proof : Let V be a fuzzy regular open set of Y . It follows from lemma 2.18, V is a θ-fuzzy open set of Y . Since f is FWCC , $f^{-1}(V)$ is fuzzy regular open in X This shows that f is an R-fuzzy map .

Theorem:3.8
If f :X→Y is FWCC and Y is a fuzzy regular space then f is fuzzy completely continuous .

Proof : Let V be an fuzzy open set of Y **.** Since Y is fuzzy regular ,V is θ-fuzzy open in Y and hence $f^{-1}(V)$ is fuzzy regular open in X . Therefore f is fuzzy completely continuous .

4. References

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