Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 1 (2015), pp. 319-322

http://www.ripublication.com

Regular Pentagon Cover for Isoperimetric Rectangles

Banyat Sroysang

Department of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University, Rangsit Center, Pathumthani 12121 Thailand
E-mail: banyat@mathstat.sci.tu.ac.th

Abstract

Aregion containing a congruent copy of every arc in a family is called a cover for the family. For the family of all rectangles ofperimeter four, we present the smallest regular pentagon cover forthe family.

Keywords: Cover, Rectangle, Pentagon

1. Introductionand Preliminaries

For a family of arcs,a*cover* for the family is a region containing a congruent copy of every arc in the family. For the familyof all triangles ofperimeter two, (1) Wetzel [10,11] gavethe smallest rectangular cover for the family and the smallest equilateral triangular cover forthe family, (2) Furedi and Wetzel [1] gave the smallest convex cover for the family, (3) Zhang and Yuan [12] gave the smallest regularized parallelogram (whose length of the smaller diagonal is not less than one) coverfor the family, and (4) Sroysang [5-7] gavethe smallest regularized trapezoid (whose length of the smaller diagonal is not less than one,and two smaller angles are opposite) cover for the family and the smallest regular pentagon cover for the family. For related papers, we refer to [2-4,8-9].

For a convex set X in the plane, we let $\omega_X(\theta)$ be the distance between the two parallel support lines of X with angle of inclination θ where$0 \leq \theta \leq \pi$. For a convex set X in the plane, the minimum of$\omega_X(\theta)$ is called the *thickness* of X, and the maximum of $\omega_X(\theta)$ is called the *diameter*of X.We note that(1) the thickness of any rectangle ofperimeter four is at most one, (2) the diameter of any rectangle of perimeter four is at most two, and (3) the diameter of any regular pentagon cover for the family of all rectangles ofperimeter four is at least two since the cover contains a congruent copy of the line segment of length two.

2. Results

Theorem *The smallest regular pentagon cover for the family of all rectangles ofperimeter fouris the regular pentagon of diameter one.*

Proof. Let *P* be the regular pentagon of diameter one and let *R* be arectangle ofperimeter four.Now, we denotethe vertices of the rectangle *T* by *A*, *B*, *C* and *D* into anticlockwise positions, where $|AB| \geq |CD|$. Let *E* be the midpoint of the segment *AB*.WLOG, we can put the rectangle *R* into the pentagon *P* where the segment *AB* lies on a diagonal of the regular pentagon *P* and the point *E* is the midpoint of the diagonal as shown in Fig. 1 or Fig. 2or Fig. 3. Now, the vertex *D* may be in the pentagon *P* or not in the pentagon *P*. Since the thickness of any rectangleofperimeter fouris at most one, we obtain that the distance between the vertex *D* and the segment *AB* is at most one. This implies that Fig. 3 is impossible. Now, we only consider Fig. 1 or Fig. 2.

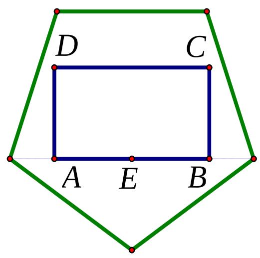

Fig. 1. A rectangle *R* inthe regular pentagon *P* where the vertex *D* is in *P*

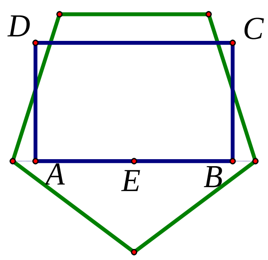

Fig. 2. A rectangle *R* inthe regular pentagon *P* where the vertex *D* is not in *P*

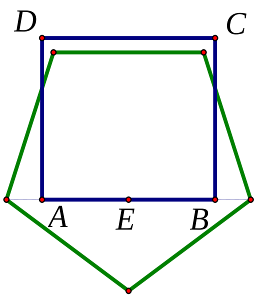

Fig. 3. A rectangle *R* inthe regular pentagon *P* where the vertex *D* is not in *P*

Suppose for a contradiction that the vertex D is not in the regular pentagon P as shown in Fig. 2. Let F be the intersection of the segment AD and the pentagon P, and let G be the intersection of the segment BC and the pentagon P as shown in Fig. 4.

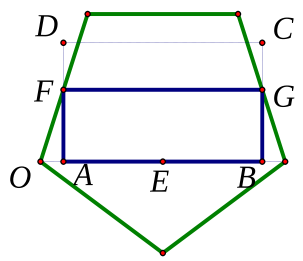

Fig. 4. A rectangle *FABG* in the rectangle *R* and the regular pentagon *P*

Let x be the length of the segment OA. Then $0 \le x \le 0.5$ and the length of the segment AE is $1-x$. We note that the angle FOA is 72°. Then the length of the segment AF is $x\sqrt{5+2\sqrt{5}}$.

Define $L(x) = 4(1-x) + 2x\sqrt{5+2\sqrt{5}}$. Then $L(x)$ is the total length of the perimeter of the rectangle $FABG$. By the calculation on $L(x)$, we obtain that $L(x) \ge L(0) = 4$ as shown in Fig. 5.

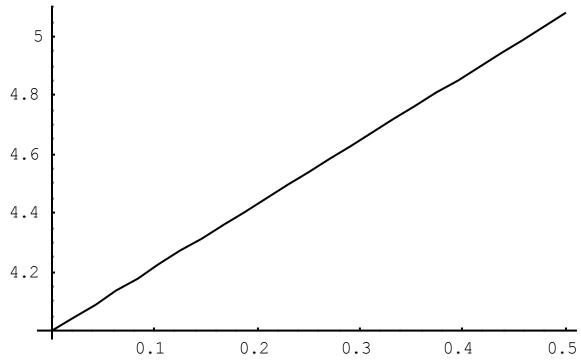

Fig. 5. The graph of $L(x)$ where $0 \le x \le 0.5$

Since the total length of the perimeter of the rectangle $ABCD$ is greater than the total length of the perimeter of the rectangle $FABG$, it follows that the total length of the perimeter of the rectangle R is greater than four. This is a contradiction. We obtain that the regular pentagon P is a cover for the family of all rectangles of perimeter four.

Next, we note that every cover for the family of all rectangles of perimeter four must cover the line segment of length two. Hence, the diagonal of any regular pentagon cover for the family of all rectangles of perimeter four must have length at least two. The diagonal of the regular pentagon P has length two. Hence, the regular pentagon P is a smallest cover for the family of all rectangles of perimeter four.

Remark.For the family of all rectangles ofperimeter four, the area of the smallest regular pentagon cover for the family is $\frac{\sqrt{50-10\sqrt{5}}}{8}$.

Acknowledgement. The authors gratefully acknowledge the financial support provided by Thammasat Universityunder the TU Research Scholar, Contract No.2/7/2557.

3. References

[1] Z. Furedi and J. E. Wetzel, The smallest convex cover for triangles of perimeter two, Geom. Dedicata, **81** (2000), 285-293.

[2] B. Sroysang, Fitting Triangle into Regular Octagon, Appl. Math. Sci., **7** (2013), 3655-3659.

[3] B. Sroysang, Regular Decagon Cover for Isoperimetric Triangles, Appl. Math. Sci., **7** (2013), 3667-3671.

[4] B. Sroysang, Regular Hexagon Cover for Isoperimetric Triangles, Appl. Math. Sci., **7** (2013), 1545-1550.

[5] B. Sroysang, Regularized Trapezoid Cover for Isoperimetric Triangles, Int. J. Comput. Appl. Math., **6** (2011), 239-245.

[6] B. Sroysang, Regular Pentagon Cover for Triangles of Perimeter Two, Appl. Math. Sci., **7** (2013), 1551-1555.

[7] B. Sroysang, Right Trapezoid cover for triangles of perimeter two, Kasetsart J. (Nat. Sci.), **45** (2011), 756-671.

[8] B. Sroysang, Triangles in Regular Heptagons, Appl. Math. Sci., **7** (2013), 1557-1561.

[9] B. Sroysang, Triangles in Regular Nonagons, Appl. Math. Sci., 7 (2013), 3661-3665.

[10] J. E. Wetzel, Boxes for isoperimetric triangles, Math. Mag., **73** (2000), 315-319.

[11] J. E. Wetzel, The smallest equilateral cover for triangles of perimeter two, Math. Mag., **70** (1997), 125-130.

[12] Y. Zhang and L. Yuan, Parallelogram cover for triangles of perimeter two, Southeast Asian Bull. Math., **33** (2009), 607-612.