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A Criterion for (m_k)-hypercyclic Operators and Weighted Shifts[1]

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Abstract

In this paper, we give a sufficient condition for (m_k)-hypercyclic operators on a separable F-space and we show that the criterion is equivalent to (m_k)-hypercyclicity for the weighted backward shifts.

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1. Introduction

A continuous linear operator T on a separable F-space X is said to be hypercyclic if the T-orbit of some vector is dense in X and thus the orbit intersects with each non-empty open subset of X. A question is that how frequently can such an orbit visit each non-empty open set and this leads to a notion of the frequent hypercyclicity guided by the Birkhoff ergodic theorem, [1]. General criterions for hypercyclcity and frequent hypercyclicity have been developed in [10, 6] and [1, 5], respectively. In [3], F. Bayart and É. Matheron introduced a notion of (m_k)-hypercyclicity by controlling the frequency of the orbit visiting each non-empty open set, where $(m_k)_{k\in\mathbb{N}}$ is an increasing sequence of positive integers. It provides us various examples of linear operators between hypercyclicity and frequent hypercyclicity. In this paper, we give a sufficient condition for (m_k)-hypercyclicity following the ideas given in [4] and [8]. As an application, we show that the criterion is equivalent to the (m_k)-hypercyclicity for the case of weighted shifts.

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2. (m_k)-hypercyclic operators

Let X be a separable F-space and let $\mathcal{L}(X)$ be the space of all continuous linear operators on X. An operator $T \in \mathcal{L}(X)$ is *hypercyclic* if there exists a vector $x \in X$ such that the T-orbit $O(T, x) = \{T^n x \mid n = 0, 1, 2, \cdots\}$ is dense in X. Such a vector $x \in X$ is called a *hypercyclic vector* for T. If T is hypercyclic, then for any non-empty open subset U of X, the set

$$\mathbf{N}(x, U) = \{n \in \mathbb{N} \mid T^n x \in U\}$$

is non-empty, where $\mathbb{N}$ is the set of all positive integers. For a subset A of $\mathbb{N}$, the lower density of A is defined by

$$\underline{\text{dens}}(A) = \liminf_{N\to\infty} \frac{|A \cap [1, N]|}{N}$$

where $|A \cap [1, N]|$ denotes the cardinality of the set $A \cap [1, N]$. An operator T is said to be *frequently hypercyclic* if there is a vector $x \in X$ such that for every non-empty open set U, $\mathbf{N}(x, U)$ has positive lower density. Such a vector x is called frequently hypercyclic for T. Let A be an infinite subset of $\mathbb{N}$ and let $(n_k)_{k\in\mathbb{N}}$ be an increasing enumeration of A. It is easy to see that A has positive lower density if and only if there is a constant C such that

$$n_k \le Ck \quad \text{for all } k \ge 1. \tag{2.1}$$

Thus, a vector $x \in X$ is frequently hypercyclic for T if and only if for each non-empty open subset U of X, there is a strictly increasing sequence $(n_k)_{k\ge1}$ and some constant C such that

$$T^{n_k} x \in U \quad \text{and} \quad n_k \le Ck$$

for all $k \in \mathbb{N}$.

We introduce a lower density of a subset of positive integers which depends on a sequence of positive integers.

Definition 2.1. Let $(m_k)_{k\in\mathbb{N}}$ be an increasing sequence of positive integers and let $A \subseteq \mathbb{N}$. The *lower (m_k)-density* of A is defined as

$$(m_k)\text{-}\underline{\text{dens}}(A) = \liminf_{k\to\infty} \frac{|\mathbf{N}(x, U) \cap [0, m_k]|}{k}. \tag{2.2}$$

Let $(n_k)_{k\in\mathbb{N}}$ be an enumeration of A and (m_k)-$\underline{\text{dens}}(A) > 0$ if and only if, for some constant $C > 0$,

$$n_k \le Cm_k \quad \text{for all } k \ge 1. \tag{2.3}$$

Definition 2.2. Let $(m_k)_{k\in\mathbb{N}}$ be an increasing sequence of positive integers. An operator T on a separable F-space X is said to be *(m_k)-hypercyclic* if there is a vector $x \in X$ such that for each non-empty open set U in X, the lower (m_k)-density of the set $\mathbf{N}(x, U)$ is positive. In that case, the vector x is said to be (m_k)-hypercyclic for T.

A vector $x \in X$ is (m_k)-hypercyclic for T if and only if for each non-empty open subset U of X, there is a strictly increasing sequence (n_k) and some constant C such that

$$T^{n_k}x \in U \quad \text{and} \quad n_k \le Cm_k$$

for all $k \in \mathbb{N}$. For example, the frequently hypercyclic operators are the (k)-hypercyclic ones. Also, q-frequently hypercyclic operators given in [8] are (k^q)-hypercyclic.

Let $\|\cdot\|$ be an F-norm defining the topology of X and let $(x_l)_{l\in\mathbb{N}}$ be a dense sequence in X. Then a vector $x \in X$ is (m_k)-hypercyclic for T if there exist subsets J_l of $\mathbb{N}$, $l \ge 1$ of positive lower (m_k)-density such that, for any $n \in J_l$ and $\epsilon > 0$

$$\|T^n x - x_l\| < \epsilon.$$

In order to show that an operator is (m_k)-hypercyclic, one may need the following lemma which is modified from the Lemma 2.5 in [5].

Lemma 2.3. There exist pairwise disjoint sequence $(J_n)_{n\ge1}$ of subsets of $\mathbb{N}$ such that

(a) for each $n \ge 1$, (m_k)-$\underline{\text{dens}}(J_n) > 0$;

(b) if $l \in J_n$, then $l \ge n$;

(c) $|p - q| \ge \max\{n, m\}$, for $(p, q) \in J_n \times J_m$.

Proof. Let $I_n = 2^n\mathbb{N} \setminus 2^{n+1}\mathbb{N} = 2^n(\mathbb{N} \setminus 2\mathbb{N})$. Since $\underline{\text{dens}}(\mathbb{N} \setminus 2\mathbb{N}) = \frac{1}{2}$, $\underline{\text{dens}}\,(I_n) > 0$ for all $n \ge 1$. For $i \ge 1$, define

$$r_i = n \quad \text{if } i \in I_n$$

and

$$n_i = 2\sum_{\nu=1}^{i-1} r_\nu + r_i. \tag{2.4}$$

For each $n \ge 1$, let

$$A_n = \{n_i \mid i \in I_n\}$$

Then, as shown in [5], each set A_n has positive lower density and satisfies the condition (b) and (c), for all $n \ge 1$. Let $I_n = \{i_k \mid k \ge 1\}$. Since $\underline{\text{dens}}\,(I_n) > 0$, by (2.1), there exists $C > 0$ such that

$$i_k \le Ck \quad \text{for all } k \ge 1$$

Since $A_n = \{n_i \mid i \in I_n\} = \{n_{i_k} \mid k \ge 1\}$, by (2.1), there exist $M > 0$ and $C > 0$ such that

$$n_{i_k} \le Mi_k \le MCk. \tag{2.5}$$

Let $I'_n = \{i_{m_k} \in I_n \mid f \ge 1\}$ and let

$$J_n = \{a_k \in A_n \mid a_k = n_{i_{m_k}}, i_{m_k} \in I'_n, k \ge 1\}.$$

Then, as in (2.5), it is easy to see that

$$a_k \leq Mm_k \quad \text{for some } M > 0.$$

In other words, (m_k)-$\underline{\text{dens}}(J_n) > 0$ for all $n \geq 1$. ■

Let us recall that a series $\sum x_n$ in a normed space is said to be *unconditionally convergent* if for every permutation σ of $\mathbb{N}$, $\sum x_{\sigma(n)}$ is convergent, see [9] for details. As given in [5], a collection of series $\sum_{n=1}^{\infty} x_{n,k}$, $k \in J$, is called *unconditionally convergent, uniformly in $k \in J$* if for any $\varepsilon > 0$ there is some $N \in \mathbb{N}$ such that for any finite set $F \subset [N, \infty) \cap \mathbb{N}$ and every $k \in J$ we have

$$\left\| \sum_{n \in F} x_{n,k} \right\| < \varepsilon.$$

We now have a sufficient condition for an operator to be (m_k)-hypercyclic and it will be referred as *(m_k)-hypercyclicity criterion.* For simplicity, a sequence $(J_n)_{n\geq 1}$, given in Lemma 2.3, will be referred as an (m_k)-hypercyclicity sequence. The proof of the following theorem is essentially the same as the proofs given in [5] and [4], and we simply sketch the proof.

Theorem 2.4. Let X be a separable F-space and let $T \in \mathcal{L}(X)$. For an increasing sequence $(m_k)_{k\geq 1}$ of positive integers, let $(J_n)_{n\geq 1}$ be an (m_k)-hypercyclicity sequence and let $J = \bigcup_{n\geq 1} J_n$. Suppose that there exist a dense subset Y_0 of X and a map $S : Y_0 \longrightarrow Y_0$ such that for all $y \in Y_0$,

1. $\sum_{n \in J_l} S^n y$ is unconditionally convergent, uniformly in $l \in \mathbb{N}$,
2. for any $i \geq 1$, there exists $l \geq i$ such that $\sum_{n \in J_l} T^k S^n y$ is unconditionally convergent, uniformly in $k \in J$,
3. there is an $l_0 \geq 1$ such that $\sum_{n \in J_l} T^k S^n y$ is unconditionally convergent, uniformly in $k \in \bigcup_{l \geq l_0} J_l$
4. $TS = I$, the identity on Y_0.

Then the operator T is (m_k)-hypercyclic.

Proof. Since X is separable, we may assume that $Y_0 = \{y_l \mid l \geq 1\}$. Let $\epsilon_l > 0$ be a real number such that $\sum_l \epsilon_l < \infty$. Then for each $\epsilon_l > 0$, there exists $N_l \in \mathbb{N}$ such that

$$\left\| \sum_{n \in J_l \cap F} S^n y_l \right\| < \epsilon_l \tag{2.6}$$

for any finite set $F \subset [N_l, \infty) \cap \mathbb{N}$. Define

$$x = \sum_{l=1}^{\infty} \sum_{n \in J_l} S^n y_l. \tag{2.7}$$

Then by assumption 1, we have

$$\sum_{l=1}^{\infty} \left\| \sum_{n \in J_l \cap F} S^n y_l \right\| < \sum_{l=1}^{\infty} \epsilon_l < \infty. \tag{2.8}$$

and hence $x \in X$.

Fix $l_0 \geq 1$ and $k \in J_{l_0}$, then

$$\begin{aligned} T^k x &= \sum_{l=1}^{\infty} \sum_{n \in J_l} T^k S^n y_l \\ &= \sum_{l=1}^{\infty} \sum_{\substack{n < k, \\ n \in J_l}} T^k S^n y_l + \sum_{l=1}^{\infty} \sum_{\substack{n > k, \\ n \in J_l}} T^k S^n y_l + y_{l_0} \\ &= \sum_{l=1}^{\infty} \sum_{n \in J_l} T^{k-n} y_l + \sum_{l=1}^{\infty} \sum_{n \in J_l} S^{n-k} y_l + y_{l_0} \end{aligned}$$

The last equality follows from the hypothesis 4. Now by the assumption 2 and 3, we have

$$\left\| T^k x - y_{l_0} \right\| \leq \epsilon_n$$

and x is a (m_k)-hypercyclic vector. ■

3. (m_k)-hypercyclic weighted shifts

Let $X = \ell^p(\mathbb{Z}_+)$ or $X = c_0(\mathbb{Z}_+)$ and let $\mathbf{w} = (w_n)_{n \geq 1}$ be a bounded sequence of positive real numbers. The weighted shifts on X is given by $B_{\mathbf{w}}(x) = (w_1 x_1, w_2 x_2, \cdots)$, where $x = (x_0, x_1, \cdots) \in X$. Let e_n be the canonical basis for X, $n \geq 0$. Then

$$B_{\mathbf{w}}^n x = \sum_{r=0}^{\infty} w_{r+1} \cdots w_{r+n} x_{r+n} e_r.$$

or

$$B_{\mathbf{w}}e_0 = 0, \quad B_{\mathbf{w}}e_r = w_r e_{r-1}$$

Suppose that the operator $B_{\mathbf{w}}$ is (m_k)-hypercyclic and let $x = (x_n)_{n\geq 0}$ be an (m_k)-hypercyclic vector for $B_{\mathbf{w}}$. Then for any $n \geq 1$ and $c_n > 0$, there exists a sequence (R_n) of subsets of $\mathbb{N}$, which is of positive lower (m_k)-density, such that for any $j \in R_n$,

$$\left\| B_{\mathbf{w}}^j x - c_n \sum_{r=0}^{n} e_r \right\| < \frac{1}{n} \tag{3.9}$$

or

$$\left\| \sum_{r=0}^{\infty} w_{r+1} \cdots w_{r+j} x_{r+j} e_r - c_n \sum_{r=0}^{n} e_r \right\| < \frac{1}{n}$$

Thus, for $0 \leq r \leq n$, we have

$$|w_{r+1} \cdots w_{r+j} x_{r+j} - c_n| < \frac{1}{n}$$

or

$$c_n - \frac{1}{n} < |w_{r+1} \cdots w_{r+j} x_{r+j}| < c_n + \frac{1}{n} \tag{3.10}$$

and for $r > n$,

$$|w_{r+1} \cdots w_{r+j} x_{r+j}| < \frac{1}{n}. \tag{3.11}$$

Lemma 3.1. If the weighted shift $B_{\mathbf{w}}$ on X is (m_k)-hypercyclic, then there exists a sequence $(R_n)_{n\geq 1}$ of disjoint subsets of $\mathbb{N}$ such that for any $j \in R_n$ and any $l \in R_m$, $j \neq l$, we have

$$|j - l| \geq \max\{m, n\}. \tag{3.12}$$

Proof. For any $j \in R_n$, suppose that $j + r \in R_m$, where $0 \leq r \leq n$ and $1 \leq m < n$. Then, for $0 \leq l \leq m$,

$$c_m - \frac{1}{m} < |w_{l+1} \cdots w_{l+j+r} x_{l+j+r}| < c_m + \frac{1}{m}$$

and for $l > m$,

$$|w_{l+1} \cdots w_{l+j+r} x_{l+j+r}| < \frac{1}{m} < c_m + \frac{1}{m}$$

Thus for any $l \geq 0$, we have

$$|w_{l+1} \cdots w_{l+j+r} x_{l+j+r}| < c_m + \frac{1}{m}$$

Let $l = n - r$. Then

$$|w_{n-r+1} \cdots w_{n+j} x_{n+j}| < c_m + \frac{1}{m}$$

and

$$|w_{n-r+1}\cdots w_n|\cdot|w_{n+1}\cdots w_{n+j}x_{n+j}| < c_m + \frac{1}{m}.$$

Since the weight sequence $(w_n)_{n\geq 1}$ is bounded, there is a number $M > 0$ such that $|w_n| \leq M$ for all $n \geq 1$. By applying (3.10) to the above inequality, we get

$$\left(c_n - \frac{1}{n}\right)\cdot M^r < c_m + \frac{1}{m}. \tag{3.13}$$

Similarly, if $j - r \in R_m$, we get

$$c_n - \frac{1}{n} < M^r \cdot \left(c_m + \frac{1}{m}\right). \tag{3.14}$$

Also, for the case when $n = m$, we have

$$c_n - \frac{1}{n} < \frac{1}{n}\cdot M^r \tag{3.15}$$

for $j - r \in R_n$. Thus, if we choose the sequence $(c_n)_{n\geq 1}$ to satisfy the condition

$$c_n - \frac{1}{n} \geq \max\left\{\frac{1}{M^r}\left(c_m + \frac{1}{m}\right),\ M^r\left(c_m + \frac{1}{m}\right),\ \frac{M^r}{n}\right\},$$

then the sequence $(R_n)_{n\geq 1}$ satisfies the separation property (3.12). ■

Let $R = \bigcup_{n\geq 1} R_n$ and let $(r_n)_{n\geq 1}$ be the enumeration of R. For each $n \geq 1$, let $(n_i)_{i\geq 1}$ be the subsequence of $(r_n)_{n\geq 1}$, which is the enumeration of R_n. Then the separation property (3.12) can be given as follows

$$|n_i - m_j| \geq \max\{m, n\} \quad \text{for all } i, j \geq 1.$$

We also may assume that the sequence $(c_n)_{n\geq 1}$ is increasing and $\lim_{n\to\infty} c_n = \infty$.

Theorem 3.2. Let $B_{\mathbf{w}}$ be the backward shift associated to the weight sequence $\mathbf{w} = (w_n)_{n\geq 1}$. If $B_{\mathbf{w}}$ is (m_k)-hypercyclic, then there exists a set $\{(n_i)_{i\geq 1} \mid n \geq 1\}$ of pairwise disjoint sequences of positive integers such that

1. for any $j \geq 0$, any $n > j$
$$\sum_{i=1}^{\infty} \frac{e_{n_i+j}}{w_1\cdots w_{n_i+j}}$$
converges unconditionally;

2. for any $j \geq 0$, any $n > j$, any $l \geq 1$

$$\left\| \sum_{\nu=1}^{\infty} \frac{e_{n_\nu - l_i + j}}{w_{1+j} \cdots w_{n_\nu - l_i + j}} \right\| \leq \frac{1}{l} M_n$$

where $M_n \to 0$ as $n \to \infty$.

Proof. (a) Let $0 \leq j \leq n$ and let x be a (m_k)-hypercyclic vector for $B_{\mathbf{w}}$. Then by (3.10),

$$c_n - \frac{1}{n} < |w_{j+1} \cdots w_{j+n_i} x_{j+n_i}| < c_n + \frac{1}{n}$$

and so

$$\frac{c_n - \frac{1}{n}}{|w_{j+1} \cdots w_{j+n_i}|} < |x_{j+n_i}| \tag{3.16}$$

Now, we have

$$\begin{aligned}
&\left\| \sum_{i=1}^{\infty} \frac{e_{j+n_i}}{w_1 \cdots w_{j+n_i}} \right\| \\
&= \left\| \sum_{i=1}^{\infty} \frac{1}{w_1 \cdots w_j} \cdot \frac{e_{j+n_i}}{w_{j+1} \cdots w_{j+n_i}} \right\| \\
&\quad \leq \left\| \sum_{i=1}^{\infty} \frac{x_{j+n_i} e_{j+n_i}}{w_1 \cdots w_j} \right\| \cdot \frac{1}{c_n - \frac{1}{n}} \quad \text{by (3.16)} \\
&\left\| \sum_{i=1}^{\infty} \frac{e_{j+n_i}}{w_1 \cdots w_{j+n_i}} \right\| \leq \frac{1}{|w_1 \cdots w_j|} \frac{\|x\|}{c_n - \frac{1}{n}} \\
&\to 0 \quad \text{as } n \to \infty
\end{aligned}$$

Thus $\sum_{i=1}^{\infty} \frac{e_{n_i+j}}{w_1 \cdots w_{n_i+j}}$ converges unconditionally.

(b) Let $n > j \geq 0$, $l \geq 1$. Then by the definition of the sequence $(l_i)_{i \geq 1}$, we have

$$\begin{aligned}
\frac{1}{l} &> \left\| B_{\mathbf{w}}^{l_i} x - c_l \sum_{\nu=0}^{l} e_\nu \right\| \\
&= \left\| \sum_{\nu=1}^{\infty} w_{\nu+1} \cdots w_{\nu+l_i} x_{\nu+l_i} e_\nu - c_l \sum_{\nu=0}^{l} e_\nu \right\| \\
&= \left\| \sum_{\nu=l+1}^{\infty} w_{\nu+1} \cdots w_{\nu+l_i} x_{\nu+l_i} e_\nu \right. \\
&\qquad \left. + \sum_{\nu=0}^{l} \{w_{\nu+1} \cdots w_{\nu+l_i} - c_l\} e_\nu \right\|
\end{aligned}$$

$$\geq \left\| \sum_{\nu=l+1}^{\infty} w_{\nu+1} \cdots w_{\nu+l_i} x_{\nu+l_i} e_\nu \right\|$$
$$\geq \left\| \sum_{\nu=l+1}^{\infty} w_{\nu+1+j} \cdots w_{\nu+l_i+j} x_{\nu+l_i+j} e_{\nu+j} \right\|$$
$$\geq \left\| \sum_{\nu=1}^{\infty} w_{n_\nu-l_i+1+j} \cdots w_{n_\nu+j} x_{n_\nu+j} e_{n_\nu-l_i+j} \right\|$$
$$= \left\| \sum_{\nu=1}^{\infty} \frac{w_{j+1} \cdots w_{n_\nu+j}}{w_{j+1} \cdots w_{n_\nu-l_i+j}} x_{n_\nu+j} e_{n_\nu-l_i+j} \right\|$$

Thus we have

$$\frac{1}{l} \geq (c_n - \frac{1}{n}) \left\| \sum_{\nu=1}^{\infty} \frac{e_{n_\nu-l_i+j}}{w_{j+1} \cdots w_{n_\nu-l_i+j}} \right\|$$

or

$$\left\| \sum_{\nu=1}^{\infty} \frac{e_{n_\nu-l_i+j}}{w_{j+1} \cdots w_{n_\nu-l_i+j}} \right\| \leq \frac{1}{l} \cdot \frac{1}{c_n - \frac{1}{n}}.$$

■

Theorem 3.3. Let $B_{\mathbf{w}}$ be a weighted shift on X. Then $B_{\mathbf{w}}$ is (m_k)-hypercyclic if and only if $B_{\mathbf{w}}$ satisfies the (m_k)-hypercyclicity criterion.

Proof. By Theorem 2.4, we have only to show that if $B_{\mathbf{w}}$ is (m_k)-hypercyclic, $B_{\mathbf{w}}$ satisfies the (m_k)-hypercyclicity criterion.

Let $B_{\mathbf{w}}$ be (m_k)-hypercyclic, then there is a set $\{(n_i)_{i\geq 1} \mid n \geq 1\}$ of disjoint sequences satisfying the conditions given in Theorem 3.2. Let $S(x_0, x_1, \cdots) = (0, \frac{x_0}{w_1}, \frac{x_1}{w_2}, \cdots)$ and let $Y_0 = \text{span}\{e_\nu \mid \nu \geq 1\}$. First note that

$$B_{\mathbf{w}}^n S^i x = \begin{cases} B_{\mathbf{w}}^{n-i} x, & \text{if } n > i, \\ S^{i-n} x, & \text{if } n < i, \\ x, & \text{if } n = i \end{cases}$$

Let $j \geq 0$. For any $n > j$, we have

$$\sum_{\nu=1}^{\infty} S^{n_\nu} e_j = \sum_{\nu=1}^{\infty} \frac{e_{n_\nu+j}}{w_{1+j} \cdots w_{n_\nu+j}}$$
$$= w_1 \cdots w_j \cdot \sum_{\nu=1}^{\infty} \frac{e_{n_\nu+j}}{w_1 \cdots w_{n_\nu+j}}.$$

By Theorem 3.2, the sum converges unconditionally, for all $n > j$ and this shows the condition 1 in Theorem 2.4.

Let $j \geq 0$. For any $n > j$, we have $l_i - n_\nu \geq \max\{l, n\} > j$. Thus

$$\sum_{\nu=1}^{\infty} B_{\mathbf{w}}^{l_i - n_\nu} e_j = 0,$$

which proves the condition 3.

Since

$$S^{n_\nu - l_i} e_j = \frac{e_{n_\nu - l_i + j}}{w_{1+j} \cdots w_{n_\nu - l_i + j}},$$

by Theorem 3.2,

$$\begin{aligned} \left\| \sum_{\nu=1}^{\infty} S^{n_\nu - l_i} e_j \right\| &= \left\| \sum_{\nu=1}^{\infty} \frac{e_{n_\nu - l_i + j}}{w_{1+j} \cdots w_{n_\nu - l_i + j}} \right\| \\ &\leq \frac{1}{l} \cdot \frac{1}{c_n - \frac{1}{n}} \end{aligned}$$

If $n \to \infty$, then $\| \sum_{\nu=1}^{\infty} S^{n_\nu - l_i} e_j \| \to 0$. Thus for any $\epsilon >$ and $l \geq 1$, we have $\left\| \sum_{\nu=1}^{\infty} S^{n_\nu - l_i} e_j \right\| < \epsilon$ and this shows the condition 2. ■

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