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Chi-squared goodness-of-fit test for transmuted generalized linear exponential distribution

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Abstract

The main purpose of this work is the construction of chi-squared goodness-of-fit tests for the transmuted generalized linear exponential distribution with unknown parameters and right censoring. The criterion test used is the modified chi-squared statistic Y^2, developed by Bagdonavicius and Nikulin (2011) for some parametric models when data are censored. The performances of the proposed test are shown by an intensive simulation study and an application to real data.

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Keywords: Censored data, chi-squared test, maximum likelihood estimation.

1. Introduction

The main problem in statistical analysis is to assess whether an observed distribution can be estimated or not by a probability distribution. Various techniques have been used to validate the model chosen such as graphical tests, chi-squared tests, Kolmogorov-Smirnov statistic, Anderson-Darling statistic and others. The principle of these tests is to measure the distance between the observed values and the expected theoretical values, If this distance is greater than the critical distance, we conclude that the model chosen should be rejected.

The literature is rich and varied when the model is well specified. However, if data are censored and the parameters are unknown, which often happens in reliability and medical studies, the problem remains open. The adequacy of many newly introduced distributions have not yet been investigated.

The aim of this paper is to study the adequacy of the transmuted generalized linear exponential distribution ($TGLE$) proposed recently by Elbatal *et al.* (2013). This distribution is very interesting in modeling many lifetime data. It can be considered as a generalization of generalized linear exponential, Weibull, transmuted Weibull, transmuted linear exponential, Rayleigh and transmuted Rayleigh distributions. The authors studied statistical properties, reliability analysis and maximum likelihood parameter's estimation for complete samples.

Censorship and goodness-of-fit tests are not investigated. After computing maximum likelihood estimators for unknown parameters, we propose the construction of chi-squared tests for the $TGLE$ model when data are right censored. We use the modified chi-squared statistic developed by Bagdonavicius and Nikulin (2011) for some parametric accelerated failure times models. This technique has been used to validate some models like, generalized Birnbaum–Saunders distribution (Nikulin and Tran, 2013), generalized inverse Weibull accelerated failure time model (Goual and Seddik-Ameur, 2014), competing risk model (Chouia and Seddik-Ameur, 2014).

Note that modified Kolmogorov-Smirnov and Anderson-Darling statistics which take into account the censorship have been proposed for testing accelerated failure time models (Galanova *et al.* 2012). Bagdonavicius *et al.* (2013) gave an overview of chi-squared tests and their different applications.

2. Transmuted generalized linear exponential distribution

The transmuted generalized linear exponential distribution ($TGLE$) proposed recently by Elbatal *et al.* (2013) has a great interest for modeling many real data whether in reliabilty, in survival analysis and biology. It can be considered as generalization of at least four probability distributions: transmuted linear exponential distribution, generalized linear exponential distribution, transmuted Weibull distribution, transmuted Rayleigh distribution, in addition to simple distributions linear exponential, Weibull and Rayleigh. This distribution is caracterized by four parameters vector $\gamma = (\alpha, \lambda, \beta, \sigma)^T$, where α and λ are scale parameters, β and σ are respectively shape and transmuted parameters. Its cumulative density and hazard functions are

$$F(t;\gamma) = \left(1 - e^{-\left(\alpha t + \frac{\lambda}{2}t^2\right)^\beta}\right)\left(1 + \sigma e^{-\left(\alpha t + \frac{\lambda}{2}t^2\right)^\beta}\right)$$

$$h(t;\gamma) = \frac{(\alpha + \lambda t)\,\beta e^{-\left(\alpha t + \frac{\lambda}{2}t^2\right)^\beta}\left(\alpha t + \frac{\lambda}{2}t^2\right)^{\beta-1}\left(1 - \sigma + 2\sigma e^{-\left(\alpha t + \frac{\lambda}{2}t^2\right)^\beta}\right)}{1 - \left(1 - e^{-\left(\alpha t + \frac{\lambda}{2}t^2\right)^\beta}\right)\left(1 + \sigma e^{-\left(\alpha t + \frac{\lambda}{2}t^2\right)^\beta}\right)}$$

The authors studied maximum likelihood estimation for complete samples, moment generating function, order statistics and derived information matrix. They showed the utility of this distribution by applications on real data sets from survival analysis and reliabilty.

3. Maximum likelihood estimaton in censored data case

Suppose that $T_1, T_2, \ldots, T_n$ is a random sample with right censoring from $TGLE(\gamma)$ distribution. The observed data $t_i = \min(T_i, C_i)$; $i = 1, 2, \ldots, n$ are the minimum of the survival time T_i and censoring time C_i for each subject in the sample. So, t_i can be written in the form $(t_i, \delta_i)_{i=1,\ldots,n}$ where, $\delta_i = 1$ if T_i is the moment of failure (complete observation) and $\delta_i = 0$ if T_i is the moment of censoring. The right censoring is assumed to be non informative, so the expression of the likelihood function is

$$l(t, \gamma) = \Pi_{i=1}^{n} f(t_i, \gamma)^{\delta_i} S(t_i, \gamma)^{1-\delta_i}, \qquad \delta_i = 1_{T_i < C_i}$$

The log-likelihood function of $TGLE(\gamma)$ distribution is

$$\begin{aligned} L(\gamma) \;=\; & \sum_{i=1}^{n} \delta_i \left[\ln(\beta) + \ln(\alpha + \lambda t_i) - \left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)^{\beta} + (\beta - 1) \ln\left(\alpha t_i + \frac{\lambda}{2} t_i^2\right) \right. \\ & + \ln\left(1 - \sigma + 2\sigma e^{-\left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)^{\beta}}\right) \\ & \left. - \ln\left[1 - \left(1 - e^{-\left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)^{\beta}}\right)\left(1 + \sigma e^{-\left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)^{\beta}}\right)\right]\right] \\ & + \sum_{i=1}^{n} \ln\left[1 - \left(1 - e^{-\left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)^{\beta}}\right)\left(1 + \sigma e^{-\left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)^{\beta}}\right)\right] \end{aligned}$$

and the score functions are obtained as follows

$$\begin{aligned} \frac{\partial L}{\partial \alpha} &= \sum_{i=1}^{n} \delta_i \left[\frac{1}{\alpha + \lambda t_i} + \frac{(\beta - 1) t_i}{u_i} + \beta\sigma\,(1 - \sigma)\, t_i u_i^{\beta - 1} M(t_i, \gamma) \right] \\ & \quad - \beta \sum_{i=1}^{n} \frac{t_i u_i^{\beta-1} \left[1 - \sigma + 2\sigma e^{-u_i^{\beta}}\right]}{1 - \sigma + \sigma e^{-u_i^{\beta}}} \\ \frac{\partial L}{\partial \lambda} &= \sum_{i=1}^{n} \delta_i \left[\frac{t_i}{\alpha + \lambda t_i} + \frac{(\beta - 1) t_i^2}{2u_i} + \beta\sigma\,(1 - \sigma)\, t_i^2 u_i^{\beta - 1} M(t_i, \gamma) \right] \\ & \quad - \frac{\beta}{2} \sum_{i=1}^{n} \frac{t_i^2 u_i^{\beta-1} \left[1 - \sigma + 2\sigma e^{-u_i^{\beta}}\right]}{1 - \sigma + \sigma e^{-u_i^{\beta}}} \\ \frac{\partial L}{\partial \beta} &= \sum_{i=1}^{n} \delta_i \left[\frac{1}{\beta} + \ln u_i + \sigma\,(1 - \sigma)\, M(t_i, \gamma) u_i^{\beta} \ln(u_i) \right] \\ & \quad - \sum_{i=1}^{n} \frac{u_i^{\beta} \ln(u_i) \left[1 - \sigma + 2\sigma e^{-u_i^{\beta}}\right]}{1 - \sigma + \sigma e^{-u_i^{\beta}}} \end{aligned}$$

$$\frac{\partial L}{\partial \sigma} = \sum_{i=1}^{n} \delta_i M(t_i, \gamma) - \sum_{i=1}^{n} \frac{1 - e^{-u_i^{\beta}}}{1 - \sigma + \sigma e^{-u_i^{\beta}}}$$

with

$$M(t_i, \gamma) = e^{-u_i^{\beta}} \left(1 - \sigma + \sigma e^{-u_i^{\beta}}\right)^{-1} \left(1 - \sigma + 2\sigma e^{-u_i^{\beta}}\right)^{-1} \quad , \quad u_i = \left(\alpha t_i + \frac{\lambda}{2} t_i^2\right)$$

Maximum likelihood estimators of the unknown parameters can be obtained using various techniques, either software R, EM algorithm or Newton Raphson method.

4. Modified chi-squared type test for right censored data

Methods for testing the validity of parametric models are in constant development, but the presence of censorship make them unavalaible. Habib and Thomas (1986), Hollander and Pena (1992) proposed modified chi-squared test based on Kaplan-Meyer estimators, Galanova *et al.* (2012) considered modifications of Kolmogorov-Smirnov statistic, Anderson-Darling statistic, Cramer Von-Mises statistic for accelerate failure models. In this work, we are interested by the modified chi-squared type test proposed by Bagdonavicius and Nikulin (2011), Bagdonavicius et al. (2013), for parametric models with right censored data. Based on maximum likelihood estimators on non grouped data, this statistic test is also based on the differences between the numbers of observed failures and the numbers of expected failures in the grouped intervals chosen. For this, random grouping intervals are considered as data functions. The description of the construction of this chi-squared type test is developed in Voinov *et al.* (2013). The statistic test is defined as follows.

Suppose that $T_1, T_2, \ldots, T_n$ is a random sample with right censoring from a parametric model, and a finite time τ.

The statistic test is defined as follows.

$$Y_n^2 = \sum_{j=1}^{n} \frac{\left(U_j - e_j\right)^2}{U_j} + Q$$

where U_j and e_j are the observed and the expected numbers of failure in grouping intervals, and Q is

$$\begin{aligned} Q &= W^T \hat{G}^- W, \quad \hat{W} = \hat{C}\hat{A}^{-1} Z = \left(\hat{W}_1, \ldots, \hat{W}_s\right)^T, \quad Z_j = \frac{1}{\sqrt{n}} \left(U_j - e_j\right) \\ W_l &= \sum_{j=1}^{r} \hat{C}_{lj} \hat{A}_j^{-1} Z_j, \quad \hat{G} = \left[\hat{g}_{ll'}\right]_{s \times s}, \quad \hat{g}_{ll'} = \hat{\imath}_{ll'} - \sum_{j=1}^{r} \hat{C}_{lj} \hat{C}_{l'j} \hat{A}_j^{-1} \\ i &= 1, \ldots, n, \quad j = 1, \ldots, r, \quad l, l\prime = 1, \ldots, s \end{aligned}$$

The limits a_j of r random grouging intervals $I_j = [a_{j-1}, a_j[$ are chosen such as the expected failure times to fall into these intervals are the same for each $j = 1, \ldots, r - 1$,

$\hat{a}_r = \max\left(t_{(l)}, \tau\right)$. The estimated $\hat{a}_j$ is defined by

$$\hat{a}_j = H^{-1}\left(\frac{E_j - \sum_{l=1}^{i-1} H\left(t_l, \theta\right)}{n-i+1}, \hat{\theta}\right), \qquad \hat{a}_k = \max\left(t_{(n)}, \tau\right)$$

where $H(t)$ is the cdf of the model distribution. This statistic test Y_n^2 follows a chi-squared distribution.

4.1. Choice of random grouping intervals

Suppose that $T_1, T_2, \ldots, T_n$ is a random sample with right censoring from $TGLE(\gamma)$ distribution and a finite time τ. In our case, the estimated $\hat{a}_j$ is obtained as follows

$$\hat{a}_j = -\frac{\hat{\alpha}}{\hat{\lambda}} + \frac{1}{\hat{\lambda}}\sqrt{\hat{\alpha}^2 + 2\hat{\lambda}\left(-\ln\left\{\frac{\hat{\sigma} - 1 + \sqrt{\left(1-\hat{\sigma}\right)^2 + 4\hat{\sigma}\exp\left(\frac{\sum_{l=1}^{i-1} H(t_l, \hat{\sigma}) - E_j}{(n-i+1)}\right)}}{2\hat{\sigma}}\right\}\right)^{1/\hat{\beta}}}$$

where $\hat{\gamma} = (\hat{\alpha}, \hat{\lambda}, \hat{\beta}, \hat{\sigma})^T$ are the maximum likelihood estimators of the unknown parameters $\gamma = (\alpha, \lambda, \beta, \sigma)^T$ on initial data, and $H(t)$ is the cumulative distribution function of the $TGLE(\gamma)$ distribution.

4.2. Quadratic form Q

To calculate the quadratic form Q of the statistic Y_n^2, and as its distribution does'nt depend on the parameters, so we can use the estimated matrices $\hat{W}$, $\hat{C}$ and the estimated information matrix $\hat{I}$. The elements of $\hat{C}$ defined by

$$\hat{C}_{lj} = \frac{1}{n}\sum_{i:t_i \in I_j}^{n} \delta_i \frac{\partial}{\partial \hat{\gamma}_l} \ln h(t; \hat{\gamma})$$

are obtained as below

$$\hat{C}_{1j} = \frac{1}{n}\sum_{i:t_i \in I_j}^{n} \delta_i \left[\frac{1}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i}{u_i} + \hat{\beta}\hat{\sigma}\left(1 - \hat{\sigma}\right) t_i u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma})\right]$$

$$\hat{C}_{2j} = \frac{1}{n}\sum_{i:t_i \in I_j}^{n} \delta_i \left[\frac{t_i}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i^2}{2u_i} + \hat{\beta}\hat{\sigma}\left(1 - \hat{\sigma}\right) t_i^2 u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma})\right]$$

$$\hat{C}_{3j} = \frac{1}{n}\sum_{i:t_i \in I_j}^{n} \delta_i \left[\frac{1}{\hat{\beta}} + \ln\left(u_i\right) + \sigma\left(1 - \sigma\right) M(t_i, \gamma) u_i^{\beta} \ln\left(u_i\right)\right]$$

$$\hat{C}_{4j} = \frac{1}{n} \sum_{i:t_i \in I_j}^{n} \delta_i M(t_i, \gamma)$$

Therefore the estimated matrix $\hat{W}$ can be deducted from $\hat{C}$.

4.2.1 Estimated information matrix $\hat{I}$

We need also the information matrix $\hat{I}$ of the $TGLE(\gamma)$ distribution with right censoring. After difficult calculations and some simplifications, we have obtained the elements of the matrix as follows:

$$\hat{\imath}_{11} = \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(\frac{1}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i}{u_i} + \hat{\beta}\hat{\sigma} \left(1 - \hat{\sigma}\right) t_i u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma}) \right)^2$$

$$\hat{\imath}_{22} = \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(\frac{t_i}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i^2}{2u_i} + \hat{\beta}\hat{\sigma} \left(1 - \hat{\sigma}\right) t_i^2 u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma}) \right)^2$$

$$\hat{\imath}_{33} = \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(\frac{1}{\hat{\beta}} + \ln(u_i) + \sigma (1 - \sigma) M(t_i, \gamma) u_i^{\beta} \ln(u_i) \right)^2$$

$$\hat{\imath}_{44} = \frac{1}{n} \sum_{i=1}^{n} \delta_i M^2(t_i, \gamma)$$

$$\begin{aligned} \hat{\imath}_{12} &= \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(\frac{1}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i}{u_i} + \hat{\beta}\hat{\sigma} \left(1 - \hat{\sigma}\right) t_i u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma}) \right) \\ &\quad \times \left(\frac{t_i}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i^2}{2u_i} + \hat{\beta}\hat{\sigma} \left(1 - \hat{\sigma}\right) t_i^2 u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma}) \right) \end{aligned}$$

$$\begin{aligned} \hat{\imath}_{13} &= \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(\frac{1}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i}{u_i} + \hat{\beta}\hat{\sigma} \left(1 - \hat{\sigma}\right) t_i u_i^{\hat{\beta}-1} M(t_i, \hat{\gamma}) \right) \\ &\quad \times \left(\frac{1}{\hat{\beta}} + \ln(u_i) + \sigma (1 - \sigma) M(t_i, \gamma) u_i^{\beta} \ln(u_i) \right) \end{aligned}$$

$$\hat{\imath}_{14} = \frac{1}{n} \sum_{i=1}^{n} \delta_i \left(\frac{M(t_i, \gamma)}{\hat{\alpha} + \hat{\lambda} t_i} + \frac{\left(\hat{\beta} - 1\right) t_i}{u_i} M(t_i, \gamma) + \hat{\beta}\hat{\sigma} \left(1 - \hat{\sigma}\right) t_i u_i^{\hat{\beta}-1} M^2(t_i, \hat{\gamma}) \right)$$

$$\hat{\imath}_{23} = \frac{1}{n}\sum_{i=1}^{n}\delta_i\left(\frac{t_i}{\hat{\alpha}+\hat{\lambda}t_i}+\frac{\left(\hat{\beta}-1\right)t_i^2}{2u_i}+\hat{\beta}\hat{\sigma}\left(1-\hat{\sigma}\right)t_i^2u_i^{\hat{\beta}-1}M(t_i,\hat{\gamma})\right) \times\left(\frac{1}{\hat{\beta}}+\ln(u_i)+\sigma(1-\sigma)M(t_i,\gamma)u_i^{\beta}\ln(u_i)\right)$$

$$\hat{\imath}_{24} = \frac{1}{n}\sum_{i=1}^{n}\delta_i\left(\frac{M(t_i,\gamma)t_i}{\hat{\alpha}+\hat{\lambda}t_i}+\frac{\left(\hat{\beta}-1\right)t_i^2}{2u_i}M(t_i,\gamma)+\hat{\beta}\hat{\sigma}\left(1-\hat{\sigma}\right)t_i^2u_i^{\hat{\beta}-1}M^2(t_i,\hat{\gamma})\right)$$

$$\hat{\imath}_{34} = \frac{1}{n}\sum_{i=1}^{n}\delta_i\left(\frac{M(t_i,\gamma)}{\hat{\beta}}+M(t_i,\gamma)\ln(u_i)+\sigma(1-\sigma)M^2(t_i,\gamma)u_i^{\beta}\ln(u_i)\right)$$

As all the components of the statistic are given explicitly, then we obtain the statistic test for the $TGLE(\gamma)$ distribution with unknown parameters and right censored data. This statistic follows a chi-squared distribution with r degrees of freedom.

$$Y_n^2(\hat{\gamma}) = \sum_{j=1}^{r}\frac{\left(U_j-e_j\right)^2}{U_j}+\hat{W}^T\left[\hat{\imath}_{ll'}-\sum_{j=1}^{r}\hat{C}_{lj}\hat{C}_{l'j}\hat{A}_j^{-1}\right]^{-1}\hat{W}$$

5. Simulations

In this section we conduct an importante simulation study to consolidate our results. For this, 10, 000 censored samples (with sizes: $n_1 = 15$,$n_2 = 25$, $n_3 = 50$, $n_4 = 130$, $n_5 = 350$, $n_6 = 500$) from $TGLE(\gamma)$ distribution are simulated.

5.1. Maximum likelihood estimation

We generate the simulated samples with the following parameters values $\alpha = 0.3$, $\lambda = 0.6$, $\beta = 0.65$, $\sigma = 0.7$. Using R software and BB algorithm, means simulated maximum likelihood estimators and their squared mean errors are calculated and given in table 1. As shown in these results, maximum likelihood estimators are convergent.

5.2. Modified chi-squared test $Y^2(\widehat{\gamma})$

Depending on the values of its parameters, the transmuted generalized linear exponential distribution $TGLE(\gamma)$ is the generalization of several distributions. For testing the null hypothesis H_0 that the correponding data belongs to the $TGLE(\gamma)$, we calculate the values of the modified chi-squared statistic Y_n^2 for all the samples generated above. Then we compute means of the numbers of rejection of H_0, when $Y^2 > \chi_\varepsilon^2(r)$ for significance levels $\varepsilon(\varepsilon = 0.10, \varepsilon = 0.05, \varepsilon = 0.01)$. We repeat the same procedure for all sub models.

Table 1: Means simulated maximum likelihood estimators $\hat{\alpha}$, $\hat{\lambda}$, $\hat{\beta}$, $\hat{\sigma}$ and their square mean errors.

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\hat{\alpha}$	0.2425	0.2541	0.2625	0.2745	0.2894	0.2978
$S.M.E$	0.0341	0.0216	0.0167	0.0098	0.0018	0.0010
$\hat{\lambda}$	0.5423	0.5525	0.5714	0.5854	0.5945	0.6012
$S.M.E$	0.0291	0.0224	0.0121	0.0054	0.0035	0.0022
$\hat{\beta}$	0.6821	0.6754	0.6625	0.6554	0.6562	0.6519
$S.M.E$	0.0152	0.0148	0.0098	0.0050	0.0035	0.0017
$\hat{\sigma}$	0.7654	0.7425	0.7315	0.7245	0.7194	0.7098
$S.M.E$	0.0135	0.0130	0.0126	0.0089	0.0052	0.0036

The results of the comparison between simulated values and their theoretical values are given below:

Case of transmuted generalized linear exponential distribution $TGLED$

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\alpha = 1\%$	0.0023	0.0054	0.0084	0.0091	0.0112	0.0128
$\alpha = 5\%$	0.0034	0.0072	0.0097	0.0108	0.0152	0.0163
$\alpha = 10\%$	0.0073	0.0120	0.0139	0.0260	0.0282	0.0319

Case of generalized linear exponential distribution $GLED$

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\alpha = 1\%$	0.0029	0.0059	0.0087	0.0094	0.0119	0.0138
$\alpha = 5\%$	0.0030	0.0079	0.0099	0.0102	0.0121	0.0168
$\alpha = 10\%$	0.0063	0.0109	0.0121	0.0258	0.0291	0.0312

Case of linear exponential distribution LED

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\alpha = 1\%$	0.0032	0.0065	0.0080	0.0105	0.0135	0.0147
$\alpha = 5\%$	0.0045	0.0087	0.0102	0.0141	0.0162	0.0174
$\alpha = 10\%$	0.0084	0.0135	0.0165	0.0198	0.0237	0.0295

Case of transmuted linear exponential distribution $TLED$

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\alpha = 1\%$	0.0036	0.0072	0.0099	0.0118	0.0149	0.0172
$\alpha = 5\%$	0.0063	0.0073	0.0143	0.0150	0.0176	0.0293
$\alpha = 10\%$	0.0089	0.0152	0.0175	0.0245	0.0286	0.0321

Case of transmuted Rayleigh distribution TRD

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\alpha = 1\%$	0.0039	0.0078	0.0092	0.0121	0.0154	0.0198
$\alpha = 5\%$	0.0051	0.0082	0.0098	0.0109	0.0134	0.0178
$\alpha = 10\%$	0.0078	0.0098	0.0145	0.0224	0.0286	0.0328

Case of Rayleigh distribution RD

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\alpha = 1\%$	0.0054	0.0068	0.0072	0.0085	0.0098	0.0109
$\alpha = 5\%$	0.0125	0.0150	0.0212	0.0250	0.0310	0.0350
$\alpha = 10\%$	0.0236	0.0335	0.0369	0.0410	0.0432	0.0496

Case of transmuted Weibull distribution TWD

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\varepsilon = 1\%$	0.0124	0.0128	0.0204	0.0213	0.0245	0.0247
$\varepsilon = 5\%$	0.0214	0.0257	0.0289	0.0312	0.0324	0.0336
$\varepsilon = 10\%$	0.0254	0.0274	0.0289	0.0345	0.0389	0.0425

Case of Weibull distribution WD

$M = 10,000$	$n_1 = 15$	$n_2 = 25$	$n_3 = 50$	$n_4 = 130$	$n_5 = 350$	$n_6 = 500$
$\varepsilon = 1\%$	0.0142	0.0182	0.0240	0.0321	0.0425	0.0427
$\varepsilon = 5\%$	0.0241	0.0327	0.0349	0.0378	0.0398	0.0433
$\varepsilon = 10\%$	0.0314	0.0374	0.0389	0.0445	0.0439	0.0589

As expected the results confirm that the statistic test proposed in this work can be used to check suitably the validity of the transmuted generalized linear exponential distribution $TGLE(\gamma)$ distribution and its sub models.

6. Application

We consider sample data of 51 patients with advanced acute myelogenous leukemia reported to the International Bone Marrow Transplant Registry.These patients had received an autologous (auto) bone marrow transplant in which, after high doses of chemotherapy, their own marrow was reinfused to replace their destroyed immune system.

Leukemia free-survival times (in months) for Autologous Transplants:

0.658, 0.822, 1.414, 2.5, 3.322, 3.816, 4.737, 4.836*, 4.934, 5.033, 5.757, 5.855, 5.987, 6.151, 6.217, 6.447*, 8.651, 8.717, 9.441*, 10.329, 11.48, 12.007, 12.007*, 12.237, 12.401*, 13.059*, 14.474*, 15*, 15.461, 15.757, 16.48, 16.711, 17.204*, 17.237, 17.303*, 17.664*, 18.092, 18.092*, 18.75*, 20.625*, 23.158, 27.73*, 31.184*, 32.434*, 35.921*,

Table 2: Values of $\widehat{a}_j, e_j, U_j, \hat{C}_{1j}, \hat{C}_{2j}, \hat{C}_{3j}, \hat{C}_{4j}, \hat{C}_{5j}$.

$\hat{a}_j$	3.5487	8.1876	13.284	21.956	56.086
U_J	5	9	6	6	2
$\hat{C}_{1j}$	−0.1025	−7.0414	−2.049	−4.0412	−9.0711
$\hat{C}_{2j}$	1.0584	0.8783	1.9278	1.3479	1.8439
$\hat{C}_{3j}$	−0.0546	−0.5044	−0.2707	−0.7086	−0.4561
$\hat{C}_{4j}$	0.4829	0.8574	0.6463	0.8575	0.7550
e_j	5.5246	5.5246	5.5246	5.5246	5.5246

Table 3: Values of the test statistics Y_n^2 for Leukemia free-survival times to fit the different distributions.

$Modeling\ distribution$	Y_n^2
$TGLED$	7.0107
$GLED$	7.9256
$TLED$	9.5748
LED	8.524
TRD	10.6358
RD	12.0659
TWD	11.6989
WD	12.5855

42.237*,44.638*, 46.48*, 47.467*, 48.322*, 56.086. (*indicates the censorship)

We use the statistic test provided above to verify if these data are modeled by the transmuted generalized linear exponential distribution $TGLE(\gamma)$, and At that end,we first calculate the maximum likelihood estimators of the unknown parameters

$$\hat{\theta} = \left(\hat{\alpha}, \hat{\lambda}, \hat{\beta}, \hat{\sigma}\right)^T = (2.534, 0.8716, 0.758, 0.0530)^T .$$

Data are grouped into $r = 5$ intervals I_j. We give the necessary calculus in the following table 2.

Then we obtain the value of the statistic test Y_n^2:

$$Y_n^2 = X^2 + Q = 5.5653 + 2.4454 = 7.0107$$

For significance level $\varepsilon = 0.05$, the critical value $\chi_5^2 = 11.0705$ is superior than the value of $Y_n^2 = 7.0107$, so we can say that the proposed model $TGLE$ fit these data. The test statistics Y_n^2 to fit these data to the sub models are also calculated and given in table 3.

7. Conclusion

The transmuted generalized linear exponential distribution is very interesting in modeling several real data in reliability study. So goodness-of-fit tests are needed in the analysis. The statistic test provided in this work can be used to fit data to this model and its sub models with unknown parameters and censorship. The performances of the results and the effectivness of the proposed test are shown by an importante simulation study.

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