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Indexed Norlund Summability Of A Factored Fourier Series-Via-Local Property

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ABSTRACT

In this paper we have established a theorem on $\left|N,\ p_n,\alpha_n,\delta,\gamma\right|_k$ -summability of a factored Fourier series-via-Local property.

KEY WORDS: $\left|N,p_n\right|_k$ -summability, $\left|N,\ p_n,\alpha_n\right|_k,\ k\geq 1$ -summability, $\left|N,\ p_n,\alpha_n,\delta\right|_k,\quad k\geq 1,\ \delta\geq 0$ -summability, $\left|N,\ p_n,\alpha_n,\delta,\gamma\right|_k,\ k\geq 1,\ \delta\geq 0$ -summability.

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1. INTRODUCTION:

Let $\sum a_n$ be a given infinite series with sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of positive real constants such that

(1.1) $$P_n=\sum_{\nu=0}^{n}p_\nu \to \infty \ as\ n\to\infty\ (P_{-i}=p_{-i}=0, i\geq 1)$$

The sequence-to-sequence transformation

(1.2) $$t_n=\frac{1}{P_n}\sum_{\nu=0}^{n}p_{n-\nu}s_\nu$$

defines (N, p_n)-mean of the sequence $\{s_n\}$ generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be summable $|N, p_n|_k$, $k \geq 1$, if

(1.3) $$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{k-1}|t_n - t_{n-1}|^k < \infty .$$

For k=1, $|\overline{N}, p_n|_k$ -summability is same as $|N, p_n|$-summability.

When $p_n = 1$, for all n and $k = 1$, $|N, p_n|_k$ -summability is same as |C, 1| summability.

Let $\{\alpha_n\}$ be any sequence of positive numbers. The series $\sum a_n$ is said to be summable $|N, p_n, \alpha_n|_k$, $k \geq 1$, if

(1.4) $$\sum_{n=1}^{\infty}\alpha_n^{k-1}|t_n - t_{n-1}|^k < \infty,$$

where $\{t_n\}$ is as defined in (5.1.2).The series $\sum a_n$ is said to be $|N, p_n, \alpha_n, \delta|_k$, $k \geq 1, \delta \geq 0$, summable if

(1.5) $$\sum_{n=1}^{\infty}\alpha_n^{\delta k+k-1}|t_n - t_{n-1}|^k < \infty.$$

For $\delta = 0$, the summability metod $|N, p_n, \alpha_n, \delta|_k$, $k \geq 1, \delta \geq 0$, reduces to the summabilty method $|N, p_n, \alpha_n|_k$, $k \geq 1$

For any real number γ, the series $\sum a_n$ is said to be summable by the summabilty method $|N, p_n, \alpha_n; \delta, \gamma|_k$, $k \geq 1, \delta \geq 0$, if

(1.6) $$\sum_{n=1}^{\infty}\alpha_n^{\gamma(\delta k+k-1)}|t_n - t_{n-1}|^k < \infty .$$

For $\gamma = 1$, the summability method $|N, p_n, \alpha_n; \delta, \gamma|_k$, $k \geq 1, \delta \geq 0$, any real γ, reduces to the method $|N, p_n, \alpha_n; \delta|_k$, $k \geq 1, \delta \geq 0$.

A sequence $\{\lambda_n\}$is said to be convex if $\Delta^2\lambda_n \geq 0$ for every positive integer n.

Let $f(t)$ be a periodic function with period 2π and integrable in the sense of Lebesgue over $(-\pi, \pi)$. Without loss of generality we may assume that the constant term in the Fourier series of $f(t)$ is zero, so that

(1.7) $$f(t) \sim \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t)$$

It is well known that the convergence of Fourier series at t = x is a local property of $f(t)$ (i.e., it depends only on the behavior of $f(t)$ in an arbitrarily small neighborhood of x) and hence the summability of the Fourier series at t = x by any regular linear method is also a local property of $f(t)$.

2. KNOWN THEOREMS:

Dealing with the $\left|\overline{N}, p_n\right|_k$ -summability of an infinite series Bor [5] proved the following theorem:

THEOREM-2.1:

Let $k \geq 1$ and let the sequences $\{p_n\}$ and $\{\lambda_n\}$ be such that

(2.1.1) $$\Delta X_n = O\left(\frac{1}{n}\right),$$

(2.1.2) $$\sum_{n=1}^{\infty} X_n^{k-1} \frac{|\lambda_n|^k + |\lambda_{n+1}|^k}{n} < \infty,$$

and

(2.1.3) $$\sum_{n=1}^{\infty} (X_n^k + 1)|\Delta\lambda_n| < \infty,$$

where $X_n = \dfrac{P_n}{np_n}$. Then the summability $\left|\overline{N}, p_n\right|_k$ of the factored Fourier series $\sum_{n=1}^{\infty} A_n(t)\lambda_n X_n$ at a point can be ensured by the local property.

Subsequently, Padhy et al [2] proved the following theorem on the local property of $\left|\overline{N}, p_n, \alpha_n, \delta\right|_k$ summability($k \geq 1, \delta \geq 0$) of a factored Fourier series.

THEOREM-2.2:

Let $k \geq 1$. Suppose $\{\lambda_n\}$ be a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent and $\{p_n\}$ be a sequence of positive numbers such that

(2.2.1) $$\Delta X_n = O\left(\frac{1}{n}\right),$$

(2.2.2) $$\sum_{n=v+1}^{m+1} \alpha_n^{\delta k+k-1} \left(\frac{p_n}{P_n}\right)^k \left(\frac{1}{P_{n-1}}\right) = O\left(\frac{1}{P_v}\right),$$

(2.2.3) $$\sum_{n=1}^{\infty} X_n^{k-1} \frac{|\lambda_n|^k + |\lambda_{n+1}|^k}{n} < \infty,$$

(2.2.4) $$\sum_{n=1}^{\infty} (X_n^k + 1)|\Delta\lambda_n| < \infty,$$

and

(2.2.5) $$\sum_{n=2}^{\infty} \alpha_n^{\delta k+k-1} \frac{|\lambda_n|^k}{n^k} < \infty,$$

where $X_n = \dfrac{P_n}{np_n}$. Then the summability $\left|\overline{N}, p_n, \alpha_n, \delta\right|_k$, $k \geq 1, \delta \geq 0$ of the factored

Fourier series $\sum_{n=1}^{\infty} A_n(t)\lambda_n X_n$ at a point can be ensured by the local property, where $\{\alpha_n\}$ is a sequence of positive numbers.
In what follows, in the present paper we establish the following theorem on $|N, p_n, \alpha_n, \delta, \gamma|_k$ -summability of a factored Fourier series through its local property.

3. MAIN THEOREM:

Let $k \geq 1$. Suppose $\{\lambda_n\}$ be a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent and $\{p_n\}$ be a sequence such that

(3.1) $$\Delta X_n = O\left(\frac{1}{n}\right),$$

(3.2) $$\frac{P_{n-r-1}}{P_n} = O\left(\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\right),$$

(3.3) $$\sum_{n=r+1}^{m+1} (\alpha_n)^{\delta k+k-1}\frac{p_{n-r}}{P_n} = O\left(\frac{p_r}{P_r}\right),$$

(3.4) $$\sum_{n=1}^{\infty} X_n^{k-1}\frac{|\lambda_n|^k}{n} < \infty,$$

and

(3.5) $$\sum_{n=1}^{\infty} X_n^{k-1}\frac{|\Delta\lambda_n|^k}{n} < \infty,$$

where $X_n = \dfrac{P_n}{np_n}$. Then the summability $|N, p_n, \alpha_n, \delta, \gamma|_k$, $k \geq 1$of the factored Fourier series $\sum_{n=1}^{\infty} A_n(t)\lambda_n X_n$ at a point can be ensured by the local property, where $\{\alpha_n\}$ is a sequence of positive numbers.

4. REQUIRED LEMMA:

In order to prove the above theorem we require the following lemma:

LEMMA-4.1:

Let $k \geq 1$and suppose $\{\lambda_n\}$ be a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent and $\{p_n\}$ be a sequence such that the conditions (3.1)-(3.5) are satisfied. If $\{s_n\}$ is bounded, then for the sequence of positive numbers $\{\alpha_n\}$ the series $\sum_{n=1}^{\infty} a_n\lambda_n X_n$ is summable$|N, p_n, \alpha_n, \delta|_k$, $k \geq 1, \delta \geq 0$.

PROOF OF THE LEMMA-4.1:

Let $\{T_n\}$ denote the (N, p_n)-mean of the series $\sum_{n=1}^{\infty} a_n \lambda_n X_n$. Then by definition we have

$$T_n = \frac{1}{P_n}\sum_{\nu=0}^{n} p_{n-\nu}\sum_{r=0}^{\nu} a_r \lambda_r X_r$$
$$= \frac{1}{P_n}\sum_{r=0}^{n} a_r \lambda_r X_r \sum_{\nu=r}^{n} p_{n-\nu}$$
$$= \frac{1}{P_n}\sum_{r=0}^{n} a_r P_{n-r}\lambda_r X_r$$

Hence

$$T_n - T_{n-1} = \frac{1}{P_n}\sum_{r=1}^{n} P_{n-r} a_r \lambda_r X_r - \frac{1}{P_{n-1}}\sum_{r=1}^{n-1} P_{n-r-1} a_r \lambda_r X_r$$
$$= \sum_{r=1}^{n}\left(\frac{P_{n-r}}{P_n} - \frac{P_{n-r-1}}{P_{n-1}}\right) a_r \lambda_r X_r$$
$$= \frac{1}{P_n P_{n-1}}\sum_{r=1}^{n}\left(P_{n-r}P_{n-1} - P_{n-r-1}P_n\right) a_r \lambda_r X_r$$
$$= \frac{1}{P_n P_{n-1}}\left[\sum_{r=1}^{n-1}\Delta\left\{\left(P_{n-r}P_{n-1} - P_{n-r-1}P_n\right)\lambda_r X_r\right\}\right]\sum_{\nu=1}^{r} a_\nu$$
$$= \frac{1}{P_n P_{n-1}}\left[\sum_{r=1}^{n-1}\left(p_{n-r}P_{n-1} - p_{n-r-1}P_n\right)\lambda_r X_r s_r\right.$$
$$+\sum_{r=1}^{n-1}\left(P_{n-r-1}P_{n-1} - P_{n-r-2}P_n\right)\Delta\lambda_r X_r s_r$$
$$\left.+\sum_{r=1}^{n-1}\left(P_{n-r-1}P_{n-1} - P_{n-r-2}P_n\right)\lambda_{r+1}\Delta X_r s_r\right]$$

(by Abel's transformation)

$= T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4} + T_{n,5} + T_{n,6}$, (say).

In order to complete the proof of the theorem by using Minokowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty} \alpha_n^{\gamma(\delta k+k-1)}\left|T_{n,i}\right|^k < \infty \quad for\ i = 1,2,3,4,5,6.$$

Now, we have

$$\sum_{n=2}^{m+1} \alpha_n^{\gamma(\delta k+k-1)}\left|T_{n,1}\right|^k = \sum_{n=2}^{m+1} \alpha_n^{\gamma(\delta k+k-1)}\left|\frac{1}{P_n P_{n-1}}\sum_{r=1}^{n-1} p_{n-r}P_{n-1}\lambda_r X_r s_r\right|^k$$
$$\le \sum_{n=2}^{m+1} \alpha_n^{\gamma(\delta k+k-1)}\frac{1}{P_n}\left(\sum_{r=1}^{n-1} p_{n-r}\left|\lambda_r\right|^k \left|s_r\right|^k X_r^k\right)\left(\frac{1}{P_n}\sum_{r=1}^{n-1} p_{n-r}\right)^{k-1}$$

$$= O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k \sum_{n=r+1}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left(\frac{p_{n-r}}{P_n}\right)$$

$$= O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k \frac{p_r}{P_r} \quad \text{, by(3.3)}$$

$$= O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^{k-1} \frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n}$$

$$= O(1)\sum_{r=1}^{m} X_r^{k-1}\frac{|\lambda_r|^k}{r}$$

$= O(1)$ $\quad as\ m \to \infty$,by (3.4).

Next,

$$\sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}|T_{n,2}|^k = \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}p_{n-r-1}P_n\lambda_r X_r s_r\right|^k$$

$$\leq \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\frac{1}{P_{n-1}}\left(\sum_{r=1}^{n-1}p_{n-r-1}|\lambda_r|^k|s_r|^k X_r^k\right)\left(\frac{1}{P_{n-1}}\sum_{r=1}^{n-1}p_{n-r-1}\right)^{k-1}$$

$$= O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k \sum_{n=r+1}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left(\frac{p_{n-r-1}}{P_{n-1}}\right)$$

$$= O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k \frac{p_r}{P_r} \quad \text{, by(3.3)}$$

$$= O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^{k-1} \frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n}$$

$$= O(1)\sum_{r=1}^{m} X_r^{k-1}\frac{|\lambda_r|^k}{r}$$

$= O(1)$ $\quad as\ m \to \infty$, by (3.4).

Further,

$$\sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}|T_{n,3}|^k = \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-1}P_{n-1}\,\Delta\lambda_r X_r s_r\right|^k$$

$$\leq \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\frac{1}{P_n}\left(\sum_{r=1}^{n-1}P_{n-r-1}|\Delta\lambda_r|^k|s_r|^k X_r^k\right)\left(\frac{1}{P_n}\sum_{r=1}^{n-1}P_{n-r-1}|\Delta\lambda_r|\right)^{k-1}$$

$$= O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^k \sum_{n=r+1}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left(\frac{P_{n-r-1}}{P_n}\right)$$

$$\left(Since\ \frac{1}{P_n}\sum_{r=1}^{n-1}P_{n-r-1}|\Delta\lambda_r| \leq \sum_{r=1}^{n-1}|\Delta\lambda_r| = O(1)\right)$$

$$= O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^k \frac{p_r}{P_r} \quad \text{, by(3.3)}$$

$$= O(1)\sum_{r=1}^{m}\left|\Delta\lambda_r\right|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n}$$

$$= O(1)\sum_{r=1}^{m} X_r^{k-1}\frac{\left|\Delta\lambda_r\right|^k}{r}$$

$= O(1) \quad as\ m \to \infty$, by (3.5).

Now,

$$\sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|T_{n,4}\right|^k = \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-2}P_n\,\Delta\lambda_r X_r s_r\right|^k$$

$$\leq \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\frac{1}{P_{n-1}}\left(\sum_{r=1}^{n-1}P_{n-r-2}\left|\Delta\lambda_r\right|^k\left|s_r\right|^k X_r^k\right)\left(\frac{1}{P_{n-1}}\sum_{r=1}^{n-1}P_{n-r-2}\left|\Delta\lambda_r\right|\right)^{k-1}$$

$$= O(1)\sum_{r=1}^{m}\left|\Delta\lambda_r\right|^k X_r^k\sum_{n=r+1}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left(\frac{P_{n-r-2}}{P_{n-1}}\right) \text{ (as above)}$$

$$= O(1)\sum_{r=1}^{m}\left|\Delta\lambda_r\right|^k X_r^k\frac{p_r}{P_r}, \text{ by (3.3)}$$

$$= O(1)\sum_{r=1}^{m}\left|\Delta\lambda_r\right|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n}$$

$$= O(1)\sum_{r=1}^{m} X_r^{k-1}\frac{\left|\Delta\lambda_r\right|^k}{r}$$

$= O(1) \quad as\ m \to \infty$, by (3.5).

Again

$$\sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|T_{n,5}\right|^k = \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-1}P_{n-1}\lambda_{r+1}\Delta X_r s_r\right|^k$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{P_{n-r-1}}{P_n}\lambda_{r+1}\Delta X_r s_r\right|^k$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\lambda_{r+1}\Delta X_r s_r\right|^k \quad \text{by (3.2)}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\lambda_{r+1}s_r\frac{1}{r}\right|^k \quad \text{by (3.1)}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\lambda_{r+1}s_r X_r\frac{p_r}{P_r}\right|^k, \text{ as } X_n = \frac{P_n}{np_n}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\left|\lambda_{r+1}\right|^k\left|s_r\right|^k X_r^k\right\}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\right\}^{k-1}$$

$$= O(1)\sum_{r=1}^{m}\left|\lambda_{r+1}\right|^{k} X_r^k \sum_{n=r+1}^{m+1} \alpha_n^{\gamma(\delta k+k-1)}\left(\frac{p_{n-r-1}}{P_{n-1}}\right)$$

$$= O(1)\sum_{r=1}^{m}\left|\lambda_{r+1}\right|^{k} X_r^{k-1} \frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{ as } X_n = \frac{P_n}{np_n} \text{ and by (3.3)}$$

$$= O(1)\sum_{r=1}^{m}\frac{\left|\lambda_{r+1}\right|^{k}}{r} X_r^{k-1},$$

$$= O(1) \quad as\ m \to \infty, \text{ by (3.4).}$$

Finally,

$$\sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|T_{n,6}\right|^{k} = \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-2}P_n\lambda_{r+1}\Delta X_r s_r\right|^{k}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{P_{n-r-2}}{P_{n-1}}\lambda_{r+1}\Delta X_r s_r\right|^{k}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\frac{P_r}{p_r}\lambda_{r+1}\Delta X_r s_r\right|^{k} \text{, by (3.2)}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\frac{P_r}{p_r}\lambda_{r+1}s_r\frac{1}{r}\right|^{k} \text{, by (3.1)}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\frac{P_r}{p_r}\lambda_{r+1}s_r X_r\frac{p_r}{P_r}\right|^{k} \text{, as } X_n = \frac{P_n}{np_n}$$

$$= \sum_{n=2}^{m+1}\alpha_n^{\gamma(\delta k+k-1)}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\left|\lambda_{r+1}\right|^{k}\left|s_r\right|^{k}X_r^k\right\}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\right\}^{k-1}$$

$$= O(1)\sum_{r=1}^{m}\left|\lambda_{r+1}\right|^{k} X_r^k \sum_{n=r+1}^{m+1} \alpha_n^{\gamma(\delta k+k-1)}\left(\frac{p_{n-r-2}}{P_{n-2}}\right)$$

$$= O(1)\sum_{r=1}^{m}\left|\lambda_{r+1}\right|^{k} X_r^{k-1} \frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{ as } X_n = \frac{P_n}{np_n} \text{ and by(3.3)}$$

$$= O(1)\sum_{r=1}^{m}\frac{\left|\lambda_{r+1}\right|^{k}}{r} X_r^{k-1},$$

$$= O(1) \quad as\ m \to \infty, \text{ by (3.4).}$$

This completes the proof of the Lemma.

5. PROOF OF THE THEOREM:

Since the behavior of the Fourier series, as far as convergence is concerned, for a particular value of x depends on the behavior of the function in the immediate neighborhood of this point only, the truth of the theorem is necessarily the consequence of the Lemma.

6. CONCLUSION:

Putting δ = 0 and $\alpha = \dfrac{P_n}{p_n}$ with δ=0, the result of Padhy et al.[2] and the result of H.Bor [1] can be achieved respectively from the result established in the present chapter under a few varying condition. Further there is a reach scope to work in this area for different indexed summability methods with additional parameter.

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