Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 4 (2016), pp. 3253-3273

http://www.ripublication.com/gjpam.htm

Paramters Estimation of Compound Rayleigh Distribution under an Adaptive Type-II Progressively Hybrid Censored Data for Constant Partially Accelerated Life Tests

G. A. Abd-Elmougod
Department of Mathematics,
Faculty of Applied Medical Sciences,
Turabah Taif University, KSA.

Emad E. Mahmoud
Department of Mathematics,
Faculty of Sciences,
Taif University, Taif, KSA.
Department of Mathematics,
Faculty of Sciences,
Sohag University, Sohag, Eygpt.

Abstract

In this paper, we considering two parameter compound Rayleigh distribution [CRD] with constant partially accelerated life tests under an adaptive Type II progressive hybrid censoring samples. The likelihood equations of the involved parameters are reduced to one non-linear equation which it is solved numerically to obtain the maximum likelihood estimates [MLEs] of the parameters. The approximate confidence intervals [CIs] and two bootstrap confidence intervals are also proposed. Bayesian point estimation and credible intervals by using MCMC method for the parameters are presented. The obtaining results discussed through analysis of simulated data set. Finally to investigate the precision and compare the performance of different corresponding confidence intervals considered, we presented Monte Carlo simulation study.

AMS subject classification:
Keywords: Constant partially accelerated life tests, Compound rayleigh distribution, Adaptive Type II progressive hybrid censoring data, Maximum likelihood estimation, Bootstrap confidence intervals, Bayesian estimation, MCMC.

1. Introduction

In manufacturing industries the accelerated life tests [ALTs] are presented to get more failure data in reducing test time which it is necessary to present inferences in use condition. Different type of ALTs presented in Nelson [1], firstly one is constant stress ALT, in which the stress saved at a constant level through testing experiment more details in [2–4]. Secondly is progressive stress ALT, stress continuously increasing in time see [5–7]. Finaly one is the step stress ALT, in which stress changes through a given interval of time or specified number of failures see [8–9]. The constant partially ALT applied in this paper items tested at both use and accelerated condition simultaneously see recently Seunggeun H. and Lee [10] and Tahani and Soliman [11].

CRD is one of models which is useful in different areas of statistics, this model under use condition have he probability density function [pdf] given by

$$f_1(x) = 2\alpha\beta^{\alpha}x(\beta + x^2)^{-(\alpha+1)},\ x > 0,\ \alpha, \beta > 0, \tag{1}$$

and the cumulative distribution function [cdf] given by

$$F_1(x) = 1 - \beta^{\alpha}(\beta + x^2)^{-\alpha}. \tag{2}$$

Where β and α defined as scale and shape parameters, respectively. The reliability and hazerd rate functions of the CRD, respectively, given by

$$S_1(t) = \beta^{\alpha}(\beta + t^2)^{-\alpha}, \tag{3}$$

and

$$h_1(t) = \frac{2\alpha t}{(\beta + t^2)}. \tag{4}$$

CRD is contained in three parameter Burr Type XII distribution as a special case. Application of randomly censored data as goodness of fit of the CRD using a medical data set Ghitany [12]. Generalization of the CRD Bekker et al. [13].

The censoring schemes which are most common in life testing experiments are called Type I and Type II censoring, firstly one, the experiment terminate at a prefixed time τ and the secondly the experiment terminate at a prefixed number r. These types of censoring don't allow to remove units of experiment at time other than the end point of the experiment. The more general censoring schemes of Type I or II called progressive Type II censoring Balakrishnan and Aggarwala [14]. The Type I censoring combined with Type II progressive censoring to introduced Type II progressive hybrid censoring scheme see Kundu and Joarder [15], in which a life testing experiment with progressive Type-II right censoring scheme $\mathbf{R} = (R_1, R_2, \ldots, R_m)$ is terminated at a prefixed time τ. The Type-II progressive hybrid censoring, is more similar to Type-I censoring, because the sample size is randomly and it can be a very small number even though equal zero. The statistical inference procedures may be request more data to be efficiency, Ng et al. [16] suggested censoring scheme which can be saving both the total test time and the cost induced by failure of the units and increase the efficiency of statistical analysis called an adaptive Type-II progressive hybrid censoring described as follows.

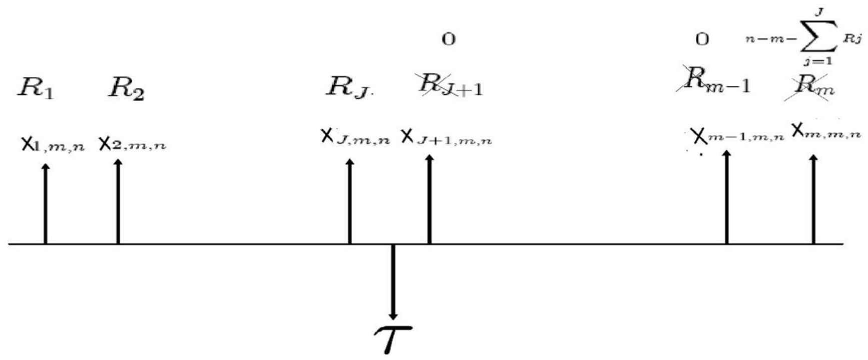

Figure 1: Description scheme of an adaptive Type II progressive hyperd censoring.

Let n units put on a life testing experiment and $T_1,\ T_2, \ldots, T_n$ are corresponding independent and identically distributed lifetimes with pdf $f(t)$ and cdf $F(t)$. An integer $m < n$ and censoring scheme $\mathbf{R} = (R_1, R_2, \ldots,\ R_m)$ satsfies $n = m + \sum_{i=1}^{m} R_i$, is specified at the prior the experiment. During the experiment, at any i-th failure, R_i items are randomly removed from the test. Ideal total test time τ, also is specified at the prior the experiment but the experiment allow to run over time τ. If $T_{m:m:n} < \tau$ the experiment stop at the time $T_{m:m:n}$. Other case the experimental time passes time τ but the number of observed failures has not reached m. Supposed number J is observed before time τ, i.e. $T_{J:m:n} < \tau < T_{J+1:m:n},\quad J = 1, 2, \ldots, m$, then after the experiment passed time τ, we set $R_{J+1} = \cdots = R_{m-1} = 0$ and $R_m = n - m - \sum_{i=1}^{J} R_i$ see Ng et al. [15]. Fig.1 discrebe schematic representation of this situation

The value of τ introduce a major role in the determination of $\mathbf{R}$ to reduce test time and a more chance to observe extreme failures. If $\tau \to \infty$, we have a progressive Type II censoring with censoring scheme $\mathbf{R} = (R_1,\ R_2, \ldots, R_m)$. If $\tau = 0$, we have Type II censoring scheme.

For given integer J, the likelihood function is given by

$$f(t_{1;m,n}, t_{2;m,n}, \ldots, t_{m;m,n}) = C \prod_{i=1}^{m} f(t_{i;m,n})[1 - F(t_{i;m,n})]^{\delta_i R_i}, \tag{5}$$

$$0 < t_{1;m,n} < t_{2;m,n} < \cdots < t_{m;m,n} < \infty,$$

where

$$C = \prod_{i=1}^{m} \left[n - i + 1 - \sum_{i=1}^{\min\{i-1,J\}} R_i \right], \tag{6}$$

and

$$\delta_i = \begin{cases} 1 & \text{if } i \leq J \\ 0 & \text{if } J < i \leq m-1 \\ \dfrac{n-m-\sum_{j=1}^{J} Rj}{R_m} & \text{if } i = m \text{ and J} < m. \end{cases} \tag{7}$$

In this paper, in Section 2, the model description and some basic assumptions. Section 3 the derivation of the ML estimators of the parameters of CRD as well as the approximate confidence intervals [CIs]. In Section 4, the two parametric bootstrap CIs are derived. In Section 5, the Bayesian approach is applied with the help of MCMC method. Data analysis is provided in Section 6. In Section 7 Monte Carlo results are presented. Section 8, is discused to the concluding remarks.

2. Model Description and Basic Assumptions

An adaptive Type II progressive hybrid censoring scheme in constant partially ALTs described as follows. Let n_1 be random units chosen from n test units and tested in use condition. The remaining $n_2 = n - n_1$ units are tested in accelerated condition. An adaptive Type II progressive hybrid censoring is described as follows. At the first failure $T_{j1;m_j,n_j}$, R_{j1} units are randomly removed from the number $n_j - 1$ surviving units. At the second failure $T_{j2;m_j,n_j}$, R_{j2} units from $n_j - 2 - R_{j1}$ units are randomly removed. Supposed number J_j is observed before time τ, i.e.$T_{J_j:m:n} < \tau < T_{J_j+1:m:n}$, $J_j = 1, 2, \ldots, m_j$, then after the experiment passed time τ, we set $R_{J_j+1} = \cdots = R_{m_j-1} = 0$ and set $R_{m_j} = n_j - m_j - \sum_{i=1}^{J_j} R_{ji}$. The test continues until the m_j-th failure $T^{Rj}_{jm_j;m_j,n_j}$ at this time, all remaining units are removed for $j = 1, 2$. In this study each of R_{ji}, τ and $m_j < n_j$ are fixed prior. If the failure unites of the n_j unites are from a continuous population with $F_j(x)$ and $f_j(x)$, the joint likelihood function given in (5) for $T_{j1;m_j,n_j}$, $T_{j2;m_j,n_j}$, ..., $T_{jm_j;m_j,n_j}$ and $j = 1, 2$ is given by

$$L(\alpha, \beta, \lambda|\underline{t}) = \prod_{j=1}^{2} C_j \left\{ \prod_{i=1}^{m_j} f_j(t_{ji;m_j,n_j}) \left(S_j(t_{ji;m_j,n_j})\right)^{\delta_{ji} R_{ji}} \right\}, \tag{8}$$

where

$$C_j = \prod_{i=1}^{m_j} \left[n_j - i + 1 - \sum_{i=1}^{\min\{i-1, J_j\}} R_{ji} \right]. \tag{9}$$

It is clear from (8) that an adaptive Type-II progressive hybrid censoring scheme under constant partially ALTs containing the following schemes:

(1) Type-II censored scheme when $\tau = 0$.

(2) Type-II progressive censoring when $\tau = \infty$.

When the lifetime of units follows a CRD, given by (1-4). The hazard rate function of units tested under accelerated condition is given by $h_2(t) = \lambda h_1(t)$, λ is called an acceleration factor satisfying $\lambda > 0$. Therefore the pdf, cdf, $S_2(t)$ and $h_2(t)$ under accelerated condition are given, respectively, by

$$h_2(t) = \lambda \frac{2\alpha t}{(\beta + t^2)}, \tag{10}$$

$$S_2(t) = \exp\left(-\int_0^t h_2(z)\right) dz = \beta^{\lambda\alpha}(\beta + t^2)^{-\lambda\alpha}, \tag{11}$$

$$F_2(t) = 1 - \beta^{\lambda\alpha}(\beta + t^2)^{-\lambda\alpha}, \tag{12}$$

and

$$f_2(t) = \alpha\lambda\beta^{\lambda\alpha} t (\beta + t^2)^{-(\lambda\alpha+1)}. \tag{13}$$

3. Maximum Likelihood Estimation

3.1. MLEs

Let, $T_{j1;m_j,n_j}, T_{j2;m_j,n_j}, \ldots, T_{jm_j;m_j,n_j}$, $j = 1, 2$, present two an adaptive Type II progressively hybrid censored data from two populations whose pdfs and cdfs given in (1), (2) and (12), (13), with $\mathbf{R}_j = (R_{j1}, R_{j2}, \ldots, R_{j1})$ the likelihood function $L(\alpha, \beta, \lambda|\underline{t})$

$$\begin{aligned} L(\alpha, \beta, \lambda|\underline{t}) \;=\; & C\alpha^{m_1+m_2}\lambda^{m_2} \left(\prod_{i=1}^{m1} t_{1i}\right)\left(\prod_{i=1}^{m2} t_{2i}\right) \\ & \times \exp\left\{-\alpha \sum_{i=1}^{m_1} (\delta_{1i} R_{1i} + 1) \log\left[1 + \frac{t_{1i}^2}{\beta}\right]\right. \\ & - \sum_{i=1}^{m_1} \log\left[\beta + t_{1i}^2\right] - \alpha\lambda \sum_{i=1}^{m_2} (\delta_{2i} R_{2i} + 1) \log\left[1 + \frac{t_{2i}^2}{\beta}\right] \\ & \left. - \sum_{i=1}^{m_2} \log\left[\beta + t_{2i}^2\right]\right\}, \end{aligned} \tag{14}$$

where $C = 2^{m_1+m_2}C_1C_2$. The log-likelihood function $\ell(\alpha, \beta, \lambda|\underline{t}) = \log L(\alpha, \beta, \lambda|\underline{t})$ without constant values is then given by

$$\begin{aligned}\ell(\alpha, \beta, \lambda|\underline{t}) &= (m_1+m_2)\log\alpha + m_2\log\lambda \\ &\quad -\alpha\sum_{i=1}^{m_1}(\delta_{1i}R_{1i}+1)\log\left[1+\frac{t_{1i}^2}{\beta}\right] \\ &\quad -\sum_{i=1}^{m_1}\log\left[\beta+t_{1i}^2\right] - \alpha\lambda\sum_{i=1}^{m_2}(\delta_{2i}R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right] \\ &\quad -\sum_{i=1}^{m_2}\log\left[\beta+t_{2i}^2\right]. \end{aligned} \tag{15}$$

The likelihood equations obtained by calculating the first partial derivatives of (15) with respect to α, β and λ and equating each to zero, as follows

$$\begin{aligned}\frac{\partial\ell(\alpha, \beta, \lambda|\underline{t})}{\partial\alpha} &= \frac{m_1+m_2}{\alpha} - \sum_{i=1}^{m_1}(\delta_{1i}R_{1i}+1)\log\left[1+\frac{t_{1i}^2}{\beta}\right] \\ &\quad -\lambda\sum_{i=1}^{m_2}(\delta_{2i}R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right] = 0, \end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial\ell(\alpha, \beta, \lambda|\underline{t})}{\partial\beta} &= \frac{\alpha}{\beta}\sum_{i=1}^{m_1}\frac{(\delta_{1i}R_{1i}+1)\,t_{1i}^2}{\beta+t_{1i}^2} \\ &\quad -\sum_{i=1}^{m_1}\frac{1}{\beta+t_{1i}^2} + \frac{\alpha\lambda}{\beta}\sum_{i=1}^{m2}\frac{(\delta_{2i}R_{2i}+1)\,t_{2i}^2}{\beta+t_{2i}^2} - \sum_{i=1}^{m_2}\frac{1}{\beta+t_{2i}^2} = 0, \end{aligned} \tag{17}$$

and

$$\frac{\partial\ell(\alpha, \beta, \lambda|\underline{t})}{\partial\lambda} = \frac{m_2}{\lambda} - \alpha\sum_{i=1}^{m_2}(\delta_{2i}R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right] = 0. \tag{18}$$

From (16) and (18) we obtain the ML estimates of α and λ as

$$\hat{\alpha}\,(\beta) = \frac{m_1}{D_1}, \tag{19}$$

and

$$\hat{\lambda}\,(\beta) = \frac{m_2D_1}{m_1D_2}, \tag{20}$$

where

$$D_1 = \sum_{i=1}^{m_1}(\delta_{1i}R_{1i}+1)\log\left[1+\frac{t_{1i}^2}{\beta}\right], \tag{21}$$

and

$$D_2 = \sum_{i=1}^{m_2} (\delta_{2i} R_{2i} + 1) \log \left[1 + \frac{t_{2i}^2}{\beta} \right]. \tag{22}$$

From (19) and (20) in (15) and (18) we obtain

$$\begin{aligned} f(\beta) &= (m_1 + m_2) \log \frac{m_1}{D_1} + m_2 \log \frac{m_2 D_1}{m_1 D_2} \\ &\quad - \sum_{i=1}^{m_1} \log \left[\beta + t_{1i}^2 \right] - \sum_{i=1}^{m_2} \log \left[\beta + t_{2i}^2 \right] - m_1 - m_2, \end{aligned} \tag{23}$$

and

$$\frac{m_1}{\beta D_1} \sum_{i=1}^{m_1} \frac{(\delta_{1i} R_{1i} + 1)\, t_{1i}^2}{\beta + t_{1i}^2} - \sum_{i=1}^{m_1} \frac{1}{\beta + t_{1i}^2} + \frac{m_2}{\beta D_2} \sum_{i=1}^{m2} \frac{(\delta_{2i} R_{2i} + 1)\, t_{2i}^2}{\beta + t_{2i}^2} - \sum_{i=1}^{m_2} \frac{1}{\beta + t_{2i}^2} = 0. \tag{24}$$

Thus, likelihoods equations (16–18) are reduced to one nonlinear equations (24) which can be solved numerically for β by using one iteration method such as quasi Newton Raphson, or fixed point solution hence the MLE, $\hat{\alpha}$ and $\hat{\beta}$, from (19) and (20).

3.2. Approximate interval estimation

The log-likelihood function given in (15), present

$$\frac{\partial^2 \ell(\alpha, \beta, \lambda | \underline{t})}{\partial \alpha^2} = -\frac{m_1 + m_2}{\alpha^2}, \tag{25}$$

$$\begin{aligned} \frac{\partial^2 \ell(\alpha, \beta, \lambda | \underline{t})}{\partial \beta^2} &= \frac{-\alpha}{\beta^2} \sum_{i=1}^{m_1} \frac{(\delta_{1i} R_{1i} + 1)\, t_{1i}^2}{\beta + t_{1i}^2} + \sum_{i=1}^{m_1} \frac{1}{\left[\beta + t_{1i}^2 \right]^2} \\ &\quad - \frac{\alpha\lambda}{\beta^2} \sum_{i=1}^{m2} \frac{(\delta_{2i} R_{2i} + 1)\, t_{2i}^2}{\beta + t_{2i}^2} \\ &\quad + \sum_{i=1}^{m_2} \frac{1}{\left[\beta + t_{2i}^2 \right]^2} - \frac{\alpha}{\beta} \sum_{i=1}^{m_1} \frac{(\delta_{1i} R_{1i} + 1)\, t_{1i}^2}{\left[\beta + t_{1i}^2 \right]^2} \\ &\quad - \frac{\alpha\lambda}{\beta} \sum_{i=1}^{m2} \frac{(\delta_{2i} R_{2i} + 1)\, t_{2i}^2}{\left[\beta + t_{2i}^2 \right]^2}, \end{aligned} \tag{26}$$

$$\frac{\partial^2 \ell(\alpha, \beta, \lambda | \underline{t})}{\partial \lambda^2} = -\frac{m_2}{\lambda^2}, \tag{27}$$

$$\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha\partial\beta} = \frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta\partial\alpha} = \frac{1}{\beta}\sum_{i=1}^{m_1}\frac{(\delta_{1i}R_{1i}+1)\,t_{1i}^2}{\beta+t_{1i}^2}$$
$$+\frac{\lambda}{\beta}\sum_{i=1}^{m2}\frac{(\delta_{2i}R_{2i}+1)\,t_{2i}^2}{\beta+t_{2i}^2} - \sum_{i=1}^{m_2}\frac{1}{\beta+t_{2i}^2}, \tag{28}$$

$$\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha\partial\lambda} = \sum_{i=1}^{m_2}(\delta_{2i}R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right], \tag{29}$$

and

$$\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta\partial\lambda} = \frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda\partial\beta} = \frac{\alpha}{\beta}\sum_{i=1}^{m2}\frac{(\delta_{2i}R_{2i}+1)\,t_{2i}^2}{\beta+t_{2i}^2}. \tag{30}$$

The Fisher information matrix $I\ (\alpha,\beta,\lambda)$, for the estimates ($\hat{\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$) Nelson [1], is given by the negative second partial derivatives of (15) with respect to (α, β and λ). In some cases , we can estimate $I^{-1}\ (\alpha,\beta,\lambda)$ by $I_0^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})$ where

$$I_0^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \begin{bmatrix} -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha 2} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha\partial\beta} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta\partial\alpha} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta 2} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta\partial\lambda} \\ -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda\partial\alpha} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda\partial\beta} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda 2} \end{bmatrix}^{-1}_{(\hat{\alpha},\hat{\beta},\hat{\lambda})}. \tag{31}$$

Hence $100(1-\gamma)\%$ approximate confidence intervals for α, β and λ are respectively given by

$$\hat{\alpha} \mp z_{\frac{\gamma}{2}}\sqrt{F_{11}},\ \hat{\beta} \mp z_{\frac{\gamma}{2}}\sqrt{F_{22}} \text{ and } \hat{\lambda} \mp z_{\frac{\gamma}{2}}\sqrt{F_{33}} \tag{32}$$

where F_{11}, F_{22} and F_{33} are the elements on the main diagonal of the covariance matrix $I^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})$ and $z_{\frac{\gamma}{2}}$ is the percentile of the standard normal distribution with any tail probability equal $\frac{\gamma}{2}$.

4. Bootstrap Confidence Intervals

In statistical inference the bootstrap method is commonly one used to estimate confidence intervals, bias and variance or calibrate hypothesis tests of estimators. Mor survey of the nonparametric and parametric bootstrap methods Davison and Hinkley [17], Efron and Tibshirani [18]. In this section, the two confidence intervals in contexts of the parametric bootstrap are discussed: (i) For percentile bootstrap method see Efron [19], (ii) For bootstrap-t method see Hall, [20]. The bootstrap algorithms for estimating the two confidence intervals are illustrated as follows.

1 For given the original an adaptive Type II progressively sample, $(t_{j1;m_j,n_j}, t_{j2;m_j,n_j}, \ldots, t_{jm_j;m_j,n_j})$, obtain $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\lambda}$, $j = 1, 2$.

2 For given n_j and m_j $(1 < m_j < n_j)$ and the same values of R_{ji}, $(i = 1, 2, \ldots, m_j)$ and $j = 1, 2$, m_1 and m_2 independent samples are generated from CRD, $\underline{t}^* = (t^*_{j1;m_j,n_j}, t^*_{j2;m_j,n_j}, \ldots, t^*_{jm_j;m_j,n_j})$ by using the algorithm presented in Ng et al. [16].

3 Based on $\underline{t}^*$ in 1 compute the bootstrap sample estimates of $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\lambda}$ say $\hat{\alpha}^*$, $\hat{\beta}^*$ and $\hat{\lambda}^*$.

4 Steps 2 and 3 are repeated N times, N may taken to be 1000.

5 The values $\hat{\alpha}^*$, $\hat{\beta}^*$ and $\hat{\lambda}^*$ are arranged in ascending order to get the bootstrap samples $(\hat{\varphi}_k^{*[1]}, \hat{\varphi}_k^{*[2]}, \ldots, \hat{\varphi}_k^{*[N]})$, $k = 1, 2, 3$ where $(\varphi_1^* = \alpha^*, \varphi_2^* = \beta^*, \varphi_3^* = \lambda^*)$.

Percentile bootstrap confidence interval:

Let distribution $G(z) = P(\hat{\varphi}_k^* \leqslant z)$ are cumulative distribution function of $\hat{\varphi}_k^*$. Define $\hat{\varphi}^*_{kboot} = G^{-1}(z)$ for each z. Hencethe approximate bootstrap $100(1-\gamma)\%$ confidence interval of $\hat{\varphi}_k^*$ given by

$$\left[\hat{\varphi}^*_{kboot}\left(\frac{\gamma}{2}\right), \hat{\varphi}^*_{kboot}\left(1-\frac{\gamma}{2}\right)\right]. \tag{33}$$

Bootstrap-*t* confidence interval

First, find the order statistics $\delta_k^{*[1]} < \delta_k^{*[2]} < \cdots < \delta_k^{*[N]}$, where

$$\delta_k^{*[j]} = \frac{\hat{\varphi}_k^{*[j]} - \hat{\varphi}_k}{\sqrt{\operatorname{var}\left(\hat{\varphi}_k^{*[j]}\right)}}, \; j = 1, 2, \ldots, N, \; k = 1, 2, 3, \tag{34}$$

where $\hat{\varphi}_1 = \hat{\alpha}$, $\hat{\varphi}_2 = \hat{\beta}$, $\hat{\varphi}_3 = \hat{\lambda}$.

Let $H(z) = P(\delta_k^* < z)$ be the cumulative distribution function of δ_k^*. For a given z, define

$$\hat{\varphi}_{k\text{boot}-t} = \hat{\varphi}_k + \sqrt{\operatorname{Var}(\hat{\varphi}_k)}H^{-1}(z). \tag{35}$$

The approximate $100(1-\gamma)\%$ confidence interval of $\hat{\varphi}_k$ is given by

$$\left(\hat{\varphi}_{k\text{boot}-t}\left(\frac{\gamma}{2}\right), \hat{\varphi}_{k\text{boot}-t}\left(1-\frac{\gamma}{2}\right)\right). \tag{36}$$

5. Bayes estimation of the model parameters

In several practical situations, the information of the parameters value are available in an independent manner Basu et al. [21]. Thus, assumed that the parameters are independent a priori and let the NIP for the acceleration factor λ is given by

$$\pi_1^*(\lambda) = \lambda^{-1},\ \lambda>0. \tag{37}$$

and for each parameter α and β be represented by independent gamma distributions presented respectively by

$$\pi_2^*(\alpha) = \frac{b^a}{\Gamma(a)}\alpha^{a-1}\exp\left(-b\alpha\right),\ \alpha>0 \text{ and } a, b>0 \tag{38}$$

and

$$\pi_3^*(\beta) = \frac{d^c}{\Gamma(c)}\alpha^{c-1}\exp\left(-d\beta\right),\ \beta>0 \text{ and } c, d>0. \tag{39}$$

Hence, the joint prior pdf of the three parameters can be expressed by

$$\pi^*(\alpha, \beta, \lambda) = \frac{b^a d^c}{\Gamma(a)\Gamma(c)}\alpha^{a-1}\beta^{c-1}\lambda^{-1}\exp\left(-b\alpha - d\beta\right),\ \alpha>0,\ \beta>0, \lambda>0. \tag{40}$$

From the joint prior pdf (40) and likelihood function (14) of parameters, the joint posterior pdf given the data, denoted by $\pi(\alpha, \beta, \lambda|\underline{t})$, presented as

$$\pi(\alpha, \beta, \lambda|\underline{t}) = \omega^{-1}L(\alpha, \beta, \lambda|\underline{t}) \times \pi^*(\alpha, \beta, \lambda)L(\alpha, \beta, \lambda|\underline{t}) \times \pi^*(\alpha, \beta, \lambda), \tag{41}$$

where $\omega = \int_0^\infty \int_0^\infty \int_0^\infty L(\alpha, \beta, \lambda|\underline{t}) \times \pi^*(\alpha, \beta, \lambda) d\alpha d\beta d\lambda$. Hence, the Bayes estimate of any function of α, β and λ say $\varphi(\alpha, \beta, \lambda)$, under squared error loss function (SEL) is

$$\begin{aligned}\hat{\varphi}(\alpha, \beta, \lambda) &= E_{\alpha,\beta,\lambda|\underline{t}}(\varphi(\alpha, \beta, \lambda)) \\ &= \omega^{-1}\int_0^\infty \int_0^\infty \int_0^\infty \varphi(\alpha, \beta, \lambda)L(\alpha, \beta, \lambda|\underline{t}) \times \pi^*(\alpha, \beta, \lambda) d\alpha d\beta d\lambda.\end{aligned} \tag{42}$$

In several cases, the ratio of two integrals given by (42) can not be obtained in closed form. Hence, different methods can be used the important one is MCMC method which selected to be used in this article then compute the Bayes estimator of any function $\varphi(\alpha, \beta, \lambda)$ under the SEL function.

MCMC Approach

The joint posterior pdf of $\alpha,\ \beta$ and λ can be written as

$$\begin{aligned}\pi(\alpha,\beta,\lambda|\underline{t}) \propto\ & \alpha^{m_1+m_2+a-1}\beta^{c-1}\\ & \times \exp\left(-b\alpha - \alpha\sum_{i=1}^{m_1}(\delta_{1i}R_{1i}+1)\log\left[1+\frac{t_{1i}^2}{\beta}\right] - \sum_{i=1}^{m_1}\log\left[\beta+t_{1i}^2\right]\right)\\ & \times \exp\left(-d\beta - \alpha\lambda\sum_{i=1}^{m_2}(\delta_{2i}R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right]\right.\\ & \left. - \sum_{i=1}^{m_2}\log\left[\beta+t_{2i}^2\right]\right)\lambda^{m_2-1}. \end{aligned} \tag{43}$$

The conditional posterior PDF's of $\alpha,\ \beta$ and λ are as follows

$$\pi_1(\alpha|\beta,\lambda,\underline{t}) \sim \text{Gamma}\left(m_1+m_2+a,\ b+\sum_{j=1}^{2}\sum_{i=1}^{m_j}\gamma_j(\delta_{ji}R_{ji}+1)\log(1+\frac{t_{ji}^2}{\beta})\right), \tag{44}$$

where

$$\gamma_j = \begin{cases} 1, & \text{if } j=1 \\ \lambda, & \text{if } j=2 \end{cases} \tag{45}$$

$$\pi_2^*(\lambda|\alpha,\beta,\underline{t}) \sim \text{Gamma}\left(m_2,\ \alpha\sum_{i=1}^{m_2}(R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right]\right), \tag{46}$$

and

$$\begin{aligned}\pi_3^*(\beta|\alpha,\lambda,\underline{t}) \propto\ & \beta^{c-1}\\ & \times \exp\left(-d\beta - \alpha\sum_{i=1}^{m_1}(\delta_{1i}R_{1i}+1)\log\left[1+\frac{t_{1i}^2}{\beta}\right] - \sum_{i=1}^{m_1}\log\left[\beta+t_{1i}^2\right]\right)\\ & \times \exp\left(-\alpha\lambda\sum_{i=1}^{m_2}(\delta_{2i}R_{2i}+1)\log\left[1+\frac{t_{2i}^2}{\beta}\right] - \sum_{i=1}^{m_2}\log\left[\beta+t_{2i}^2\right]\right). \end{aligned} \tag{47}$$

The plot of $\pi_3^*(\beta|\alpha,\lambda,\underline{t})$ is approximately similar to normal distribution. Hnce MH method Metropolis et al.[22] with normal proposal distribution used to generate random sample from $\pi_3^*(\beta|\alpha,\lambda,\underline{t})$.

Gibbs with MH algorithm given as follows

Step 1: Beginning with initial values as ($\beta^{(0)}=\hat{\beta}$ and $\lambda^{(0)}=\hat{\lambda}$) and set $I=1$.

Table 1: Simulated progressively censored samples with constant PALTs.

0.1038	0.2140	0.2364	0.2903	0.3150	0.3685	0.3838	0.4342	0.5273
0.5420	0.7577	1.0719	1.14888	1.16728	1.17777			
0.0965	0.1152	0.1544	0.1591	0.1890	0.2112	0.2220	0.2424	0.2457
0.3553	0.3586	0.3887	0.3977	0.4396	0.5134	0.5645	0.5806	0.6552
0.6729	0.7193	0.7382	0.7441	0.7697	0.7890	1.2187		

Step 2: By using Gamma distribution $\pi_1(\alpha|\beta^{(I-1)}, \lambda^{(I-1)}, \underline{t})$ given in (44) generate $\alpha^{(I)}$.

Step 3: Also from Gamma distribution $\pi_2(\lambda|\alpha^{(I)}, \beta^{(I-1)}, \underline{t})$ given in (45) generate $\lambda^{(I)}$.

Step 4: MH with $N(\beta^{(I-1)}, \sigma)$ proposal distribution generate $\beta^{(I)}$ using (47), Where σ is computed as $\sqrt{F_{22}}$ from variances-covariances matrix.

Step 5: Compute $\alpha^{(I)}$, $\beta^{(I)}$ and $\lambda^{(I)}$.

Step 6: Put $I = I + 1$ and Repeat steps $2 - 5$ N times.

Step 7: The MCMC point estimate of φ_l ($\varphi_1 = \alpha$, $\varphi_2 = \beta$ and $\varphi_3 = \lambda$) as

$$E(\varphi_l|\underline{t}) = \frac{1}{N-M}\sum_{i=M+1}^{N} \varphi_l^{(i)}, \tag{48}$$

where M is the number of generated parameters which deleted before the stationary distribution is achieved and called burn-in. and posterior variance of φ_l becomes

$$\hat{V}(\varphi_l|\underline{t}) = \frac{1}{N-M}\sum_{i=M+1}^{N} \left(\varphi_l^{(i)} - \hat{E}(\varphi_l|\underline{t})\right)^2. \tag{49}$$

Step 8: By ordering the values $\varphi_l^{(M+1)}, \varphi_l^{(M+2)}, \ldots, \varphi_l^{(N)}$ as $\varphi_{l(1)}, \varphi_{l(2)}, \ldots, \varphi_{l(N-M)}$. The $100(1-\gamma)\%$ symmetric credible interval is given by

$$\left(\varphi_{l(\frac{\gamma}{2}(N-M))}, \varphi_{l((1-\frac{\gamma}{2})(N-M))}\right). \tag{50}$$

6. Illustrative Example

Estimation procedures developed in this paper can be illustrated by consideration simulated samples of size ($m_1 = 15$ and $m_2 = 25$ of $n_1 = 25$, and $n_2 = 40$ with adaptive

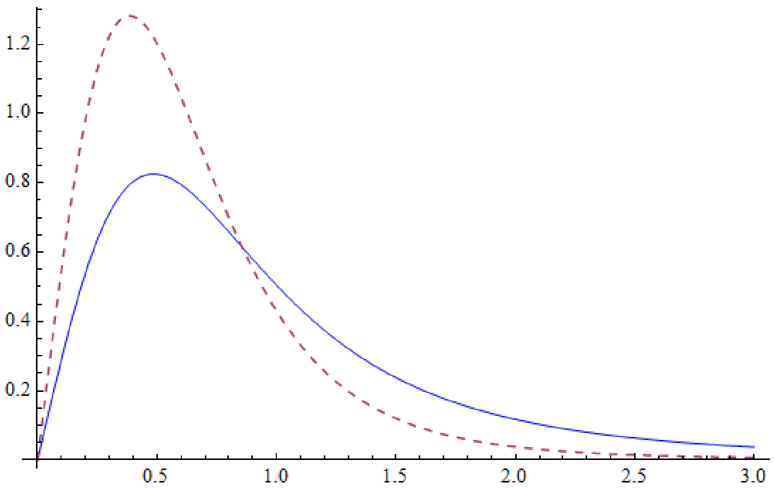

Figure 2: The plot of $f_1(t)$ with black line and $f_2(t)$ with dashed line.

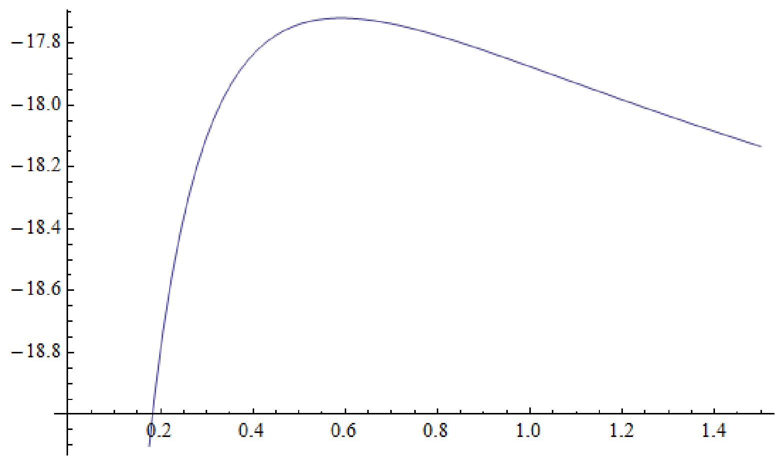

Figure 3: Profile log-likelihood function of β.

Table 2: MLEs, bootstrap and 95% confidence intervales

Pa.s	$(.)_{ML}$	$(.)_{Boot}$	95% AC	Len.	95% PBCI	Len.	95% BTCI	Len.
$\alpha = 1.2$	1.1173	1.3254	(-0.2422, 2.4768)	2.719	(0.2458, 4.0124)	3.7666	(0.4325, 3.1245)	2.6920
$\beta = 0.8$	0.5912	0.7012	$(-0.3968, 1.5790)$	1.9761	(0.1245, 2.2149)	2.0904	(0.1458, 1.9989)	1.8531
$\lambda = 2.0$	1.6660	1.8952	(0.5767, 2.7554)	2.1787	(0.3254, 3.5842)	3.2588	(0.8364, 3.0012)	2.1648

Table 3: MCMC, and 95% credible intervales.

Pa.s	$(.)_{\text{MCMC}}$	95% CI	Len.
$\alpha = 1.2$	1.43442	(0.5562, ,3.0607)	2.5045
$\beta = 0.8$	0.857068	(0.2351, ,2.1536)	1.9185
$\lambda = 2.0$	1.70629	(0.9053, ,2.9344)	2.0291

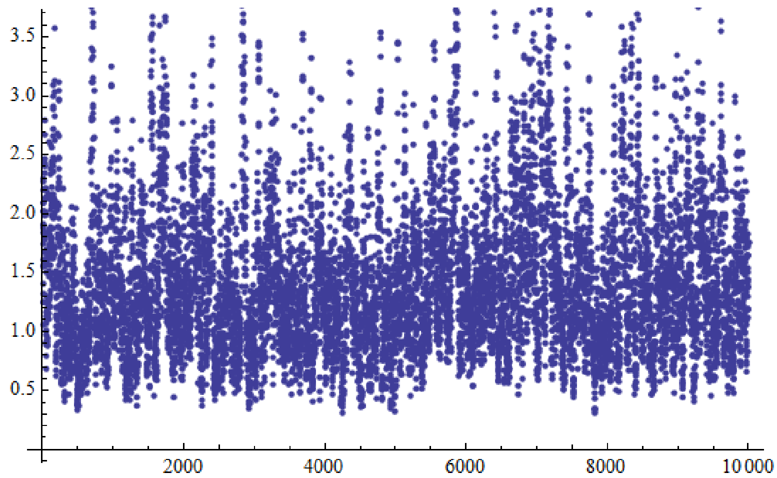

Figure 4: Simulation number of α generated by MCMC method.

parameter $\tau = 0.6$) using algorithm introduced by Ng et al. [16]. CRD with parameters vector $[\alpha, \beta, \lambda] = [1.2, 0.8, 2]$ and two progressive censoring scheme (CSs) $\mathbf{R}_1 = \{1, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2\}$ and $\mathbf{R}_2 = \{2, 0, 0, 0, 2, 0, 1, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2\}$. Figure 1 show the different cases of probability density functions with use conditions or accelerate conditions. The samples generated from this distribution are presented in Table 1. Figure 2 show the plot of profile log-likelihood function of β given in (23), it is a unimodal function. The MLE of β from (24) with the initial guess of β say 2.5 and the MLE of α and λ from (21) and (22). The point estimates as well as 95% approximate, Percentile bootstrap (PBCIs) and bootstrap-t (BTCIs) confidence intervals are presented in Table 2. In Bayesian context the hypered parameters of gamma distribution are selected to satisfies E(α or β) $\simeq \frac{a}{b}$ or $\frac{c}{d}$, respectively. In MCMC method, we run the Gibbs for 11, 000 times and discard the first $M = 1000$ values as 'burn-in'. The simulation number of α, β and λ generated by MCMC method and the corresponding histogram are shown in Fig. (3–9). The Bayes point estimation and corresponding 95% credible intervals of parameters α, β and λ are computed and presented in Table 3. From the results in Table 2 and 3, we observed that the BTCIs and credible intervals are narrower than the approximate and PBCIs confidence intervals.

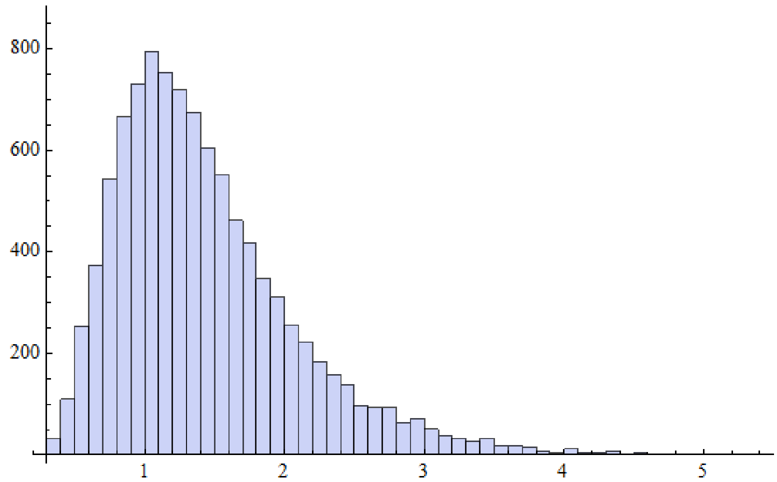

Figure 5: Histogram of α generated by MCMC method.

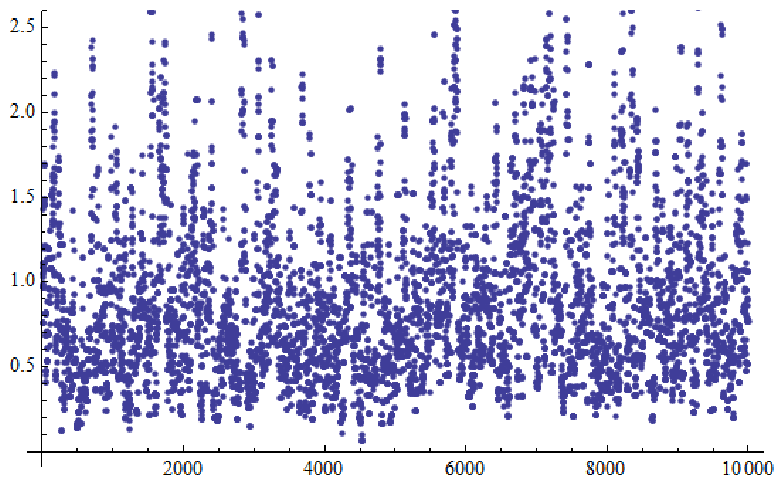

Figure 6: Simulation number of β generated by MCMC method.

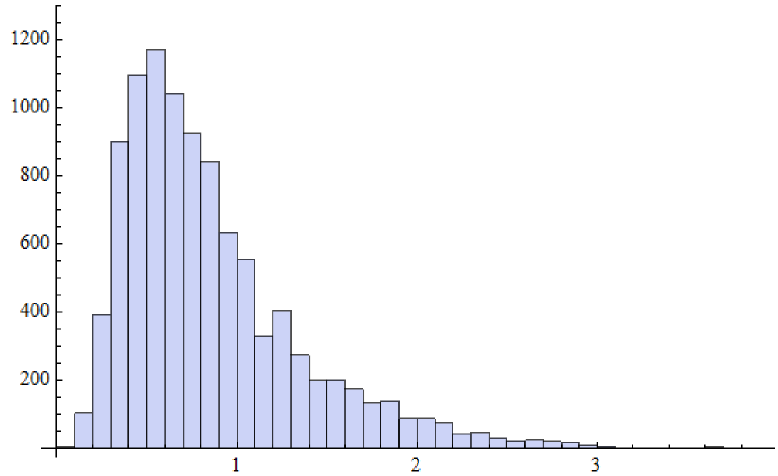

Figure 7: Histogram of β generated by MCMC method.

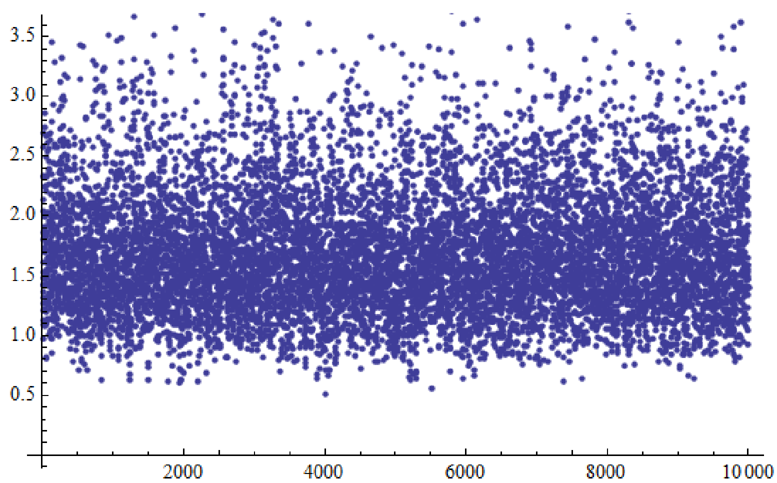

Figure 8: Simulation number of λ generated by MCMC method.

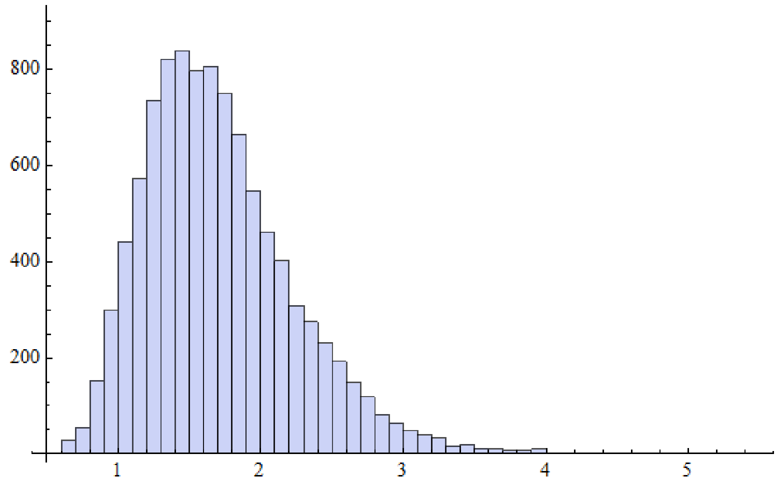

Figure 9: Histogram of λ generated by MCMC method.

Table 4: AVG and (MSEs) for the estimate of the parameters $[\alpha, \beta, \lambda] = [0.5, 0.1, 1.5]$.

τ	(n, m)	CS	MLE			Boot			MCMC(Prior 0)		
			α	β	λ	α	β	λ	α	β	λ
1.5	(40,20)	$(20,0^{19})$	0.5545	0.1021	1.5424	0.6141	0.1329	1.8547	0.5241	0.1197	1.5599
			0.1112	0.0534	0.4736	0.3235	0.1021	0.5840	0.1100	0.0516	0.4514
		(1^{20})	0.5754	0.1216	1.5828	0.6288	0.1489	1.6627	0.5351	0.1533	1.4994
			0.2001	0.0614	0.5398	0.4479	0.0906	0.6438	0.1801	0.0636	0.5130
		$(0^5,2^{10},0^5)$	0.5664	0.1441	1.5967	0.6388	0.1504	1.7714	0.5461	0.1621	1.5977
			0.2121	0.0707	0.5537	0.4531	0.1069	0.7741	0.2111	0.0605	0.5425
	(50,30)	$(20,0^{20})$	0.5200	0.1023	1.5365	0.5970	0.1332	1.8327	0.5205	0.1121	1.4998
			0.0888	0.0423	0.3890	0.1282	0.0874	0.3347	0.0859	0.0396	0.3379
		$(0^5,1^{20},0^5)$	0.5351	0.1124	1.5273	0.5899	0.1404	1.7234	0.5151	0.1119	1.5122
			0.1101	0.0570	0.4028	0.2220	0.0903	0.4500	0.1102	0.0511	0.3843
		$(0^{10},1^{20})$	0.6164	0.1168	1.5482	0.6062	0.1400	1.8204	0.5346	0.1143	1.4875
			0.1325	0.0567	0.4191	0.2335	0.0893	0.6194	0.1222	0.0537	0.4071
0.6	(40,20)	$(20,0^{19})$	0.5428	0.1124	1.5498	0.6000	0.1329	1.8445	0.5119	0.1087	1.5402
			0.1047	0.0527	0.4645	0.3002	0.1124	0.5745	0.0999	0.0499	0.4334
		(1^{20})	0.5684	0.1154	1.5742	0.6124	0.1500	1.6054	0.5248	0.1379	1.5008
			0.1987	0.0604	0.5390	0.4442	0.0802	0.6125	0.1024	0.0640	0.5223
		$(0^5,2^{10},0^5)$	0.5789	0.1333	1.5475	0.6038	0.1447	1.7001	0.5339	0.1225	1.5789
			0.2002	0.0685	0.5449	0.3987	0.1124	0.6698	0.2008	0.0598	0.5455
	(40,30)	$(20,0^{20})$	0.5356	0.0996	1.5881	0.6357	0.1297	1.4828	0.5277	0.1020	1.4934
			0.1100	0.0440	0.4534	0.2102	0.0707	0.6296	0.0985	0.0418	0.4377
		$(0^5,1^{20},0^5)$	0.5360	0.0990	1.5606	0.5399	0.1099	1.5464	0.5355	0.1154	1.5575
			0.1840	0.0447	0.5018	0.2990	0.0570	0.6999	0.1500	0.0430	0.5539
		$(0^{10},1^{20})$	0.5321	0.0950	1.5049	0.6355	0.1320	1.4581	0.5133	0.1051	1.4861
			0.2014	0.0548	0.5074	0.3554	0.0741	0.6381	0.2000	0.0528	0.5215

7. Simulation Studies

Theoretical results of estimation problem discussed in this paper have been illustrating throw simulation studies using (Mathematica ver. 8.0). The performance of different estimators of acceleration, shape and scale parameters assessment in terms of average [AVG] and mean square error [MSE], where

$$\text{AVG} = \overline{\hat{\omega}_k} = \frac{1}{s}\sum_{i=1}^{s} \hat{\omega}_k^{(i)}, \text{ where } \omega_1 = \alpha,\ \omega_2 = \beta,\ \omega_3 = \lambda, \text{ and } k = 1,\ 2,\ 3, \tag{51}$$

and

$$\text{MSE} = \frac{1}{s}\sum_{i=1}^{s} \left(\hat{\omega}_k^{(i)} - \omega_k\right)^2. \tag{52}$$

Different confidence intervals compared in terms of the average confidence lengths [AC] and coverage percentages [CP]. For each sample computed 95% confidence interval and checked whether it contain the true value and recorded the length of the interval. This procedure was repeated 1000 times. Coverage probability estimated as the number of confidence intervals that covered the true values divided by 1000 and sum of the lengths for all intervals divided by 1000 estimated as expected width of the confidence interval. Different censoring schemes [C.S] and different adaptive parameter τ used in this studies. In this studies, case considered case, [α, β, λ]= [1.5, 2.0, 2.0], ($n_1 = n_2 = n$) and ($m_1 = m_2 = m$) and non-informative prior which $a = b = c = d = 0.0001$[prior 0] the results presented in Table 1 and 2. **(ii)** [α, β, λ]= [0.5, 0.7, 0.5], $\left(n_2 = \frac{3}{2}n_1, n_1 = n\right)$ and $\left(m_2 = \frac{3}{2}m_1, m_1 = m\right)$ the results presented in Table 3 and 4.

8. Concluding Remarks

In situations which the experimenter need to reducing the cost and time associated with testing in life-testing and reliability studies or in cases which experimenter may be unable to obtain complete information on failure times for all experimental units. In this paper, we considered here a more general CS which can be balance between the total test time and the cost induced by failure of the units and increase the efficiency of statistical analysis called an adaptive Type-II progressive hybrid censoring.

A simulation study was conducted to examine and compare the performance of the proposed methods for different sample sizes, different censoring schemes. From the results in Tables 4 and, we observe the following.

1 Results for increasing values of an adaptive parameters τ is acceptable than decreasing values of τ.

2 The more accurate results through the MSEs and average confidence interval presented from scheme in which the censoring occurs after the first observed failure.

Table 5: The coverage percentages and average confidence interval of the parameters. $[\alpha, \beta, \lambda] = [0.5, 0.1, 1.5]$

τ	(n,m)	CS	MLE			PBCIs			BTCIs			MCMC(Prior 0)		
			α	β	λ	α	β	λ	α	β	λ	α	β	λ
0.15	(40,20)	$(20,0^{19})$	0.90	0.89	0.91	0.89	0.86	0.91	0.90	0.93	0.91	0.91	0.92	0.94
			0.750	0.251	1.64	0.984	0.350	2.820	0.700	0.250	1.561	0.63	0.250	1.551
		(1^{20})	0.90	0.88	0.90	0.87	0.89	0.96	0.91	0.95	0.95	0.92	0.94	0.96
			0.882	0.266	1.95	0.999	0.445	3.012	0.811	0.223	1.878	0.700	0.213	1.772
		$(0^5,2^{10},0^5)$	0.89	0.89	0.90	0.87	0.89	0.91	0.90	0.93	0.93	0.90	0.91	0.93
			0.992	0.357	2.292	1.001	0.457	3.5313	0.882	0.329	2.126	0.800	0.299	2.287
	(50,30)	$(20,0^{20})$	0.91	0.90	0.95	0.93	0.89	0.94	0.92	0.93	0.94	0.93	0.97	0.95
			0.700	0.202	1.404	0.880	0.333	1.533	0.6001	0.203	1.352	0.604	0.193	1.252
		$(0^5,1^{20},0^5)$	0.92	0.93	0.96	0.91	0.90	0.92	0.93	0.94	0.95	0.94	0.93	0.95
			0.772	0.213	1.518	0.872	0.344	1.668	0.702	0.177	1.474	0.602	0.187	1.474
		$(0^{10},1^{20})$	0.90	0.97	0.96	0.90	0.89	0.98	0.90	0.96	0.96	0.97	0.96	0.96
			0.882	0.224	1.605	0.997	0.2399	1.935	0.702	0.191	1.603	0.602	0.202	1.581
0.6	(40,20)	$(20,0^{19})$	0.91	0.91	0.92	0.90	0.90	0.90	0.92	0.92	0.92	0.93	0.96	0.93
			0.730	0.244	1.61	0.884	0.351	2.810	0.701	0.230	1.560	0.620	0.240	1.540
		(1^{20})	0.93	0.90	0.92	0.89	0.89	0.93	0.96	0.94	0.96	0.94	0.92	0.96
			0.827	0.259	1.92	0.979	0.439	2.911	0.800	0.201	1.870	0.701	0.210	1.752
		$(0^5,2^{10},0^5)$	0.90	0.91	0.97	0.89	0.91	0.90	0.93	0.95	0.91	0.92	0.95	0.94
			0.888	0.348	2.187	1.002	0.422	3.522	0.870	0.328	2.202	0.810	0.288	2.284
	(40,30)	$(20,0^{20})$	0.93	0.91	0.94	0.90	0.92	0.95	0.93	0.94	0.94	0.95	0.94	0.95
			0.600	0.194	2.494	0.688	0.199	2.888	0.500	0.201	1.456	0.559	0.201	2.456
		$(0^5,1^{20},0^5)$	0.92	0.90	0.95	0.93	0.91	0.92	0.96	0.97	0.95	0.95	0.96	0.96
			0.752	0.130	3.421	0.758	0.118	1.421	0.612	0.223	1.414	0.6100	0.213	1.413
		$(0^{10},1^{20})$	0.90	0.90	0.95	0.92	0.95	0.94	0.93	0.95	0.95	0.95	0.94	0.95
			0.792	0.136	1.199	0.892	0.188	1.199	0.698	0.260	1.051	0.6044	0.233	1.050

3 The MCMC credible intervals and BTCIs presented more suitable results than the approximate CIs and PTCIs since have small lengths than the lengths of latter, for different sample sizes, observed failures and schemes.

4 The efficiency of each estimates increasing for the increasing sample size and affected sample size.

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