Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 4 (2016), pp. 3285–3295

http://www.ripublication.com/gjpam.htm

Designing Basic Variables as Linguistic States with Multiple Channel Queueing Models and Supply Priorities In Uncertainty Environment

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Abstract

This paper develops a new contribution for constructing an architecture that describes the linguistic states as intervals with two basic variables; arrival rates and service rates, while increasing the supplying order of demands in the manufacturing production line under uncertainty environment. The basic idea for this paper considers classifying these basic values as linguistic values represented by three ranges for each value, in a multiple channel queueing model with two class of priority having ranges in each interval. Trapezoidal fuzzy numbers are considered as they are efficient to build these intervals. Adopting robust ranking index and α-cut confidence interval are suitable approaches for converting the fuzzy arrival and service values into crisp values. Numerical application is shown for the application of this new approach in evaluating the exact real waiting time of customers in the queue for each interval, which further contributes to evaluation of the whole system in uncertainty environment. Hence, an easy way for dealing with fuzzy numbers in decision-making problems is developed.

AMS subject classification:
Keywords: Linguistic states, multiple channel queueing system, two class priority, uncertainty environment.

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1. Introduction

Due to its diverse application areas, such as manufacturing/production, inventory control and logistic supply demands, queueing models (especially priority queuing models) have been extensively studied in the queuing literature due to the fact that priority queues arises with respect to desire of customerâŁ™s preference. Hence, priority queueing models are the most popular models used to solve and evaluate these types of systems [1–3]. Within the context of classical queueing theory, the inter arrival time and service times follow certain probability theories and concepts. However, in many real life applications the statistical information may be derived subjectively, that is, the arrival and service mode are more correctly described by linguistic terms such as fast, slow or moderate, with the use of probability distributions. Zadeh [4] refers to the linguistic terms bringing a merge between possibility and probability theories, and with this concept, fuzzy queues become more realistic than the commonly used crisp queues. Many researchers have adopted fuzzy queues and these include Kao et al [5] who proposed a general approach for queuing systems in a fuzzy environment based on Zadeh's extension principle. Also, other authors have studied queuing systems in non preemptive priority under two classes like Huang et al [6–8] and Palpandi [9] who studied another approach called robust ranking method with single channel priority under two classes non preemptive.

Most existing literature has focused on fuzzy queueing models with single channel and priorities two class or more. The work presented has described the linguistic terms as individual one with fuzzy sets numbers with arrival and service patterns described using linguistic quantifiers, with statements like the mean arrival rate is approximately 5 customers per minute, the mean service rate is approximately 12 customers per minute'. Although, to the best of our knowledge, fuzzy numbers, and more generally linguistic values, are approximate assessments, given by experts and accepted by decision-makers when obtaining more accurate values is impossible or unnecessary. This implies that these linguistic terms can be represented by one fuzzy set as a single linear membership function whether trapezoidal or triangular membership functions.

Another group of researchers [10–11] focused on representing fuzzy logic via controller interference under input and output membership functions by using rule base fuzzy numbers. This can be represented by three or more intervals such as low, medium or high in the priority queuing systems. To simplify the task of representing and handling fuzzy numbers, several authors have introduced real indices in order to capture the information contained in a fuzzy number [12]. However, no previous study has considered the uncertainty of the parameters by using more than one membership function with mathematical fuzzy sets and multiple channel queueing models having priority under two classes. The lack of research on this topic gave the motivation to design the linguistic values (states) represented as three sub ranges (Low, Medium & High) which are directly related to the basic variables, such as arrival and service rates by using mathematical fuzzy sub sets for each state. These ranges can be classified into three trapezoidal fuzzy numbers with overlapping amongst intervals. Also, we use one of the new multiple channel queueing and two class's priorities model to present these intervals and analyze the evaluation of this type of models via conventional performance measurements. The methodology also included robust ranking approach for converting fuzzy arrival and service rates into crisp values for each interval.

The remainder of this paper is organized as follows. Section 2 briefly describes the foundation of fuzzy set theory and Section 3 outlines a solution procedure to design and analyze the intervals with multiple channel queueing system. Section 4 explains the robust ranking method, while Section 5 provides a practical example to illustrate the applicability of the proposed design. Finally, discussion of results and conclusion with insights for future research are presented in Section 6 and 7 respectively.

2. Foundations of Fuzzy Set Theory

In this section we investigate some of the mathematical preliminaries of fuzzy set theory as:

Definition 2.1. (Fuzzy Set) A fuzzy set $\tilde{A}$ in a universe of discourse X is refers to the set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\,; x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0, 1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$. The fuzzifier transforms crisp data into suitable linguistic values, corresponding to fuzzy sets, so that these data become compatible with the fuzzy antecedent-consequent mechanism. Thus, for a crisp value x_0, we obtain a fuzzy set X via

$$X = \text{fuzzifier}\,(x_0) \tag{2.1}$$

Definition 2.2. (Fuzzy Number) A fuzzy number is an extension of a regular number in the sense that it does not refer to one single value but rather to connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. Thus a fuzzy number is a convex and normal fuzzy set.

If $\tilde{A}$ is a fuzzy number then $\tilde{A}$ is a fuzzy convex set and if $\mu_{\tilde{A}}\,(x_0) = 1$ then $\mu_{\tilde{A}}(x)$ is non-decreasing for $x \geq x_0$.

Definition 2.3. (α-cut of a Fuzzy Set) The α-cut set or confidence at level α of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in A greater than or equal to α i.e $A_\alpha = \{(x : \mu_{\tilde{A}}(x) \geq \alpha)\,, x \in X_\alpha \in [0, 1]\}$.

Definition 2.4. (Trapezoidal Fuzzy Number) A trapezoidal fuzzy number (TpFn) is denoted by $\tilde{A}(x)$, and it can be represented by $\tilde{A} = (a, b, c, d)$ where a, b, c and d are the points inside the interval [13]. The membership function $\mu_{\tilde{A}}(x)$ is defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \dfrac{(x-d)}{(c-d)} & c \leq x \leq d \\ 0 & o.w \end{cases} \tag{2.2}$$

For the trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$. The α-cut of $\tilde{A}$ is $[A_L(\alpha), A_U(\alpha)]$, $\alpha \in [0, 1]$, where $A_L(\alpha)$ represent the lower bound and, $A_U(\alpha)$ represents the upper bound, with $A_L(\alpha) = a + (b - a)\alpha$ and $A_U(\alpha) = d - (d - c)\alpha$.

3. Designing of Linguistic States with Multiple Channel System

We design and formulate the linguistic values (states) for each base variable such as arrival and service rates with multiple channel queueing models and two class disciplines of priority non preemptive. It can be defined as where a customer, once in service, will leave the facility only after the service is completed, regardless of the priority of newly arriving customers.

From previous knowledge and results in Adan and Resing [14]. We have the expected waiting time of customer in the crisp of priority with two classes $(M_1, M_2)/G/C/PR$ queuing system and stability state $\rho = \lambda_1 + \lambda_2/c\mu < 1$ as follows:

$$W_{q_1} = \frac{\prod w}{1 - \rho_1} \cdot \frac{E[R]}{c} \tag{3.3}$$

and

$$W_{q_2} = \frac{\prod w}{(1 - \rho)(1 - \rho_1)} \cdot \frac{E[R]}{c} \tag{3.4}$$

where

$$\prod w = \frac{(c\rho)^c}{c!}(1 - \rho) \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!(1 - \rho)} \right]^{-1}; \quad n = 0, 1, 2, \ldots \tag{3.5}$$

and

$$E[R] = \frac{E\left[B^2\right]}{2E[B]} \tag{3.6}$$

In this model $\prod w$ represents the probability of customerâŁ™s waiting time in the queue, and $E[R]$ represents the residual processing time needed to complete the service. By observation, it is clear that the service time of customer will be random variable. Hence, we can calculate the mean residual processing time via the first two moments of the processing time as a general distribution [15].

According to Little's formula, when we obtain one of the performance measures, the others performance measurements can be computed (see [16]) for each class in the multiple channel model. Generally, 3 to 7 fuzzy sets are normally used to describe each parameter and in this model we designed the linguistic terms as three fuzzy subsets, with each basic value represented into three trapezoidal fuzzy numbers [16-17]. Hence the membership functions interrupted, which are overlapping between them as a closed interval can be described as:

Figure 1 above explains the operation of overlapping between real numbers ranging from a to h being represented by three trapezoidal membership functions. Note that

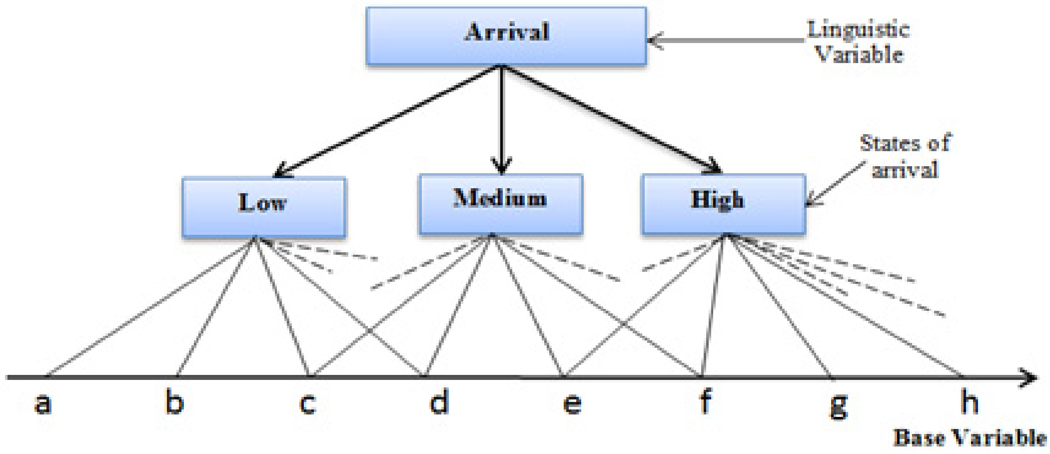

Figure 1: Overlapping Linguistic Base Variable with States.

the concept of fuzzification is closely related to knowledge because the membership functions used in equation (2.1) are the result of deep system knowledge, be it mathematical or experimental, and it can be put under α-cut closed interval with boundaries $[A_L(\alpha), A_U(\alpha)]$. Hence, the next step is to convert these fuzzy values into crisp values.

4. Robust Ranking Method

To find the characteristics of the system of interest in terms of crisp value, we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking method [18]. Robust ranking method which satisfies compensation, linearity and additive properties [19] is adopted. This provides results which are consistent with human perceptions. Given a convex fuzzy number $\tilde{a}$, the robust ranking method is defined by:

$$R(\tilde{a}) = \int_0^1 0.5 \left(\alpha_\alpha^L + \alpha_\alpha^U\right) d\alpha \tag{4.7}$$

where, $\tilde{a}$ is a convex fuzzy number and $\left(\alpha_\alpha^L, \alpha_\alpha^U\right)$ is the α-cut of $\tilde{a}$. This method is called robust ranking method $R(\tilde{a})$ .This approach possesses the properties of compensation as aforementioned as it gives the representative value of the fuzzy number $\tilde{a}$.

5. Numerical Application

Consider a factory with 5 servers following the queuing model $(FM_1, FM_2)/FM/5/PR$. The customers arrive in fuzzy arrival under Poisson process per hour represented by class one

$$\left(\tilde{\lambda}_1\right)_L = [11, 12, 13, 14],$$

$$\left(\tilde{\lambda}_1\right)_M = [13, 14, 16, 17],$$

$$\left(\tilde{\lambda}_1\right)_H = [16, 17, 18, 19],$$

and class two by

$$\left(\tilde{\lambda}_2\right)_L = [4, 5, 6, 7],$$

$$\left(\tilde{\lambda}_2\right)_M = [6, 7, 9, 10],$$

$$\left(\tilde{\lambda}_2\right)_H = [9, 10, 11, 12],$$

while the service times are also fuzzy variable with exponential distribution defined as $(\tilde{\mu})_L = [20, 21, 22, 23]$, $(\tilde{\mu})_M = [22, 23, 25, 26]$ and $(\tilde{\mu})_H = [25, 26, 27, 28]$ for all five servers. Note that these three intervals are classified as linguistic terms; low(L), medium(M) and high(H) respectively.

In this model, the mean residual time is exponential distribution given by $E[B] = \frac{1}{\mu}$ and $E\left[B^2\right] = \frac{2}{\mu^2}$. Then, $E[R] = E[B]$, where the number of channel for first class is three server, and second class are two server. The aim of this scenario considered is for the system manager to evaluate the system via performance measures of the system under uncertainty environment.

To start we need to apply α-cut level as $\left(\tilde{\lambda}_1\right)_L$, $\left(\tilde{\lambda}_1\right)_M$, $\left(\tilde{\lambda}_1\right)_H$, $(\tilde{\mu}_2)_L$, $(\tilde{\mu}_2)_M$ and $(\tilde{\mu}_2)_H$ by using trapezoidal fuzzy numbers to build lower bound and upper bound as:

$$\left(\tilde{\lambda}_{1\alpha}\right)_L = [11+\alpha, 14-\alpha], \left(\tilde{\lambda}_{1\alpha}\right)_M = [13+\alpha, 17-\alpha], \left(\tilde{\lambda}_{1\alpha}\right)_H = [16+\alpha, 19-\alpha] \tag{5.8}$$

$$\left(\tilde{\lambda}_{2\alpha}\right)_L = [4+\alpha, 7-\alpha], \left(\tilde{\lambda}_{2\alpha}\right)_M = [6+\alpha, 10-\alpha], \left(\tilde{\lambda}_{2\alpha}\right)_H = [9+\alpha, 12-\alpha] \tag{5.9}$$

$$(\tilde{\mu}_\alpha)_L = [20+\alpha, 23-\alpha], (\tilde{\mu}_\alpha)_M = [22+\alpha, 26-\alpha], (\tilde{\mu}_\alpha)_H = [25+\alpha, 28-\alpha] \tag{5.10}$$

The next step is to explain the designing and overlapping for these three linguistic values (states) as three fuzzy subsets (Low, Medium, and High). This is shown in Figures 2, 3 and 4.

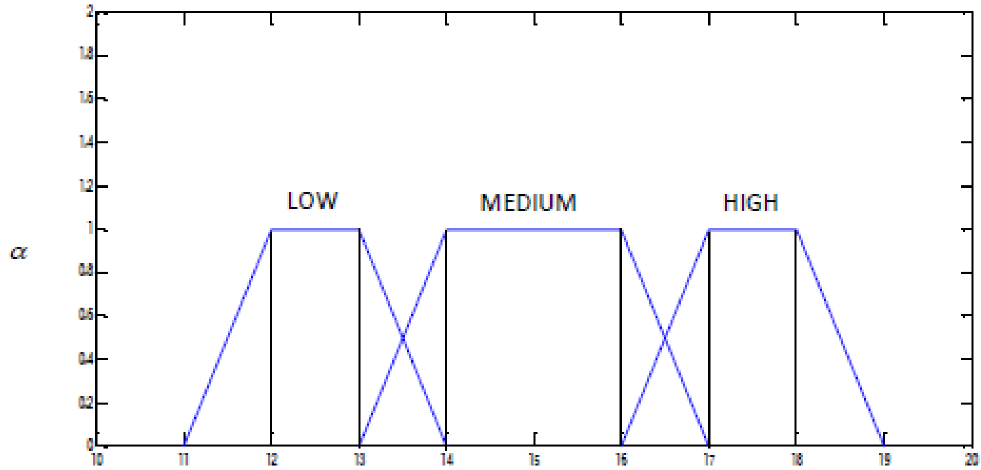

Figure 2: Membership functions of the arrival rates ($\tilde{\lambda}_1$) class one.

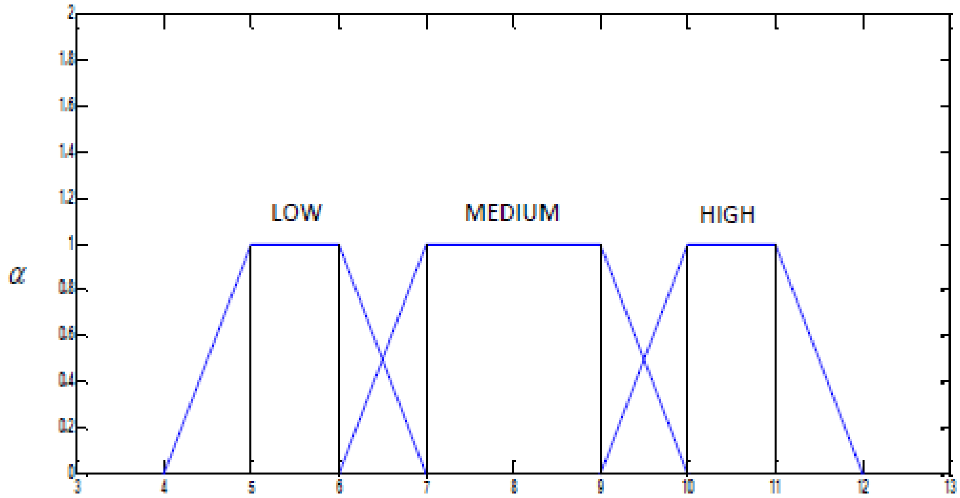

Figure 3: Membership functions of the arrival rates ($\tilde{\lambda}_2$) class two.

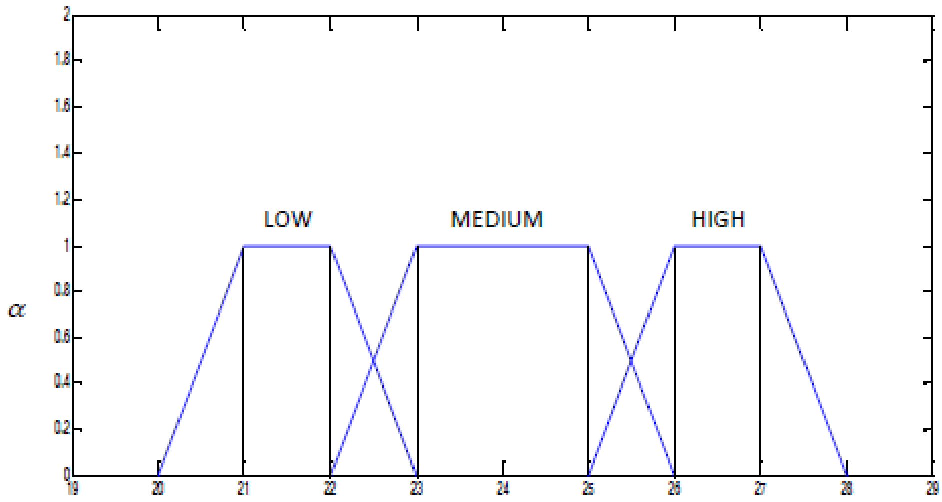

Figure 4: Membership functions of the arrival rates ($\tilde{\mu}$) class one & two.

To compute the exact real number, we rank all values according to equation (4.7) as:

Table 1: Ranking Values for Each Interval.

Ranking values	$R(\tilde{\lambda}_1)$	$R(\tilde{\lambda}_2)$	$R(\tilde{\mu})$
Low Interval	13	6	22
Medium Interval	15	8	24
High Interval	18	11	27

Hence, with reference to equations (3.3) and (3.4), the expected waiting time of customer in queue both class one and class two respectively can be obtained. Also, by adopting little's formula, other measures obtained as in table 2 is shown which represents the performance measures for each state in the system.

Table 2: The Levels of Performance Measures for Each Interval.

Low Interval $(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu})$							
W_{q1}	0.165	L_{q1}	2.156	L_{s1}	2.747	W_{s1}	0.211
W_{q2}	0.248	L_{q2}	1.493	L_{s2}	1.765	W_{s2}	0.294
Medium Interval $(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu})$							
W_{q1}	0.449		6. 742		7. 367	W_{s1}	0.491
W_{q2}	0.719		5. 753		6. 086	W_{s2}	0.760
High Interval $(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu})$							
W_{q1}	0.519	L_{q1}	8. 090		10. 015	W_{s1}	0.556
W_{q2}	0.904	L_{q2}	9. 952		10. 359	W_{s2}	0.941

6. Discussion and Analysis

The following highlights are important observations that need to be discussed:

- It is clear that the procedure for choosing three intervals under trapezoidal membership functions is more flexible for account varying precision of individual membership function.

- The boundaries of each interval which represent the ranges should not necessarily be standard shaped with other queuing systems.

- The fuzzification design for Medium interval is allowed to overlap the right and left interval states; low and high intervals under the same base values.

- The representation for this design leads to analysis of three intervals as a closed interval to obtain the evaluation performance measures of the system for each interval; low, medium and high respectively under uncertainty environment.

- By applying the α-cut approach, the range of fuzzy arrival and service rates at different possibility levels can be derived as closed interval [0, 1].

- Ranking for three parameters are located inside each interval which is a good indicator to determine the best value for all periods.

  The number of customers in the queue L_q, and in the system L_s are convergent for every class in the intervals.

7. Conclusion

In practical situations, the parameters in the queueing decision model may not be known in a precise manner due to uncontrollable factors. Consequently, the arrival rates for both classes, together with the service rates become fuzzy. In this paper, we have discussed two techniques for arrival and service rates under uncertainty environment. Firstly we have designed the basic values which are called linguistic variables as three intervals; this leads to analyzing each interval separately. This is very important to evaluate the system according to three intervals as the decision maker can evaluate their service system by three ranges rather than one range. This further implies that the efficiency of the server can be determined for each period with indication of the peak congestion period in the system. The multiple channel queueing system considered in this article has had its conventional fuzzy values converted into crisp values with help of ranking index. This obtained exact real number both two class of discipline priority. However in future, it would be interesting to analyze and design these intervals for more than three states with other types of fuzzy queueing systems.

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