

Composite Mendeleev's Quadratures for Solving a Linear Fredholm Integral Equation of The Second Kind

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Abstract

This paper discusses composite Mendeleev's quadratures for solving a linear Fredholm integral equation of the second kind. The numerical results show that our estimation have a good degree of accuracy.

AMS subject classification:

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1. Introduction

Consider a linear Fredholm integral equation of the second kind as follows:

$$u(x) = f(x) + \lambda \int_a^b k(x, t)u(t)dt, \quad a \leq x \leq b. \quad (1.1)$$

where $k(x, t)$ and $f(x)$ are known functions, but $u(x)$ is function to be determined [1, 4].

Many authors have considered solving problem (1.1) with different methods such as Taylor's expansion [6, 8, 14], Adomian's decomposition [2], wavelet [3, 7], Sinc collocation [10, 13], homotopy perturbation [5] and quadrature methods [9, 11].

In this paper, we solve problem (1.1) using composite Mendelev's quadratures. The derivation of the method is given in the second section. The numerical approach and result of this method respectively is presented by third and fourth section. In the end of the discussions is given the conclusion.

2. Composite Mendelev's Quadratures

Mendelev's quadratures for approximating the solution of $\int_a^b f(x)dx$ has two forms [12]:

$$\int_a^b f(x)dx \approx \frac{b-a}{4} \left(f(a) + 3f\left(a + \frac{2}{3}(b-a)\right) \right), \quad (2.2)$$

and

$$\int_a^b f(x)dx \approx \frac{b-a}{4} \left(3f\left(a + \frac{1}{3}(b-a)\right) + f(b) \right). \quad (2.3)$$

We call formula (2.2) and (2.3) respectively the left and right Mendelev's quadrature.

Suppose the interval $[a, b]$ divided into n subintervals, that is

$$a = x_0 < x_2 < x_4 \cdots < x_{2n} = b, \quad \text{and} \quad h = \frac{b-a}{n},$$

where

$$x_{2i} = x_0 + ih, \quad i = 1, 2, \dots, n.$$

The definite integral $\int_a^b f(x)dx$ can be written as

$$\int_{x_0}^{x_{2n}} f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \cdots + \int_{x_{2n-2}}^{x_{2n}} f(x)dx. \quad (2.4)$$

If we apply the left Mandelev's quadrature (2.2) to approximate the equation (2.4), then we have

$$\int_{x_0}^{x_{2n}} f(x)dx \approx \frac{h}{4} \sum_{i=1}^n (f(x_{2i-2}) + 3f(x_{2i-1})), \quad (2.5)$$

where

$$x_{2i-1} = \frac{x_{2i-2} + 2x_{2i}}{3}, \quad i = 1, 2, \dots, n.$$

By the same way, we use the right Mendelev's quadrature (2.2), we obtain

$$\int_{x_0}^{x_{2n}} f(x)dx \approx \frac{h}{4} \sum_{i=1}^n (3f(x_{2i-1}^*) + f(x_{2i})), \quad (2.6)$$

where

$$x_{2i-1}^* = \frac{2x_{2i-2} + x_{2i}}{3}, \quad i = 1, 2, \dots, n.$$

Formula (2.5) and (2.6), we call the composite left and right Mendelev's quadrature.

3. Solving a Linear Fredholm Integral Equation of The Second Kind with Composite Mendelev's Quadratures

To solve (1.1), we approximate the integral in the righth-side of (1.1) with the composite Mendelev's quadratures (2.5) and (2.6), we have

$$u(x) = f(x) + \frac{h}{4} \sum_{j=1}^n (k(x, t_{2j-2})u(t_{2j-2}) + 3k(x, t_{2j-1})u(t_{2j-1})), \quad (3.7)$$

and

$$u(x) = f(x) + \frac{h}{4} \sum_{j=1}^n (3k(x, t_{2j-1}^*)u(t_{2j-1}^*) + k(x, t_{2j})u(t_{2j})). \quad (3.8)$$

For $x = x_i, i = 0, 1, 2, \dots, 2n$ in (3.7) and (3.8), we get the following system:

$$u(x_i) = f(x_i) + \frac{h}{4} \sum_{j=1}^n (k(x_i, t_{2j-2})u(t_{2j-2}) + 3k(x_i, t_{2j-1})u(t_{2j-1})), \quad (3.9)$$

and

$$u(x_i) = f(x_i) + \frac{h}{4} \sum_{j=1}^n (3k(x_i, t_{2j-1}^*)u(t_{2j-1}^*) + k(x_i, t_{2j})u(t_{2j})). \quad (3.10)$$

For simplicity, we define the following notation:

$$u_i = u(x_i), \quad k_{ij} = k(x_i, t_j), \quad f_i = f(x_i), \quad (3.11)$$

so equation (3.9) and (3.10) can be written as

$$u_i = f_i + \frac{h}{4} \sum_{j=1}^n (k_{i,2j-2}u_{2j-2} + 3k_{i,2j-1}u_{2j-1}), \quad (3.12)$$

and

$$u_i = f_i + \frac{h}{4} \sum_{j=1}^n (3k_{i,2j-1}^* u_{2j-1}^* + k_{i,2j} u_{2j}), \quad (3.13)$$

Equation (3.12) and (3.13) are the approximated solution of the linear Fredholm integral equation of the second kind using the composite Mendelev's quadratures.

4. A Numerical Result

In this section, we solve the Love integral equation using the composite left Mendelev's quadrature (3.9).

Example 4.1. Love's integral equation, is defined as follows:

$$u(x) = f(x) + \frac{1}{\pi} \int_{-1}^1 \frac{s}{s^2 + (x-t)^2} u(t) dt, \quad -1 \leq x \leq 1. \quad (4.14)$$

We consider this equation in particular case when $s = 1$ and $f(x) = 1$. The numerical results are show in Table 1 and Figure 1.

Table 1: The numerical solutions of (4.14) with the composite left Mendelev's quadrature by varying n

x	$n = 8$	$n = 16$	$n = 32$	$n = 64$
-1	1.63960090042	1.63968349373	1.63969375500	1.63969503826
$-\frac{3}{4}$	1.75189402812	1.75194693332	1.75195371945	1.75195457760
$-\frac{1}{2}$	1.84236720540	1.84238235249	1.84238447820	1.84238475481
$-\frac{1}{4}$	1.89961585080	1.89961510820	1.89961515336	1.89961516544
0	1.91903290064	1.91903204797	1.91903199653	1.91903199334
$\frac{1}{4}$	1.89961699263	1.89961537985	1.89961519130	1.89961517031
$\frac{1}{2}$	1.84240678715	1.84238750615	1.84238512855	1.84238483630
$\frac{3}{4}$	1.75202094790	1.75196280399	1.75195570320	1.75195482556
1	1.63979247040	1.63970712368	1.63969669933	1.63969540601

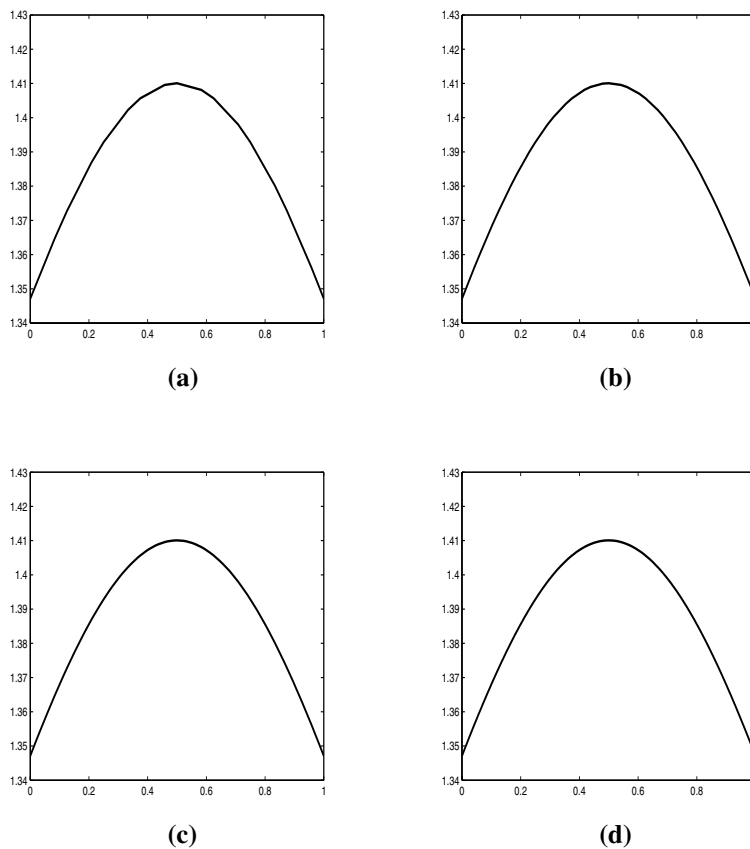


Figure 1: The numerical solutions of (4.14) with the composite left Mendeleev's quadrature for (a) $n = 8$, (b) $n = 16$, (c) $n = 32$ and (d) $n = 64$

In Table 1 we present the numerical solutions of (4.14) by varying the number of subintervals, n , in points $x = \pm 1, \pm \frac{3}{4}, \pm \frac{1}{2}, \pm \frac{1}{4}, 0$. Then we plot each numerical solution as depicted in Figure 1. From Table 1 and Figure 1 we see that, for sufficiently small h , we get a good accuracy.

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