Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 4 (2016), pp. 3775-3786

http://www.ripublication.com/gjpam.htm

Partially Statistical Test of Parameter Spline Truncated in Nonlinear Structural Equation Modeling (SEM)

Ruliana[a,b*], I Nyoman Budiantara[b], Bambang Widjanarko Otok[b], Wahyu Wibowo[b]

[a]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar (UNM), Makassar, Indonesia.

[b]Department of Statistics, Faculty of Mathematics and Natural Science, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia.

Abstract

Structural equation modeling (SEM) based on covariant (CBSEM), in defining a structural model which assumes that it is linear, can be modified to the form of nonlinear structural model that is called the spline truncated SEM $\boldsymbol{\omega}_{\boldsymbol{\eta}} = \boldsymbol{\Gamma}_{\xi\kappa}\boldsymbol{\gamma} + \boldsymbol{\zeta}$, for the $\boldsymbol{\gamma}$ is vector of parameter and a particular matrix $\boldsymbol{\Gamma}_{\xi\kappa}$ which contains factor scores of exogenous latent variable and knot point. To test the hypothesis of $\mathrm{H}_0 : \boldsymbol{\upsilon}'_j\boldsymbol{\gamma} = 0$ against $\mathrm{H}_0 : \boldsymbol{\upsilon}'_j\boldsymbol{\gamma} \neq 0$, for a vector line $\boldsymbol{\upsilon}'_j$ in which the element j is 1 and the other is 0, by using likelihood ratio test (LRT), is obtained the statistical test of hypothesis namely $\mathrm{T} = \dfrac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}}$ distributed by t with independent degree s_2. Area of rejection H_0 for this hypothesis fulfills the equation of $\mathrm{P}(\mathrm{T} < -\mathrm{K}^{**}) = \dfrac{\alpha}{2}$ or $\mathrm{P}(\mathrm{T} > \mathrm{K}^{**}) = \dfrac{\alpha}{2}$, for a constant K^{**}.

Keywords: Spline Truncated, Nonlinear SEM, Partially Statistical Test, Critical Region

---

*Corresponding Author: Ruliana, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar (UNM), Makassar, Indonesia,90222 and Ph.D Student at Department of Statistics, Faculty of Mathematics and Natural Science, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia, 60111, e-mail: ruliana.t@gmail.com

1. INTRODUCTION

Structural equation modeling (SEM), is one of the statistical approaches used for research that uses a variable that cannot be measured directly (latent variable) but it is conducted through indicators. There are two important things, which are carried out in SEM namely: i) how to explain the relationship between the latent variables with the indicators (measurement model) and ii) how to explain the relationship between the latent variables (structural model) as previously reported by Bollen [1]. In the measurement model can be used confirmatory factor analysis (CFA) and in structural model can be used, path analysis. Generally, in the present study of SEM used, is based on covariance (CBSEM) applying the software of AMOS and LISREL, which assumes that the structural model is linear, Bollen [1] Joreskog [2], and Bentler [3]. For certain cases, linearity in structural model can be modified into nonlinear in order to clarify the relationship between the latent variables that are not linear, Ruliana, Budiantara, Otok and Wibowo [4], Lee and Zhu [5], Lee and Song [6], Lee, Song and Lee [7], Lee, Song and Poon [8], Bollen and Current [9], Lee and Tang [10], Wall and Amemiya [11], Klein and Muthen [12], Mooijaart and Satorra [13], Wall and Amemiya [14], and Harring [15]. By getting a nonlinear structural model in SEM, furthermore, it is important to know what the parameters in the modified model are significant, or not. Simultaneous hypothesis testing of spline truncated model in nonlinear SEM was studied by Ruliana, Budiantara, Bambang and Wibowo [16]. In this paper aims at developing a partially statistical test to analyze the significance of structural parameters in the model of nonlinear SEM, partial distribution of the statistical test and the critical area.

2. HYPOTHESIS MODEL AND PARAMETER ESTIMATION OF SPLINE TRUNCATED MODEL IN NONLINEAR SEM

Ruliana et al (2015) had written down the measurement model of exogenous and endogenous latent variable as follows:

$$\mathbf{x} = \mathbf{\Lambda_x \xi} + \mathbf{\delta} \tag{1}$$

$$\mathbf{y_\eta} = \mathbf{\Lambda_{y,\eta} \eta} + \mathbf{\varepsilon_\eta} \tag{2}$$

with the factor scores for ξ is $\mathbf{\omega}_{\xi_1}$ and $\mathbf{\omega}_{\xi_2}$ and for $\mathbf{\eta}$ is $\mathbf{\omega_\eta}$. The model of spline truncated in nonlinear SEM for two variables of latent exogenous is written down the following

$$\omega_{\eta i} = \sum_{r=0}^{m_1} \alpha_{1r} \omega_{\xi_1 i}^{r} + \sum_{s=1}^{K_1} \beta_{1s} (\omega_{\xi_1 i} - \kappa_s)_+^{m_1} + \sum_{t=0}^{m_2} \alpha_{2t} \omega_{\xi_2 i}^{t} + \sum_{u=1}^{K_2} \beta_{2u} (\omega_{\xi_2 i} - \kappa_u)_+^{m_2} + \zeta_i \tag{3}$$

In the form of matrix, model (3) can be written down such as the following:

$$\mathbf{\omega_\eta} = \mathbf{\Gamma_{\xi\kappa} \gamma} + \mathbf{\zeta} \tag{4}$$

where $\mathbf{\Gamma_{\xi\kappa}}$ is the matrix which contains exogenous latent variable and knot [4]. To know whether the parameters in the model(3) above, partially affect to the model or not, it is carried out a test of hypothesis with the formula as written below:

$$\mathrm{H}_0 : \boldsymbol{\upsilon}'_j \boldsymbol{\gamma} = 0 \text{ versus } \mathrm{H}_0 : \boldsymbol{\upsilon}'_j \boldsymbol{\gamma} \neq 0 \quad (5)$$

where $\boldsymbol{\upsilon}'_j$ is the vector line $1 \times s_1$, $s_1 = m_1 + m_2 + k_1 + k_2 + 2$ in which the element -*j* is 1 and the other is zero, $\boldsymbol{\gamma}$ is the parameter vector $s_1 \times 1$. The form of (5) is the formulation of hypothesis for test of partial models(3), with sets of parameters below population or space Ω could be found in Ruliana, Budiantara, Bambang and Wibowo [16] and the sets of parameters below H_0, or space Ψ are

$$\Psi = \{\boldsymbol{\gamma} = (\alpha_{10}, \alpha_{11}, \ldots, \alpha_{1m_1}, \beta_{11}, \ldots, \beta_{1K_1}, \alpha_{20}, \alpha_{21}, \ldots, \alpha_{2m_2}, \beta_{21}, \ldots, \beta_{2K_2}), \sigma^2_\Psi \,|\, \boldsymbol{\upsilon}'_j \boldsymbol{\gamma} = 0\} \quad (6)$$

Estimation of parameter γ in the area of parameter Ψ for model (3)is given in Lemma 1 as stated in sentence below

Lemma1

If $\hat{\gamma}_\Omega$ and $\hat{\gamma}_\Psi$, respectively, are the estimator parameters under the area of Ω and Ψ for hypotheses model (5), then

$$\hat{\boldsymbol{\gamma}}_\Psi = \hat{\boldsymbol{\gamma}}_\Omega - (\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega \quad (7)$$

Proof:

Given the lagrange function

$$\mathrm{R}(\boldsymbol{\gamma}_\Psi, \boldsymbol{\theta}) = \mathrm{V}(\boldsymbol{\gamma}_\Psi) + 2\boldsymbol{\theta}'(\boldsymbol{\upsilon}'_j\boldsymbol{\gamma}_\Psi) \quad (8)$$

where $\mathrm{V}(\boldsymbol{\gamma}_\Psi) = (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\boldsymbol{\gamma}_\Psi)'\,(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\boldsymbol{\gamma}_\Psi)$ (9)

with the vector of lag range multipliers $2\boldsymbol{\theta}'$ and constraint $\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Psi$. Estimation for $\boldsymbol{\gamma}_\Psi$ is $\hat{\boldsymbol{\gamma}}_\Psi$ by minimizing the form of

$$\mathrm{V}(\hat{\boldsymbol{\gamma}}_\Psi) = (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_\Psi)'\,(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_\Psi) \quad (10)$$

$\hat{\boldsymbol{\gamma}}_\Psi$ is obtained with decomposing the equation of (8)and becomes

$$\mathrm{R}(\boldsymbol{\gamma}_\Psi, \boldsymbol{\theta}) = \boldsymbol{\omega}'_{\boldsymbol{\eta}}\boldsymbol{\omega}_{\boldsymbol{\eta}} - 2\boldsymbol{\gamma}'_\Psi\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\omega}_{\boldsymbol{\eta}} + \boldsymbol{\gamma}'_\Psi\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa}\boldsymbol{\gamma}_\Psi + 2\boldsymbol{\theta}'(\boldsymbol{\upsilon}'_j\boldsymbol{\gamma}_\Psi) \quad (11)$$

Furthermore, by derivation partially the equation of (11) with respect to γ_Ψ and $\boldsymbol{\theta}$ equalizing the result to zero, it is obtained:

$$\hat{\boldsymbol{\gamma}}_\Psi = \hat{\boldsymbol{\gamma}}_\Omega - (\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j\boldsymbol{\theta} \quad (12)$$

and

$$\boldsymbol{\theta} = [\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega \quad (13)$$

By substituting the equation of (13)in(12), then $\hat{\gamma}_\Psi$ is obtained for the model of (3), is:

$$\hat{\boldsymbol{\gamma}}_\Psi = \hat{\boldsymbol{\gamma}}_\Omega - (\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega$$

3. PARTIAL STATISTICAL TESTS AND STATISTICAL DISTRIBUTION MODEL TEST OF SPLINE TRUNCATED IN NONLINEAR SEM.

To obtain the statistical test of hypothetical model (5) can be used method of likelihood ratio test (LRT). The likelihood ratio is obtained by preparing a ratio of the likelihood maximum function in parameter area Ω and Ψ, and to obtain the function of likelihood maximum needs an estimation of parameter of $\boldsymbol{\gamma}$ and parameter of $\sigma^2_{\omega_\eta}$ below the area of parameter Ω and Ψ.

Theorem 1

Let a model is given in the equation of (3) with hypothesis formulation in the equation of(5), thus statistical test for this hypothesis is given by:

$$\mathrm{T} = \frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}} \tag{14}$$

where:

$$\boldsymbol{\Lambda}^* = \sqrt{\boldsymbol{\Lambda}_1^*} = \sqrt{\left((\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)'[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}'\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}})^{-1}\boldsymbol{\upsilon}_J]^{-1}(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)\right)} \tag{15}$$

$$\boldsymbol{\Lambda}_2^* = \boldsymbol{\omega}_{\boldsymbol{\eta}}'[\mathbf{I} - \boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}(\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}'\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}})^{-1}\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}']\boldsymbol{\omega}_{\boldsymbol{\eta}} \tag{16}$$

$$s_2 = n - (m_1 + m_2 + k_1 + k_2 + 2) \tag{17}$$

Proof:

Estimator for γ and $\sigma^2_{\omega_\eta}$ below the area of parameter Ω sequentially is given below

$$\hat{\boldsymbol{\gamma}}_\Omega = (\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}'\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}})^{-1}\boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}'\boldsymbol{\omega}_{\boldsymbol{\eta}} \tag{18}$$

$$\hat{\sigma}^2_{\omega_\eta\Omega} = \frac{(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}\hat{\boldsymbol{\gamma}}_\Omega)'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}\hat{\boldsymbol{\gamma}}_\Omega)}{n} \tag{19}$$

Based on Lemma 1 above, estimator for $\boldsymbol{\gamma}$ below Ψ is $\hat{\boldsymbol{\gamma}}_\Psi$ such as, in the equation of(7). Furthermore, by using a maximum likelihood estimation (MLE) is obtained estimator for $\sigma^2_{\omega_\eta}$ as follows

$$\hat{\sigma}^2_{\omega_\eta\Psi} = \frac{(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}\boldsymbol{\gamma}_\Psi)'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}\boldsymbol{\gamma}_\Psi)}{n} \tag{20}$$

Thus, the maximum likelihood functions under Ω and Ψ, respectively, in the equation of (21) and (22) is obtained as formulated in two equations:

$$\mathrm{L}(\hat{\Omega}) = \mathrm{L}(\hat{\boldsymbol{\gamma}}_\Omega, \hat{\sigma}^2_{\omega_\eta\Omega}) = (2\pi\hat{\sigma}^2_{\omega_\eta\Omega})^{-n/2}\,\mathrm{e}^{-n/2} \tag{21}$$

$$\mathrm{L}(\hat{\Psi}) = \mathrm{L}(\hat{\boldsymbol{\gamma}}_\omega, \hat{\sigma}^2_{\omega_\eta\Psi}) = (2\pi\hat{\sigma}^2_{\omega_\eta\Psi})^{-n/2}\,\mathrm{e}^{-n/2} \tag{22}$$

Furthermore in the equation of (21) and (22) are obtained *likelihood ratio* as follows

$$\mathrm{L}_{\mathrm{ratio}} = \frac{L(\hat{\Psi})}{L(\hat{\Omega})} = \left(\frac{(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Omega})'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Omega})}{(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Psi})'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Psi})} \right)^{\frac{n}{2}} = \left(\frac{\boldsymbol{\Lambda}_2^*}{\mathbf{A}} \right)^{\frac{n}{2}} \tag{23}$$

where

$$\boldsymbol{\Lambda}_2^* = (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Omega})'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Omega}) \tag{24}$$

$$\mathbf{A} = (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Psi})'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Psi}) \tag{25}$$

with modifying of matrix **A** into the form as follow:

$$\begin{aligned} \mathbf{A} &= (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Omega})'(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\xi\kappa}\hat{\boldsymbol{\gamma}}_{\Omega})(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_{\Omega})'[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_{\Omega}) \\ &= \boldsymbol{\Lambda}_2^* + \boldsymbol{\Lambda}_1^* \end{aligned} \tag{26}$$

where

$$\boldsymbol{\Lambda}_1^* = (\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_{\Omega})'[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_{\Omega}) \tag{27}$$

Furthermore by substituting (26) in (23) and decomposing it, obtained

$$\mathrm{L}_{\mathrm{ratio}} = (1 + \frac{\boldsymbol{\Lambda}_1^*}{\boldsymbol{\Lambda}_2^*})^{-\frac{n}{2}} \tag{28}$$

From the equation of (28) for $\boldsymbol{\Lambda}^* = \sqrt{\boldsymbol{\Lambda}_1^*}$, then subsequently, the statistical test for hypotheses model (5) is discovered, in form as follows:

$$\mathrm{T} = \frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}}$$

Furthermore, the distribution of the statistical test that has been obtained above is described in the following theorem:

Theorem 2

If $\boldsymbol{\Lambda}^*$ and $\boldsymbol{\Lambda}_2^*$, respectively, are written in the equations of (15) and (24) then

$$\mathrm{T} = \frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}} \sim t(s_2) \tag{29}$$

Proof:

To prove Theorem 2 above is performed in 3 points, namely 3.1, 3.2 and 3.3 with the description three elaborations as described below:

3.1 Form of distribution from $\boldsymbol{\Lambda}^*$

From the equation of (3), with the assumption of error $\boldsymbol{\zeta} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$, then the component of $(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_{\Omega})$ in the equation of (27) is written as

$$(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega) \sim N(\boldsymbol{\upsilon}_j'\boldsymbol{\gamma}_\Omega\,,\ \boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j\sigma^2) \tag{30}$$

for

$$\mathbf{B} = \frac{[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}}{\sigma^2}$$

$$\mathbf{V}^{**} = \sigma^2[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]$$

and

$(\mathbf{BV}^{**})$ is the idempotent matrix, then:

$$\frac{\boldsymbol{\Lambda}_1^*}{\sigma^2} \sim \chi^2\,(\mathrm{rank}(\mathbf{B}), \mathbf{D}^*) \tag{31}$$

with

$$\mathbf{D}^* = (\boldsymbol{\upsilon}_j'\boldsymbol{\gamma})'[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}\,\frac{(\boldsymbol{\upsilon}_j'\boldsymbol{\gamma})}{2\sigma^2} \tag{32}$$

Because $\mathbf{B}$ is the vector line, and below hypotheses of H_0 matrix $\mathbf{D}^* = \mathbf{0}$ then the form of (31) becomes

$$\frac{\boldsymbol{\Lambda}_1^*}{\sigma^2} \sim \boldsymbol{\chi}^2(1) \tag{33}$$

By dividing the components of $\boldsymbol{\Lambda}_1^*$ contained in the equation of (15) with σ^2 and writing down as follows

$$\boldsymbol{\Lambda}^* = \sqrt{\left((\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)'[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)\right)/\,\sigma^2)}$$

$$= \sqrt{\left(\frac{\left((\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)'(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)\right)}{[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}\,/\,\sigma^2\right)}$$

$$= \frac{\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega}{\sigma\sqrt{[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}} \tag{34}$$

The expected value from the equation (34) is:

$$\mathrm{E}(\boldsymbol{\Lambda}^*)=\mathrm{E}\left(\frac{\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega}{\sigma\sqrt{[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}}\right)$$

$$= \frac{1}{\sigma\sqrt{[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}}\,\mathrm{E}(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega) \tag{35}$$

And the expected and variance from $\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega$ can be derived in the form of:

$$\begin{aligned}\mathrm{E}[(\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega)\,] &= \mathrm{E}(\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\omega}_{\boldsymbol{\eta}})\\ &= \boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\Gamma}'_{\xi\kappa}\,\mathrm{E}(\boldsymbol{\omega}_{\boldsymbol{\eta}})\\ &= \boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\,\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa}\,\boldsymbol{\gamma}_\Omega\\ &= \boldsymbol{\upsilon}'_j\boldsymbol{\gamma}_\Omega\end{aligned} \tag{36}$$

$$\begin{aligned}\mathrm{Var}[(\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega)\,] &= \mathrm{Var}(\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\omega}_{\boldsymbol{\eta}})\\ &= \boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\,\boldsymbol{\Gamma}'_{\xi\kappa}\,\mathrm{Var}[\boldsymbol{\omega}_{\boldsymbol{\eta}}]\,\boldsymbol{\Gamma}_{\xi\kappa}(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j\\ &= \sigma^2\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\,\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa}\,(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j\\ &= [\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]\sigma^2\end{aligned} \tag{37}$$

The substitution of equation (36) to equation (35):

$$\mathrm{E}(\boldsymbol{\Lambda}^*) = \frac{\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega}{\sigma\sqrt{[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}} \tag{38}$$

The variance of the equations (34) is:

$$\mathrm{Var}(\boldsymbol{\Lambda}^*) = \mathrm{Var}\left(\frac{\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega}{\sigma\sqrt{[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}}\right) = \left(\frac{1}{\sigma\sqrt{[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}}\right)^2 \mathrm{Var}(\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega) \tag{39}$$

Variance from $\boldsymbol{\upsilon}'_j\hat{\boldsymbol{\gamma}}_\Omega$ in equation (37) is inserted to equation (39), and is obtained

$$\mathrm{Var}(\boldsymbol{\Lambda}^*) = \frac{1}{[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}'_j]\sigma^2}[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}'_j]\sigma^2] = 1 \tag{40}$$

From equation (38) and (40) with $\hat{\boldsymbol{\gamma}}_\Omega$ are the linear functions of $\boldsymbol{\omega}_{\boldsymbol{\eta}}$, then

$$\boldsymbol{\Lambda}^* \sim N\left(\frac{\boldsymbol{\upsilon}'_j\boldsymbol{\gamma}}{\sigma\sqrt{[\boldsymbol{\upsilon}'_j(\boldsymbol{\Gamma}'_{\xi\kappa}\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]}}\,,1\right) \tag{41}$$

Under the hypothesis $\mathrm{H}_0 : \boldsymbol{\upsilon}'_j\boldsymbol{\gamma} = 0$, form of (41) is rewritten:

$$\boldsymbol{\Lambda}^* \sim N(0,1) \tag{42}$$

From the equation of (42) can be known the distribution of $\boldsymbol{\Lambda}^*$ namely $\boldsymbol{\Lambda}^* \sim N(0,1)$.

3.2 Form of distribution from $\boldsymbol{\Lambda}_2^*$

Form of $\mathbf{\Lambda}_2^*$ contained in (24) can be modified into a quadratic form, which is expressed in relations:

$$\mathbf{\Lambda}_2^* = \boldsymbol{\omega}_{\boldsymbol{\eta}}'[\mathbf{I} - \mathbf{\Gamma}_{\xi\kappa}(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\mathbf{\Gamma}_{\xi\kappa}']\boldsymbol{\omega}_{\boldsymbol{\eta}} \quad (43)$$

with

$$\boldsymbol{\omega}_{\boldsymbol{\eta}} \sim N(\mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\gamma}, \sigma^2\mathbf{I})$$

for

$$\mathbf{C} = \frac{[\mathbf{I} - \mathbf{\Gamma}_{\xi\kappa}(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\mathbf{\Gamma}_{\xi\kappa}'}{\sigma^2}$$

$$\mathbf{V}^* = \sigma^2\mathbf{I}$$

where, $(\mathbf{CV}^*)$ is an idempotent matrix, and then

$$\frac{\mathbf{\Lambda}_2^*}{\sigma^2} \sim \chi^2(\text{rank}(\mathbf{C}), \mathbf{E}) \quad (44)$$

with

$$\mathbf{E} = (\mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\gamma})'[\mathbf{I} - \mathbf{\Gamma}_{\xi\kappa}(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\mathbf{\Gamma}_{\xi\kappa}']\frac{(\mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\gamma})}{2\sigma^2} \quad (45)$$

For $\mathbf{C}$ and $\mathbf{V}^*$ are respectively nonsingular matrix thus they are valid for $\text{rank}(\mathbf{C}) = \text{rank}(\mathbf{CV}^*) = \text{tr}(\mathbf{CV}^*) = s_2$, where $s_2 = n - (m_1 + m_2 + k_1 + k_2 + 2)$. The results of description of $\mathbf{E}$ matrix is equal to $\mathbf{0}$, then the form of (44) becomes

$$\frac{\mathbf{\Lambda}_2^*}{\sigma^2} \sim \chi^2(s_2) \quad (46)$$

3.3 Independence of $\mathbf{\Lambda}^*$ and $\mathbf{\Lambda}_2^*$

Independences of $\mathbf{\Lambda}^*$ and $\mathbf{\Lambda}_2^*$ can be identified from the results of matrix multiplication of quadratic form with the vectors of quadratic form from $\mathbf{\Lambda}_1^*$ and $\mathbf{\Lambda}_2^*$, with $\mathbf{\Lambda}^* = \sqrt{\mathbf{\Lambda}_1^*}$. Initiated by modifying $\mathbf{\Lambda}_1^*$ and $\mathbf{\Lambda}_2^*$, respectively, in the following forms

$$\mathbf{\Lambda}_1^* = [\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}_j'\boldsymbol{\upsilon}_j]^{-1}]'\mathbf{P}[\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}_j'\boldsymbol{\upsilon}_j]^{-1}] \quad (47)$$

with

$$\mathbf{P} = \mathbf{\Gamma}_{\xi\kappa}(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}_j'(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_j]^{-1}\boldsymbol{\upsilon}_j'(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\mathbf{\Gamma}_{\xi\kappa}' \quad (48)$$

and

$$\mathbf{\Lambda}_2^* = [\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}_j'\boldsymbol{\upsilon}_j]^{-1}]'\mathbf{Q}[\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{\Gamma}_{\xi\kappa}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}_j'\boldsymbol{\upsilon}_j]^{-1}] \quad (49)$$

with

$$\mathbf{Q} = [\mathbf{I} - \mathbf{\Gamma}_{\xi\kappa}(\mathbf{\Gamma}_{\xi\kappa}'\mathbf{\Gamma}_{\xi\kappa})^{-1}\mathbf{\Gamma}_{\xi\kappa}' \quad (50)$$

for

$$\begin{aligned}\mathbf{V} &= \mathbf{Var}[\boldsymbol{\omega}_{\boldsymbol{\eta}} - \boldsymbol{\Gamma}_{\boldsymbol{\xi\kappa}}\boldsymbol{\upsilon}_j[\boldsymbol{\upsilon}'_j\boldsymbol{\upsilon}_j]^{-1}] \\ &= \sigma^2\mathbf{I}\end{aligned} \tag{51}$$

Furthermore, the result of multiplying the equations(50), (51) and (48) is written down in formula as derived in following form

$$\begin{aligned}\mathbf{QVP} &= \mathbf{Q}(\sigma^2\mathbf{I})\mathbf{P}, \ \mathbf{I} \text{ is identity matrix} \\ &= \sigma^2\mathbf{QIP} \\ &= \mathbf{0}\end{aligned} \tag{52}$$

From the equation of (52)is seen that $\boldsymbol{\Lambda}_1^*$ and $\boldsymbol{\Lambda}_2^*$ are independent, which mean that $\sqrt{\boldsymbol{\Lambda}_1^*} = \boldsymbol{\Lambda}^*$ and $\boldsymbol{\Lambda}_2^*$ are also independent. Based on the equations of (42), (46) and (52), the distribution of the statistical test in the equation of (3)is:

$$\mathrm{T} = \frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}} \sim t(s_2)$$

4. CRITICAL AREA FOR PARTIAL TEST OF MODEL SPLINE TRUNCATED IN NONLINEAR SEM

Critical areas for the partial test model parameters truncated in nonlinear SEM are given in the following theorem:

Theorem 3

If given partial test hypothesis formulation in the equation of (5) with statistical test in the equation of (14) and the distribution as in the equation of (29), then the critical area is:

$$\mathrm{P}(\mathrm{T} < -\mathrm{K}^{**}) = \frac{\alpha}{2} \text{ or } \mathrm{P}(\mathrm{T} > \mathrm{K}^{**}) = \frac{\alpha}{2}; \quad \mathrm{T} \sim t(s_2) \tag{53}$$

Proof:

Critical areas for the partial test on the model of (5) is to solve a form that meets the following equation

$$\mathrm{L}_{\text{ratio}} = \frac{L(\hat{\Psi})}{L(\hat{\Omega})} < \mathrm{K} \tag{54}$$

Equation (54)can be rewritten as:

$$\left(1 + \frac{\boldsymbol{\Lambda}_1^*}{\boldsymbol{\Lambda}_2^*}\right)^{-\frac{n}{2}} < \mathrm{K} \tag{55}$$

By expanding the equation (55), the new form is obtained as

$$\left(1 + \frac{\boldsymbol{\Lambda}_1^*}{\boldsymbol{\Lambda}_2^*}\right) < \mathrm{K}^{-\frac{2}{n}}$$

$$\Rightarrow\left(\sqrt{\frac{\boldsymbol{\Lambda}_1^*}{\boldsymbol{\Lambda}_2^*}}\right)^2 > (\mathrm{K}^{-\frac{2}{n}}-1)$$

$$\Rightarrow\frac{1}{1/s_2}\left(\frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}}\right)^2 > (\mathrm{K}^{-\frac{2}{n}}-1)$$

$$\Rightarrow\left(\frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}}\right)^2 > \mathrm{K}^*;\ \text{with };\mathrm{K}^* = \frac{(\mathrm{K}^{-\frac{2}{n}}-1)}{s_2} \tag{56}$$

From equation (56) can be written as follows:

$$\mathrm{T}^2 > \mathrm{K}^*,\ \text{with}\,\mathrm{T} = \frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}} \sim t(s_2) \tag{57}$$

By writing an equation (57) in the following form

$$|\mathrm{T}| > \pm\mathrm{K}^{**}\ \ ,\mathrm{K}^{**} = \sqrt{\frac{(\mathrm{K}^{-\frac{2}{n}}-1)}{s_2}} \tag{58}$$

And then referring the equation of (58), it can be developed an expression, if given significant rate of α, then it is obtained the critical area as follows:

$$\mathrm{P}(\mathrm{T} < -\mathrm{K}^{**}) = \frac{\alpha}{2}\ \text{or}\,\mathrm{P}(\mathrm{T} > \mathrm{K}^{**}) = \frac{\alpha}{2};\ \ \mathrm{T}\sim t(s_2)$$

5. CONCLUSION

By using likelihood ratio test for the hypotheses of $\mathrm{H}_0 : \boldsymbol{\upsilon}_j'\boldsymbol{\gamma} = 0$ versus $\mathrm{H}_0 : \boldsymbol{\upsilon}_j'\boldsymbol{\gamma} \neq 0$ in the model of spline truncated in nonlinear SEM, can be concluded as follows

1. Partially statistical test for spline truncated in nonlinear SEM is

$$\mathrm{T} = \frac{\boldsymbol{\Lambda}^*}{\sqrt{(\boldsymbol{\Lambda}_2^*)/s_2}}$$

With:

$$\boldsymbol{\Lambda}^* = \sqrt{\boldsymbol{\Lambda}_1^*} = \sqrt{\left((\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)'[\boldsymbol{\upsilon}_j'(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\upsilon}_J]^{-1}(\boldsymbol{\upsilon}_j'\hat{\boldsymbol{\gamma}}_\Omega)\right)}$$

$$\boldsymbol{\Lambda}_2^* = \boldsymbol{\omega}_{\boldsymbol{\eta}}'[\mathbf{I} - \boldsymbol{\Gamma}_{\xi\kappa}(\boldsymbol{\Gamma}_{\xi\kappa}'\boldsymbol{\Gamma}_{\xi\kappa})^{-1}\boldsymbol{\Gamma}_{\xi\kappa}']\boldsymbol{\omega}_{\boldsymbol{\eta}}$$

$$s_2 = n - (m_1 + m_2 + k_1 + k_2 + 2)$$

2. Distribution of the partially statistical test to spline truncated in nonlinear SEM is

$$\mathrm{T} = \frac{\mathbf{\Lambda}^{*}}{\sqrt{(\mathbf{\Lambda}_{2}^{*})/s_{2}}} \sim t(s_{2})$$

3. Rejection area of H_0, for partial test to spline truncated in nonlinear SEM is

$$\mathrm{P}(\mathrm{T} < -\mathrm{K}^{**}) = \frac{\alpha}{2} \text{ or } \mathrm{P}(\mathrm{T} > \mathrm{K}^{**}) = \frac{\alpha}{2}; \quad \mathrm{T} \sim t(s_{2})$$

REFERENCES

[1] K.A. Bollen, *Structural Equations with Latent Variables*, John Wiley & Sons, New York, 1989. http://dx.doi.org/10.1002/ 9781118619179

[2] K.G. Joreskog, A General Method for Estimating a Linear Structural Equation System, in A.S. Goldberger, O.D. Duncan, (Eds.), *Structural Equation Models in the Social Sciences* 85-112, Academic Press, New York, 1973.

[3] P.M. Bentler, Multivariate Analysis with Latent Variables: Causal Modeling, *Annual Review of Psychology*, 31 (1980), 419-456. http://dx.doi.org/10.1146/annurev.ps.31.020180.002223

[4] Ruliana, I.N. Budiantara, B.W. Otok and W.Wibowo, Parameter Estimation of Nonlinear Structural Model SEM Using Spline Approach, *Applied Mathematical Sciences*, 9 (2015), 7439-7451. http://dx.doi.org/10.12988/ams.2015.510660

[5] S.Y. Lee, and H.T. Zhu, Statistical Analysis of Nonlinear Structural Equation Models with Continuous and Polytomous Data, *British Journal of Mathematical and Statistical Psychology*, 53 (2000), 209-232. http://dx.doi.org/10.1348/000711000159303

[6] S.Y. Lee, and X.Y. Song, Maximum Likelihood Estimation and Model Comparison of Nonlinear Structural Equation Models with Continuous and Polytomous Variables, *Computational Statistics & Data Analysis*, 44 (2003), 125-142. http://dx.doi.org/10.1016/s016 7-9473(02)00305-5

[7] S.Y. Lee, X.Y. Song and J.C.K. Lee, Maximum Likelihood Estimation of Nonlinear Structural Equation Models with Ignorable Missing Data, *Journal of Educational and Behavioral Statistics*, 28 (2003), 111-134. http://dx.doi.org/10.3102/10769986028002111

[8] S.Y. Lee, X.Y. Song, and W.Y. Poon, Comparison of Approaches in Estimating Interaction and Quadratic Effects of Latent Variables, *Multivariate Behavioral Research*,39 (2004), 37-67. http://dx.doi.org/10.1207/s15327906mbr3901_2

[9] K.A. Bollen, and P.J. Curran, *Non Linear Trajectories and The Coding of Time, Latent Curve Models: a Structural Equation Perspectiv*e, John Wiley & Sons, 2006.

[10] S.Y. Lee, and N.S. Tang, Analysis of Nonlinear Structural Equation Models with Nonignorable Missing Covariates and Ordered Categorical Data, *Statistica Sinica*, 16 (2006) 1117-1141.

[11] M.M. Wall, and Y. Amemiya, *Nonlinear Structural Equation Modeling as a Statistical Method*, Handbook of Computing and Statistics With Applications, ISSN: 1871-0301 1 (2007), 321-343 http://dx.doi.org/10.1016/b978-044452044-9/50018-5

[12] A.G. Klein, and B.O Muthen, Quasi Maximum Likelihood Estimation of Structural Equation Models with Multiple Interaction and Quadratic Effects, *Multivariate Behavioral* Research, 42 (2007), 647-673. http://dx.doi.org/10.1080/00273170701710205

[13] A. Mooijaart, and A. Satorra, *Moment Testing for Interaction Terms in Structural Equation Modeling*, Leiden University, 2011.

[14] M.M. Wall, and Y. Amemiya, Estimation for Polynomial Structural Equation Models, *Journal of the American Statistical Association*, 9 (2000), 929-940. http://dx.doi.org/10.1080/01621459.2000.10474283

[15] J.R. Harring, a Spline Model for Latent Variable, *Educational and Psychological Measurement, 20 (2013), 1-7.*

[16] Ruliana, I.N., Budiantara, B.W. Otok, W., Wibowo, Simultaneous Hypothesis Testing of Spline Truncated Model in Nonlinear Structural Equation Modeling (SEM), *Journal of Theoretical and Applied Information Technology* (JATIT), 89(2016) 372-380

[17] W., Wibowo, S., Haryatmi, and I.N., Budiantara, On Multiresponse Semiparametric Regression Model, *Journal of Mathematics and Statistics* (JMS), 8(4), 2012, 489-499

[18] I.N., Budiantara, V. Ratnasari, I. Zain, The Combination of Spline and Kernel Estimation for Nonparametric Regression and It's Properties, Applied Mathematical Sciences (AMS), 9(2015), 6083-6094