

A Typology of Strategies in Solving Algebraic Equations Problems

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Abstract

Problem solving is an important activity in mathematics and its learning. Strategies in solving mathematical problems have always been attracting researchers. The objective of this present article is to analyze a typology in solving algebraic equations problems. The data were collected through written responses from 41 mathematics teachers in junior and senior high schools to the problems given and the data were then descriptively analyzed. The results of the analysis showed that the typology of strategies in solving equation problems is computation and transposition (CT) and relating elements and transformations (RT). The CT typology is very dominantly used by teachers than the RT one.

Keywords: strategy, problems solving, algebra equations

INTRODUCTION

Problem solving is a crucial matter in learning mathematics. Students with good problem solving skills are also smart in other mental abilities such as in analogy, reasoning, critical thinking, perception, memory and creative thinking. They are also good readers and possess some knowledge of different approaches to planning that they may use to solve problems [8]. Good problem solvers own meta-cognitive skills, an ability to monitor and to evaluate their thoughts [6, 9]. However, many researchers state that students' mathematical knowledge in problem solving is merely mechanical [11, 7]. From elementary schools, the mathematical procedural understanding in the form of calculative skills for four numerical basic operation is always given an emphasis in learning mathematics. Most information at school is provided by teachers, so that they as educators more or less contribute to their students' ability. As a result, teachers' ability in solving mathematical problems is very important and should always be improved.

Ability to solve problems develops slowly and takes a very long time. Problem solving is an activity that involves a problem solver in various cognitive actions, including accessing and using some knowledge and previous experiences [13,1, 18]. Treffinger and Selby [24] state that a problem solving style is one's consistent differences in which one chooses to plan, implement and focus in order to get some clarity, to result in ideas or to prepare oneself to make an action when solving a problem. This problem solving style is a very important dimension of creative productivity [13].

In general, there are some strategies in solving mathematical problems [17, 1, 18]. Some researchers have studied students' strategies in solving mathematical problems [12, 10, 7, 14, 2, 3], but few studied what strategies mathematics teachers adopt in solving mathematical problems, while they are one of students' learning sources. Therefore, it is important to study the typology of strategies of mathematics teachers in solving problems in general, and especially algebraic equations.

METHOD

Participant

The participants were 41 high school teachers of mathematics who were voluntarily solving problems given. They consist of 10, 22, and 9 teachers of junior (SMP), senior (SMA) and vocational (SMK) high schools, respectively.

Instrument

The instrument consisted of algebraic equation problems containing three problems referring to [2,3]. These problems are expected to give a consistent typology of strategies mathematics teachers adopt in solving equation problems. For each problem, it is possible for the subjects to solve it in various ways with its own reasons.

Data Collection and Analysis

The data collection was focused on responses from mathematics teachers when they were solving the given problems. The problems should be solved in not more than 10 minutes. The data were analyzed by coding each teachers' answer to each problem given like in [2,3]. The data credibility was made by asking two persons to code answers. The agreement between the two coders was 95% for each problem with the same coding. Then the valid data were descriptively analyzed.

RESULTS

For the written responses to the given problems, coding was focused on the strategies the teachers adopted in solving equations problems. Based on the results of coding, a typology of strategies of mathematics teachers in solving T1 problems was classified into one of the two categories: 1) computation and transposition (CT) and 2) paying attention to the relation of numerical values or elements from two sides (relations) or transforming the equation so that two identical equations are obtained (RT).

CT Typology

This typology is categorized into two strategies: CT-1 and CT-2. A Teacher's response is represented by CT-1 if he focuses on doing a counting operation followed by a side change, sign change or he does a side change, in the next sign changes, he then does the counting operation. CT-2 if a teacher does a CT-1 but in the computation he employs a simpler approach.

In problem P1. Find the value of n so that $57 + 86 = n + 84$ is true! and in problem P2. Find the value of d so that $345 + 576 = 342 + 574 + d$ is true! The solution adopting the strategy CT-1 is presented in Figure 1.

$$\begin{array}{r}
 57 + 86 = n + 84 \\
 143 = n + 84 \\
 143 - 84 = n \\
 n = 59.
 \end{array}
 \qquad
 \begin{array}{r}
 345 + 576 = 342 + 574 + d \\
 921 = 916 + d \\
 921 - 916 = d \\
 d = 5.
 \end{array}$$

Figure 1: CT-1 Strategy in P1 and P2

A CT-1 strategy to obtain a n or d value is made by adding a number at the next side, then reduced by a number at the other side of the equation. This method gives a priority to procedural computation, and does not demand any high thinking process. This strategy was dominantly employed by mathematics teachers: namely 70%, 77.27% and 77.78% for F1 and for F2 80%, 81.82% and 100% for junior, senior, and vocational high school teachers, respectively. It means that most teachers are still possessing procedural comprehension and computation in solving mathematical problems. The CT-2 strategy in solving P1 and P2 is presented in Figure 2.

$$\begin{array}{r}
 57 + 86 = n + 84 \\
 57 + 86 - 84 = n \\
 57 + 2 = n \\
 59 = n
 \end{array}
 \qquad
 \begin{array}{r}
 345 + 576 = 342 + 574 + d \\
 345 - 342 + 576 - 574 = d \\
 3 + 2 = d \\
 5 = d
 \end{array}$$

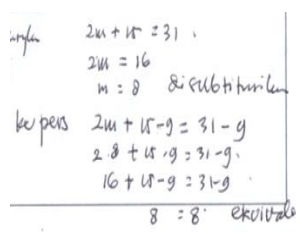
Figure 2: CT-2 strategy in P1 and P2

The CT-2 strategy to obtain a n or d value is done by transposition, namely moving a number at the right side to the left one of the equation, considering that the computation is easier to make. In the CT-2 strategy, a computation and "equal or appropriate" numbers should be paid more attention. Moreover, a high thinking process is required. This method was employed by mathematics teachers in junior high school, 30%, senior high school 31.82% and vocational high school, 33.33% for P1, while 30%, 27.28% and 11.11% for F2.

In problem P3. Does m possess the same value in $2m + 15 = 31$ and $2m + 15 - 9 = 31 - 9$? Give reasons! Solutions using CT-1 and CT-2 strategies are presented at Picture 3 (a) and (b). The CT-1 strategy to obtain an m value is adopted by solving the two equations, P1 and P2, then the results of the the two solutions are compared. This strategy was adopted by 90% junior high school teachers and 100% senior high school/*madrasah* and vocational high school teachers. The CT-2 strategy is used to solve one of the equations, then the result is distributed to another equation, so that an equation is obtained. This strategy was adopted by 10% of junior high schools teachers, but not by senior high school/*madrasah* and vocational high school teachers.

$$\begin{aligned} 2m + 15 &= 31 & 2m + 15 - 9 &= 31 - 9 \\ 2m &= 16 & 2m &= 22 - 6 \\ m &= 8 & 2m &= 16 \\ & & m &= 8 \end{aligned}$$

(a)



(b)

Figure 3: CT-1 and CT-2 Strategies in P3

RT Typology

Like CT, an RT typology is also classified into two approaches, RT-1 and RT-2. A response is categorized into the RT-1 if a focus is made on the relationship between numerical values or elements of two sides, the characteristics of number are made use of and then some computation or equation transformations are made so that an identical equation of the two equations is obtained. Meanwhile RT-2 is adopted if the numerical values or elements of the two sides are focused on, then characteristics of numbers are used, and then some computations or without explicit computations are made.

In problems P1 and P2. The solution using the RT-1 strategy is shown in Figure 4.

$$\begin{aligned} 57 + 86 &= n + 84 \\ 57 + 2 + 84 &= n + 84 \\ 59 + 84 &= n + 84 \\ 59 &= n \end{aligned}$$

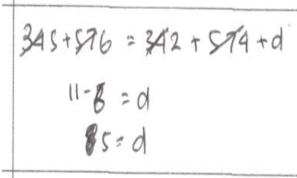


Figure 4: RT-1 Strategy in P1 and P2

The RT-1 strategy to obtain an n or d value is done using the characteristics of number and computation. In solving P1, the $86 = 2 + 84$ is described since at the right side there is number 84, so that the relationship between the two sides of the equation

is obtained. This strategy is adopted by 10% and 13.74% junior and senior high school teachers, respectively. Whereas in P1, hundreds and tens numbers of the two equation sides are crossed and then a computation is made. This strategy was adopted by 11.11% of Vocational high school teachers.

Dealing with the RT-2 strategy in solving P1 and P2, a special attention is paid to the relationship between numbers in the two sides of the equation, then an implicit computation is made, as presented in Figure 5. In P1, this strategy was adopted by 11.11% vocational high school teachers, while in P2, 12.64% senior high school/madrasah teachers.

Figure 5: RT-2 strategy in P1 and P2

The RT-1 and RT-2 strategies in solving P1 are each presented in Figure 6 (a) and (b). The RT-1 strategy is made by transforming a equation so that an identical equation is obtained. This was adopted by 0.09% senior high school/madrasah teachers, while the RT-2 strategy is done by paying attention to two sides of the second equation and the characteristics of the equation. This strategy was employed by 20% of junior high school teachers, 1.18% of junior high school/madrasah teachers, and 11.11% of vocational high school teachers.

(a)

(b)

Figure 6 6: RT-1 and RT-2 Strategies in P3

DISCUSSION

The CT typology adopted by mathematics teachers in solving equation problems possesses long steps and requires the mastery of computation. This strategy also needs a long time to obtain a correct answer. This typology, especially the CT-1 strategy most teachers adopt is a conventional strategy [21] and its thinking process is still procedural. Therefore, this strategy is also called a procedural one [4, 7]. The process

of problem solving focusing on computation and procedures makes the mathematical knowledge is mechanical in nature [11, 7].

In the RT typology, to build a strategy of problem solving, teachers do not focus on computation, but create a whole picture of a problem in their mind, make some analyses to find a core structure, and look for some important elements or relations among elements given. The process of this problem solving is called an analysis of expression [15] or relational thinking [5, 22, 23, 14]. The strategy of problem solving in the RT type has simple steps so that it needs a better concept of understanding, beside computational abilities. As a result, this type of strategy is also called a meta-conceptual strategy [7] that brings the doer to a high thinking level in problem solving than the CT type. RT-1 and CT-2 strategies possess a little bit similarity, but the difference is in the main focus of the computation. In CT-2, the focus is on the computation, while in RT-1 on the relation between two sides of the equation. The Thinking process in the RT type in problem solving tends to be divergent, while in the CT type, convergent.

In the dimension oriented to change in the problem solving style, problem solvers adopting the CT typology are included into types of “developer”, and those adopting the RT typology, an “explorer” type [20]. Those with the ‘developer’ type tend to choose a solution under their present experience and to be easily assimilate this present reality, but they are still under the existing paradigm or system and adhere rules or procedures given. While the problem solvers with the “explorer” type leave from the system, know new possibilities and tendencies, adopt in-conventional ways, and produce ideas which others are difficult to understand.

CONCLUSION

Problem solving is a base for all mathematical activities [19] and integral part of all mathematical learning [116]. Therefore, learning problem solving strategies should be paid attention, especially mathematics teachers. Most mathematics teachers still give an emphasis on procedural and computational aspects in solving problems (CT type) from all aspects of conceptual comprehension. One of strategies emphasizing conceptual and holistic aspects in problem solving is the relational and transformation (RT) type that should be taken into account as the materials in education and training for improving pedagogical and professional competences of mathematics teachers. Moreover, the RT type is more elegant than the CT type considered from the processes or stages in problem solving. The RT type enables to develop creativities and high thinking levels that are really needed by teachers in doing their jobs.

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