

A Comparative Study on ZSM and LCM in Fuzzy Transportation Problem

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Abstract

The transportation model is a special class of the linear programming problem. In real world problems, optimization techniques are useful for solving problems like, project schedules, assignment problems and network flow analysis. The objective is to minimize the transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Here, we are proposed a ranking technique for solving fuzzy transportation problem, where fuzzy demand and supply all are in the form of triangular fuzzy numbers. In this paper, a new method named ZSM-method for finding an optimal solution for a transportation problem and finally this method is compared to the LCM method with the help of numerical examples.

Keywords: Fuzzy Transportation Problem, Membership function, Robust Ranking, ZSM-(Zero Suffix Method), LCM - (Least Cost Method).

1. INTRODUCTION

Transportation problem is a special class of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values. But in real life applications supply, demand and unit transportation cost

may be uncertain due to several factors. These data may be represented by fuzzy numbers. The idea of fuzzy set was introduced by Zadeh in 1965.

A fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. The function is also considered as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. On the basis of this idea the Roubast Ranking method with the help of α solution has been adopted a transform the fuzzy transportation problem. The idea is to transform a problem with fuzzy parameters in the form of Linear programming problem and solve it by the ZSM (Zero Suffix Method) and LCM (Least Cost Method).

2. DEFINITIONS

FUZZY SET

A fuzzy set is characterized by a membership function mapping element of a domain, Space of the universe of discourse X to the unit interval $[0, 1]$ i.e. $A = \{(x, \mu_A(x)); x \in X\}$. Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A . These membership grades are often represented by real numbers ranging from $[0, 1]$.

TRIANGULAR FUZZY NUMBER

For a triangular fuzzy number $A(x)$, it can be represented by $A(a, b, c; 1)$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(x-b)} & x \leq a \leq b \\ 1 & x = b \\ \frac{(c-x)}{(c-d)} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

3. PROPOSED METHODOLOGY

Robust Ranking Technique

Robust ranking technique which satisfy compensation, linearity, and additively properties and provides results which are consist human intuition. If \tilde{a} is a fuzzy number then the Robust Ranking is defined by

$$R(\bar{a}) = \int_0^1 (0.5) (a_\alpha^L, a_\alpha^U) d\alpha$$

Where, (a_α^L, a_α^U) is the α level cut of the fuzzy number \tilde{a} In this paper we use this method for ranking the objective values. The Robust ranking index $R(\tilde{a})$ gives the representative value of fuzzy number \tilde{a} . Where, $(a_\alpha^L, a_\alpha^U) = \{(b-a) \alpha + a, c - (c-b)\alpha\}$

4. DIFFERENT METHODS TO FINDING OPTIMAL SOLUTION

Here we now introduce a new method called ZSM (Zero Suffix Method) for finding an optimal solution .This method is compared to the LCM (Least Cost Method). Zero suffix method algorithms also given below.

Zero Suffix Method

Step 1

Construct the transportation table.

Step 2

Subtract each row entries of the transportation table from the corresponding row minimum after that subtract each column entries of the transportation table from the corresponding column minimum.

Step 3

In the reduced cost matrix there will be the atleast one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S, Therefore

$$S = \{\text{Add the costs of nearest adjacent sides of zero's} | \text{No - of costs added}\}$$

Step 4

Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select $\{a_i, b_j\}$ and supply to that demand maximum possible.

Step 5

After the above step, the exhausted demands (column) or supplies (row) to be trimmed. The resultant matrix must possess at least one zero in each row and column, else repeat Step 2.

Step 6

Repeat Step 3 to Step 5 until the optimal solution is obtained.

Least Cost Method

The Least -cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out, the same as in the northwest – corner method .Next ,look for the uncrossed-out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out .

5. NUMERICAL EXAMPLES

The fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is C_{ij} . where,

$$[C_{ij}] = \begin{matrix} (-2,3,8) & (-2,3,8) & (-2,3,8) & (-1,1,4) \\ (4,9,16) & (4,8,12) & (2,5,8) & (1,4,7) \\ (2,7,13) & (0,5,10) & (0,5,10) & (4,8,12) \end{matrix}$$

Fuzzy availability of the product at source are ((0,3,6) (2,7,13) (2,5,8)) and the Fuzzy demand of the product at destinations are ((1,4,7) (0,3,5) (1,4,7) (2,4,8)) respectively.

Solution**Table I**

	D_1	D_2	D_3	D_4	Supply
S_1	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	(0,3,6)
S_2	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(2,7,13)
S_3	(2,7,13)	(0,5,10)	(0,5,10)	(4,8,12)	(2,5,8)
Demand	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

Using **Robust Ranking** Methodology,

$$\begin{aligned} \text{Min } Z = & R(-2,3,8)X_{11} + R(-2,3,8)X_{12} + R(-2,3,8)X_{13} + R(-1,1,4)X_{14} + \\ & R(4,9,16)X_{21} + \\ & R(4,8,12)X_{22} + R(2,5,8)X_{23} + R(1,4,7)X_{24} + R(2,7,13)X_{31} + \\ & R(0,5,10)X_{32} + \\ & R(0,5,10)X_{33} + R(4,8,12)X_{34} \end{aligned}$$

$$R(\bar{a}) = \int_0^1 (0.5) (a_\alpha^L, a_\alpha^U) d\alpha$$

Where, $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, c - (c-b)\alpha\}$

$$R(-2, 3, 8) = \{3 - (-2)\alpha - 2, 8 - (8-3)\alpha\} = \{5\alpha - 2, 8 - 5\alpha\} = 6 = \int_0^1 (0.5)(6) = 3$$

$$R(-2, 3, 8) = 3, R(-2, 3, 8) = 3, R(-2, 3, 8) = 3, R(-1, 1, 4) = 1.5;$$

$$R(4, 9, 16) = 10, R(4, 8, 12) = 8, R(2, 5, 8) = 5, R(1, 4, 7) = 4;$$

$$R(2, 7, 13) = 7.5, R(0,5,10) = 5, R(0,5,10) = 5, R(4,8,12) = 8.$$

Rank of all supply

$$R(0,3,6) = 3, R(2,7,13) = 7.5, R(2,5,8) = 5$$

Rank of all Demands

$$R(1,4,7) = 4, R(0,3,5) = 2.5, R(1,4,7) = 1.5, R(2,4,8) = 2.5$$

Table II

	D₁	D₂	D₃	D₄	Supply
S₁	3	3	3	1.5	3
S₂	10	8	5	4	7.5
S₃	7.5	5	5	8	5
Demand	4	2.5	4	5	

(i) Using Zero Suffix Method**Step 1**

Row reduced matrix.

1.5	1.5	1.5	0	3
6	4	1	0	7.5
2.5	0	0	3	5
4	2.5	4	5	

Step 2

Column reduced matrix.

0	1.5	1.5	0	3
4.5	4	1	0	7.5
1	0	0	3	5
4	2.5	4	5	

Final Table

3	3	3	1.5	3
10	8	5	4	7.5
7.5	5	5	8	5
4	2.5	4	5	

Total Transportation cost

$$\begin{aligned}
 Z &= (3 * 3) + (5 * 2.5) + (4 * 5) + (7.5 * 1) + (5 * 2.5) + (5 * 1.5) \\
 &= 9+12.5+20+7.5+12.5+7.5 \\
 Z &= 69.
 \end{aligned}$$

Table III

	D₁	D₂	D₃	D₄	Supply
S₁	(-2,3,8) (0,3,6)	(-2,3,8)	(-2,3,8)	(-1,1,4)	(0,3,6)
S₂	(4,9,16)	(4,8,12)	(2,5,8) (0,3,5)	(1,4,7) (2,4,8)	(2,7,13)
S₃	(2,7,13) (1,1,1)	(0,5,10) (0,3,5)	(0,5,10) (1,1,2)	(4,8,12)	(2,5,8)
Demand	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,7,27)

The total fuzzy optimal cost by applying the Robust’s ranking method. For the fuzzy transportation problem with fuzzy objective function min z= 69.

(ii) Using Least Cost Method

$\overset{?}{\boxed{0.5}}$	$\overset{?}{\boxed{2.5}}$	3	1.5	3
10	8	$\overset{5}{\boxed{2.5}}$	4 $\boxed{5}$	7.5
7 $\overset{5}{\boxed{3.5}}$	5	$\overset{5}{\boxed{1.5}}$	8	5
4	2.5	4	5	

Total Transportation cost

$$\begin{aligned}
 Z &= (3 * 0.5) + (3 * 2.5) + (5 * 2.5) + (4 * 5) + (7.5 * 3.5) + (5 * 1.5) \\
 &= 1.5+7.5+12.5+20+26.25+7.5 \\
 Z &= 75.25
 \end{aligned}$$

To finding the optimal solution for fuzzy transportation problem by using LCM (Least Cost Method) is **Rs. 75.25** and ZSM (Zero Suffix Method) is **Rs.69**.For comparing these two methods ZSM provides the minimum transportation cost. Thus the ZSM is optimal.

CONCLUSION

In this paper, fuzzy transportation problem has been transformed into crisp transportation problem by using Robust Ranking Method. We used ZSM and LCM to solve the fuzzy transportation problem get the optimal solution. Moreover, using ZSM method provides best optimal solution for the fuzzy transportation problem with effectively.

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