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Effects of Viscosity and Thermal Conductivity on Convective Flow of Two Viscous Immiscible Dusty and Pure Fluids in a Vertical Corrugated Wall and a Parallel Flat Wall

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Abstract

This paper presents the effects of viscosity and thermal conductivity on flow and heat transfer of two viscous immiscible dusty and pure fluids confined between a vertical corrugated wall and a parallel flat wall. The nonlinear partial differential equations governing the flow have been reduced to nonlinear ordinary differential equations using the regular perturbation method. The transformed nonlinear ordinary differential equations have been solved numerically using the linear approximation theorem. The effects of the governing parameters on the velocity and temperature fields for the two fluids and the dust particles have been obtained and graphically represented using Matlab.

Keywords: Immiscible fluids, Dusty fluid, Viscosity, Thermal conductivity, Perturbation method, corrugated wall.

1. INTRODUCTION

The flow and heat transfer of immiscible dusty fluids are applicable in areas such as petroleum extraction, purification of crude oil, chemical distillation processes, design of heat exchangers, petroleum reservoirs, filtration, nuclear waste treatment. Corrugated surfaces are, for example, utilized in compact heat exchangers and in industrial processes to enhance heat transfer efficiency. Abdul [1] investigated the transient Magneto hydrodynamic flow of two immiscible fluids through a horizontal channel. Champati and Ramana [3] analyzed the problem of laminar flow of two immiscible viscous liquids in a saturated porous medium through a rotating channel. Umavathi et al [4, 5] studied mixed convective flow of two immiscible fluids in a vertical channel and parallel flat wall with travelling thermal waves.

Mahadev [6] investigated on magneto convective flow and heat transfer of two immiscible fluids using a vertical wavy wall and a parallel flat wall. Flow and heat transfer of two immiscible fluids in the presence of uniform inclined magnetic field was investigated by Nikodijevic´ et al. [7]. Umavathi et al [8, 9] studied unsteady flow and heat transfer of three immiscible fluids. Vajravelu and Sastri [10] investigated free convective heat transfer in a viscous incompressible fluid between a vertical wavy wall and a parallel flat wall. Verma and Bhatt [11] considered the steady flow of two immiscible incompressible fluids with suction at the stationary plate.

Most recently, Attia et al [2] used a porous medium in a circular pipe to study unsteady dusty Bingham fluid flow. Abba et al [12] also used two parallel plates with heat transfer to investigate Couette flow of two immiscible dusty fluids. All the above cited references except Abba et al [12], investigated on dusty fluids and pure immiscible fluids through different channels but none studied flow and heat transfer of two viscous immiscible dusty and pure fluids between a corrugated wall and a parallel flat wall. Thus, the objective of the present work is to study the effects of viscosity and thermal conductivity on convective flow of two viscous immiscible dusty and pure fluids between a vertical corrugated wall and a parallel flat wall. The flow is taken to be steady, two dimensional and the fluid is liquid and not gas, incompressible and electrically non-conducting. The governing nonlinear equations for the dusty and pure fluids are solved numerically by Perturbation Method with linear approximation theorem.

2. MATHEMATICAL FORMULATION

A two dimensional steady laminar flow of two electrically non-conducting immiscible dusty and pure fluids in a vertical channel with one wavy wall and another flat wall is considered as shown in figure 1. The X- axis which represented by $Y = -w^{(2)} + \eta \cos\left(\lambda X\right)$ is taken parallel to the flat wall, while the Y- axis represented by $Y = w^{(1)}$ is taken to be perpendicular. The wavy and flat walls are maintained at constant temperatures $\tilde{T}_2$ and $\tilde{T}_1$ respectively. Region I is occupied by a fluid of density $\rho^{(1)}$, viscosity $\mu^{(1)}$, thermal conductivity $k^{(1)}$, thermal expansion coefficient $\beta^{(1)}$, specific heat at constant pressure $c_p^{(1)}$ and Region II is occupied by the fluid of density $\rho^{(2)}$, viscosity $\mu^{(2)}$, thermal conductivity $k^{(2)}$, thermal expansion coefficient $\beta^{(2)}$, specific heat at constant pressure $c_p^{(2)}$.

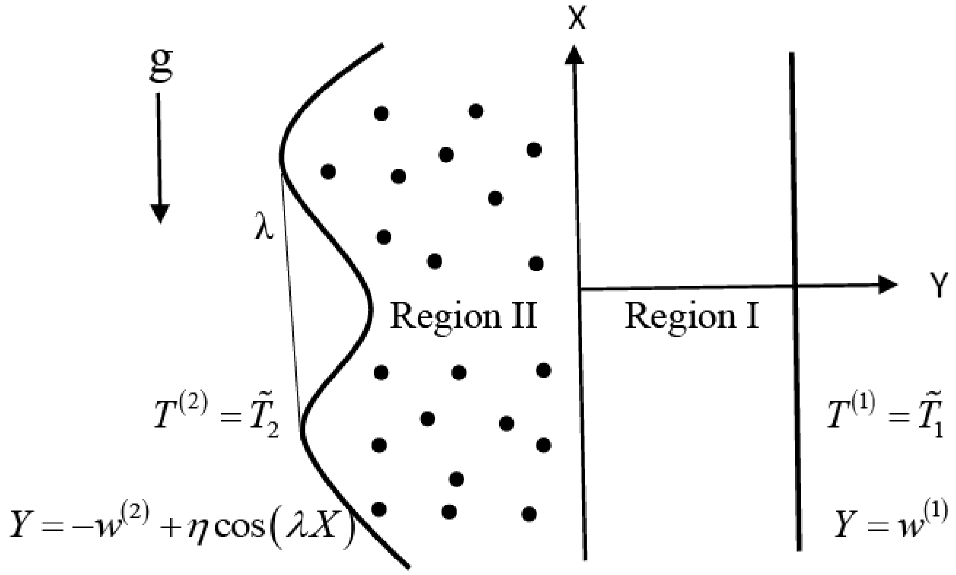

Figure 1: Physical configuration

The following assumptions are considered in this study. The fluid in region-II is dusty while the fluid in region-I is considered to be a pure fluid. Except the density in the buoyancy term in the momentum equation, all fluid properties are assumed constant. The transport properties of both fluids are assumed to be constant and the fluid rises in the channel driven by buoyancy forces. The dust particles in region II are assumed to be electrically non-conducting, spherical in shape, solid, same radius and mass (uniform in size), un-deformable, and uniformly distributed throughout the flow. This means that, by conduction through their spherical surface, the dust particles gain heat energy from the fluid. The number density N of the particles is constant throughout the flow and volume fraction of the dust particles is neglected and the temperature between the particles is uniform throughout the motion. The concentration of particles is very small that it is not interfering with the continuity and the net effect of the dust on the fluid particles is equivalent to $FN\left(u^{(2)} - u^{p}\right)$ per unit volume. Where F is stoke's law (drag force) where $F = 6\pi\mu r v$ and r is average radius of the dust particles, μ is coefficient of fluid viscosity (dynamic viscosity, v is flow velocity relative to the object and N is density number of particles per unit volume of the fluid.

3. GOVERNING EQUATIONS

Region I (Pure fluid)

$$\frac{\partial u^{(1)}}{\partial x^{(1)}}+\frac{\partial v^{(1)}}{\partial y^{(1)}}=0 \qquad \text{Continuity} \qquad (1)$$

$$\rho^{(1)}\left(u^{(1)}\frac{\partial u^{(1)}}{\partial x^{(1)}}+v^{(1)}\frac{\partial u^{(1)}}{\partial y^{(1)}}\right) =-\frac{\partial p^{(1)}}{\partial x^{(1)}}+\mu^{(1)}\nabla^2 u^{(1)}+\rho^{(1)}g\beta^{(1)}\left(T^{(1)}-T_s\right) \qquad \text{X-Momentum} \qquad (2)$$

$$\rho^{(1)}\left(u^{(1)}\frac{\partial v^{(1)}}{\partial x^{(1)}}+v^{(1)}\frac{\partial v^{(1)}}{\partial y^{(1)}}\right)=-\frac{\partial p^{(1)}}{\partial y^{(1)}}+\mu^{(1)}\nabla^2 v^{(1)} \qquad \text{Y-Momentum} \qquad (3)$$

$$\rho^{(1)}c_p^{(1)}\left(u^{(1)}\frac{\partial T^{(1)}}{\partial x^{(1)}}+v^{(1)}\frac{\partial T^{(1)}}{\partial y^{(1)}}\right)=k^{(1)}\nabla^2 T^{(1)} \qquad \text{Energy} \qquad (4)$$

Region II (Dusty fluid)

$$\frac{\partial u^{(2)}}{\partial x^{(2)}}+\frac{\partial v^{(2)}}{\partial y^{(2)}}=0 \qquad \text{Continuity} \qquad (5)$$

$$\rho^{(2)}\left(u^{(2)}\frac{\partial u^{(2)}}{\partial x^{(2)}}+v^{(2)}\frac{\partial u^{(2)}}{\partial y^{(2)}}\right) =-\frac{\partial p^{(2)}}{\partial x^{(2)}}+\mu^{(2)}\nabla^2 u^{(2)}+\rho^{(2)}g\beta^{(2)}\left(T^{(2)}-T_s\right)-FN\left(u^{(2)}-u^{p}\right) \qquad \text{X-Momentum} \qquad (6)$$

$$\rho^{(2)}\left(u^{(2)}\frac{\partial v^{(2)}}{\partial x^{(2)}}+v^{(2)}\frac{\partial v^{(2)}}{\partial y^{(2)}}\right)=-\frac{\partial p^{(2)}}{\partial x^{(2)}}+\mu^{(2)}\nabla^2 v^{(2)}-FN\left(v^{(2)}-v^{p}\right) \qquad \text{Y-Momentum} \qquad (7)$$

The equation of motion of the dust particles by taking Newton's second law in the X direction is given by

$$m_p\left(u^{p}\frac{\partial u^{p}}{\partial x}+\frac{\partial u^{p}}{\partial y}\right)=FN\left(u^{(2)}-u^{p}\right) \qquad (8)$$

m_p is average mass of dust particles.

$$\rho^{(2)}c_p^{(2)}\left(u^{(2)}\frac{\partial T^{(2)}}{\partial x^{(2)}}+v^{(2)}\frac{\partial T^{(2)}}{\partial y^{(2)}}\right)=k^{(2)}\nabla^2 T^{(2)}+\frac{\rho^p c_s}{\gamma_t}\left(T^p-T^{(2)}\right)$$ Energy equation of the fluid (9)

$$u^{(2)}\frac{\partial T^p}{\partial x^{(2)}}+v^{(2)}\frac{\partial T^p}{\partial y^{(2)}}=\frac{-1}{\gamma_t}\left(T^p-T^{(2)}\right)$$ Energy equation of the particles (10)

For both the velocity and temperature, the relevant boundary and interface conditions used to solve

Eqns. (1) to (10) are

$u^{(2)}=v^{(2)}=u^P=v^P=0$ at $Y=-w^{(2)}+\eta\cos(\lambda X)$, $u^{(1)}=v^{(1)}=0$ at $Y=w^{(1)}$,

$u^{(1)}=u^{(2)}=u^p$,

$v^{(1)}=v^{(2)}=v^p$ at $Y=0$, $\mu^{(1)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(1)}=\mu^{(2)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(2)}=\mu^{(2)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{p}$ at

$Y=0$

$\frac{\partial p^{(1)}}{\partial x^{(1)}}=\frac{\partial p^{(2)}}{\partial x^{(2)}}$ at $Y=0$, $T^{(2)}=\tilde{T}_2$ at $Y=-w^{(2)}+\eta\cos(\lambda X)$, $T^{(1)}=\tilde{T}_1$ at

$Y=w^{(1)}$

$T^{(1)}=T^{(2)}=T^p$; $k^{(1)}\left(\frac{\partial T}{\partial y}+\frac{\partial T}{\partial x}\right)^{(1)}=k^{(2)}\left(\frac{\partial T}{\partial y}+\frac{\partial T}{\partial x}\right)^{(2)}$ at $Y=0$

The non-dimensional flow variables are:

$$\hat{x}^{(1)}=\frac{x^{(1)}}{w^{(1)}},\ \hat{y}^{(1)}=\frac{y^{(1)}}{w^{(1)}},\ \hat{x}^{(2)}=\frac{x^{(2)}}{w^{(2)}},\ \hat{y}^{(2)}=\frac{y^{(2)}}{w^{(2)}},\ \hat{u}^{(1)}=\frac{w^{(1)}}{v^{(1)}}u^{(1)},\ \hat{v}^{(1)}=\frac{w^{(1)}}{v^{(1)}}v^{(1)},$$

$$\hat{u}^{(2)} = \frac{w^{(2)}}{v^{(2)}}u^{(2)},\ \hat{v}^{(2)} = \frac{w^{(2)}}{v^{(2)}}v^{(2)}\ ,\ Gr = \frac{w^{(1)^3} g\beta^{(1)}\Delta T}{v^{(1)^2}},\ \ \Delta T = T_2 - T_s\ ,\ \ \Pr = \frac{c_p^{\ (1)}\mu^{(1)}}{k^{(1)}}\ ,\ \hat{\lambda} = \lambda w^{(2)}$$

$$\hat{p}^{(1)} = \frac{p^{(1)}}{\rho^{(1)}\left(v^{(1)}/w^{(1)}\right)^2},\ \ \hat{p}^{(2)} = \frac{p^{(2)}}{\rho^{(2)}\left(v^{(2)}/w^{(2)}\right)^2},$$

$$\theta^{(1)} = \frac{T^{(1)} - T_s}{T_2 - T_s}, \theta^{(2)} = \frac{T^{(2)} - T_s}{T_2 - T_s}\ , \theta^p = \frac{T^p - T_s}{T_2 - T_s}, T_0 = \frac{T_1 - T_s}{T_2 - T_s}$$

$$\beta_0 = \frac{\beta^{(2)}}{\beta^{(1)}},\ w_0 = \frac{w^{(2)}}{w^{(1)}},\ \mu_0 = \frac{\mu^{(1)}}{\mu^{(2)}},\ \rho_0 = \frac{\rho^{(2)}}{\rho^{(1)}},\ k_0 = \frac{k^{(2)}}{k^{(1)}},\ c_p = \frac{c_p^{(1)}}{c_p^{(2)}},$$

$$v^{(1)} = \frac{\mu^{(1)}}{\rho^{(1)}}\ ,\ v^{(2)} = \frac{\mu^{(2)}}{\rho^{(2)}}$$

The non-dimensional variables are substituted in to Eqns. (3.1) to (3.10) and dropping the (caps) for simplicity, the equations obtained are as follows

Region I (Pure fluid)

$$\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y} = 0 \tag{11}$$

$$u^{(1)}\frac{\partial u^{(1)}}{\partial x} + v^{(1)}\frac{\partial u^{(1)}}{\partial y} = -\frac{\partial p^{(1)}}{\partial x} + \frac{\partial^2 u^{(1)}}{\partial x^2} + \frac{\partial^2 u^{(1)}}{\partial y^2} + Gr\theta^{(1)} \tag{12}$$

$$u^{(1)}\frac{\partial v^{(1)}}{\partial x} + v^{(1)}\frac{\partial v^{(1)}}{\partial y} = -\frac{\partial p^{(1)}}{\partial y} + \frac{\partial^2 v^{(1)}}{\partial x^2} + \frac{\partial^2 v^{(1)}}{\partial y^2} \tag{13}$$

$$u^{(1)}\frac{\partial \theta^{(1)}}{\partial x} + v^{(1)}\frac{\partial \theta^{(1)}}{\partial y} = \frac{1}{\Pr}\left(\frac{\partial^2 \theta^{(1)}}{\partial x^2} + \frac{\partial^2 \theta^{(1)}}{\partial y^2}\right) \tag{14}$$

Region II (Dusty fluid)

$$\frac{\partial u^{(2)}}{\partial x} + \frac{\partial v^{(2)}}{\partial y} = 0 \tag{15}$$

$$u^{(2)}\frac{\partial u^{(2)}}{\partial x}+v^{(2)}\frac{\partial u^{(2)}}{\partial y}=-\frac{\partial p^{(2)}}{\partial x}+\frac{\partial^2 u^{(2)}}{\partial x^2}+\frac{\partial^2 u^{(2)}}{\partial y^2}+Gr\beta_0 w_0^3\mu_0^2\rho_0^2\theta^{(2)}-R_0\left(u^{(2)}-u^p\right) \quad (16)$$

$$u^{(2)}\frac{\partial v^{(2)}}{\partial x}+v^{(2)}\frac{\partial v^{(2)}}{\partial y}=-\frac{\partial p^{(2)}}{\partial y}+\frac{\partial^2 v^{(2)}}{\partial x^2}+\frac{\partial^2 v^{(2)}}{\partial y^2}-R_0\left(v^{(2)}-v^p\right) \quad (17)$$

$$u^p\frac{\partial u^p}{\partial x}+v^p\frac{\partial u^p}{\partial y}=\frac{1}{G_0}\left(u^{(2)}-u^p\right) \quad (18)$$

$$u^{(2)}\frac{\partial \theta^{(2)}}{\partial x}+v^{(2)}\frac{\partial \theta^{(2)}}{\partial y}=\frac{k_0\mu_0 c_p}{\mathrm{Pr}}\left(\frac{\partial^2\theta^{(2)}}{\partial x^2}+\frac{\partial^2\theta^{(2)}}{\partial y^2}\right)+\frac{2R_0}{3\mathrm{Pr}}\left(\theta^p-\theta^{(2)}\right) \quad (19)$$

$$u^p\frac{\partial \theta^p}{\partial x}+v^p\frac{\partial \theta^p}{\partial y}=-L\left(\theta^{(2)}-\theta^p\right) \quad (20)$$

The boundary and interface conditions are non - dimensionalized as follows

$$u^{(2)}=v^{(2)}=u^p=v^p \quad \text{at} \quad y=-1+\eta\cos(\lambda x),\ u^{(1)}=v^{(1)}=0 \ \text{at} \ y=1$$

$$u^{(1)}=\frac{u^{(2)}}{\mu_0 w_0\rho_0}=\frac{u^p}{\mu_0 w_0\rho_0},\ v^{(1)}=\frac{v^{(2)}}{\mu_0 w_0\rho_0}=\frac{v^p}{\mu_0 w_0\rho_0} \quad \text{at } y=0$$

$$\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(1)}=\frac{1}{w_0^2\mu_0^2\rho_0}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{(2)}=\frac{1}{w_0^2\mu_0^2\rho_0}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^{p} \quad \text{at } y=0$$

$$\frac{\partial p^{(1)}}{\partial x}=\frac{1}{\rho_0\mu_0^2 w_0^2}\frac{\partial p^{(2)}}{\partial x} \ \text{at} \ y=0 \ ,\ \theta^{(2)}=1 \ \text{at} \ y=-1+\eta\cos(\lambda x)\ ,\quad \theta^{(1)}=T \ \text{at} \ y=1$$

$$\theta^{(1)}=\theta^{(2)}=\theta^p,\ \left(\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial x}\right)^{(1)}=\frac{k_0}{w_0}\left(\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial x}\right)^{(2)}=\frac{k_0}{w_0}\left(\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial x}\right)^{p} \ \text{at} \ y=0$$

4. RESULTS AND DISCUSSIONS

The partial differential Equations (11-20) are reduced into non-linear ordinary differential equations. The obtained ordinary differential equations are solved numerically using Perturbation method and the linear approximation theorem. In each graph, Grashof number, viscosity ratio, width ratio and conductivity ratio are fixed at 6, 3, 3, and 3 respectively except the temperature ratio, T_0 and the parameter in question. The temperature ratio is increasing from -2 to 2 in all the graphs.

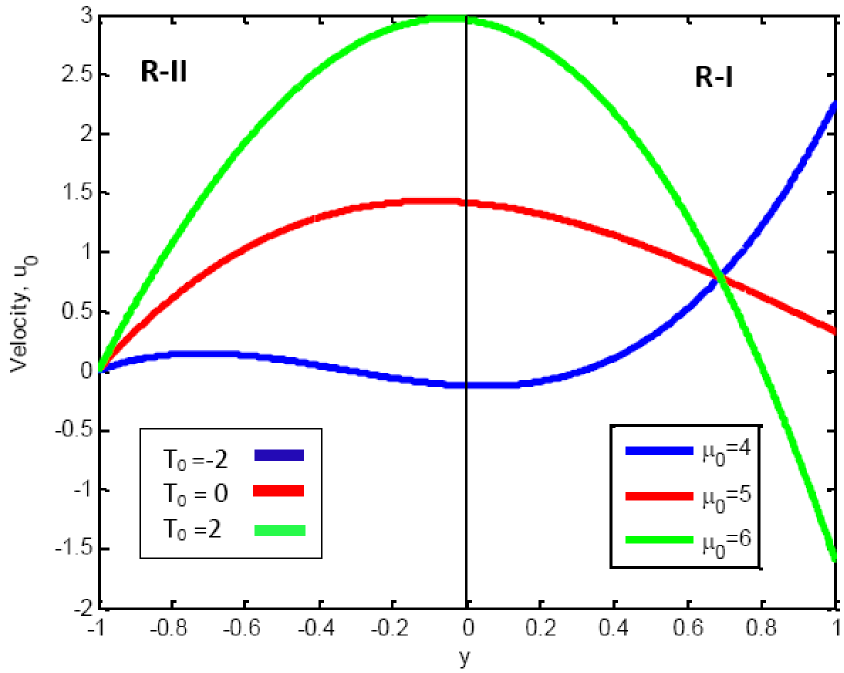

Figure 2(a): Effect of viscosity ratio on the velocity profiles.

From Figure 2(a), it is observed that, as the viscosity ratio and temperature ratio increases, the zeroth order velocity increases in both regions. For $\mu_0 = 4$, the velocity decreases in region II and increases significantly in region I. As temperature increases, physically this means that, the fluid becomes less viscous and hence increases the velocity.

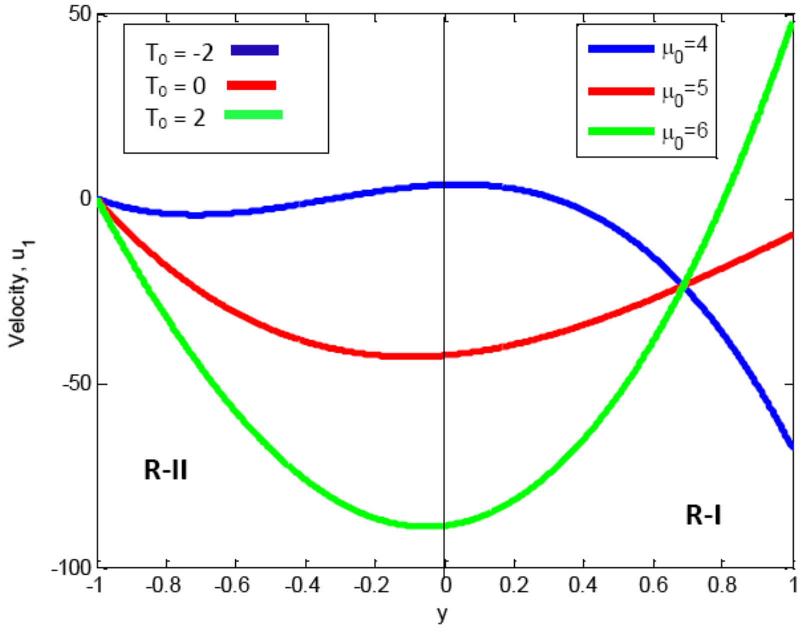

Figure 2(b): Effect of viscosity ratio on the velocity profiles.

From Figure 2(b), it is observed that, as the viscosity ratio and temperature ratio increases, the first order velocity decreases in both regions. For $\mu_0 = 4$, the velocity increases in region II and decreases significantly in region I.

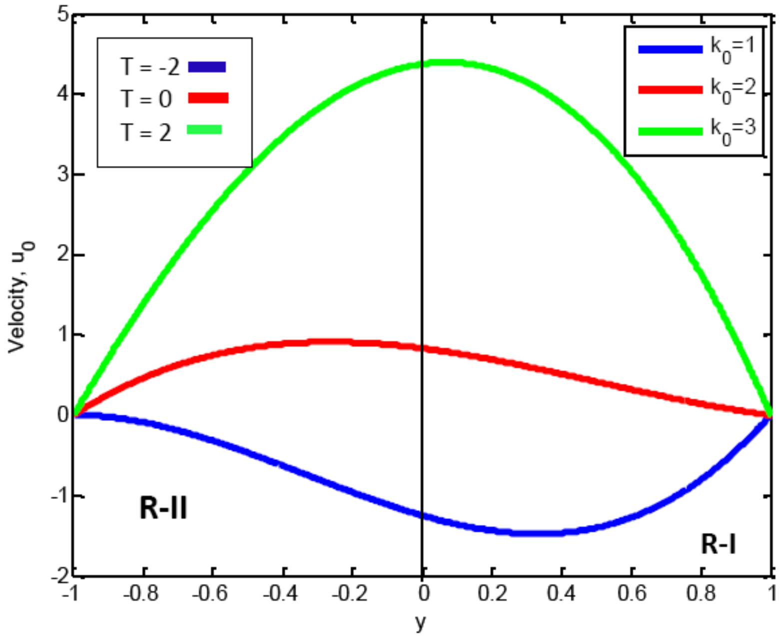

Figure 3(a): Effect of conductivity ratio on the velocity profiles.

From figure 3(a), it is observed that, the zeroth order velocity increases in both regions as the conductivity ratio and temperature ratio increases. For $T_0 = -2$, the velocity decreases significantly from region II up to the middle of region I and then increases towards the end of region I. Physically, velocity increases with increase in conductivity ratio.

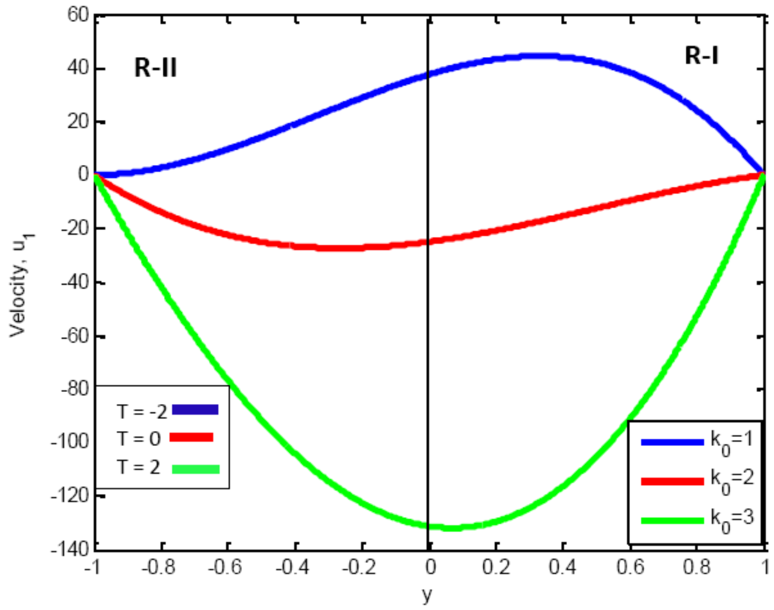

Figure 3(b): Effect of conductivity ratio on the velocity profiles.

From figure 3(b), it is observed that, the first order velocity decreases in both regions as the conductivity ratio and temperature ratio increases. For $T_0 = -2$, the velocity increases significantly from region II up to the middle of region I and then decreases towards the end of region I.

CONCLUSIONS

As the viscosity ratio and temperature ratio increases, the zeroth order velocity increases in both regions as the first order velocity decreases in both regions. As the conductivity ratio and temperature ratio increases, the zeroth order temperature increases in both regions as the first order velocity decreases in both regions.

REFERENCES

[1] Abdul Mateen, (2014): Transient Magneto hydrodynamic flow of two immiscible Fluids through a horizontal channel. *International Journal of Engineering Research,* **3(1):** 13-17.

[2] Attia, H.A., W. Abbas, A.L. Aboul-Hassan, M.A.M. Abdeen, M.A. Ibrahim,

(2016): Unsteady Flow of a Dusty Bingham Fluid through a Porous Medium in a Circular Pipe. *Journal of Applied Mechanics and Technical Physics,* 57(4): 596-602.

[3] Champati, S.L. and V.V. Ramana Rao, (2013): Laminar flow of two immiscible viscous liquids in a saturated porous medium through a rotating straight. *Annals of faculty engineering Hunedoara, International Journal of Engineering,* ISSN 1584-2673.

[4] J.C Umavathi, M. Shekar., (2011): Mixed convective flow of two immiscible viscous fluids in a vertical wavy channel with traveling thermal waves. *Int. J. Heat Mass Transfer*, Vol. 40, pp. 721-743.

[5] J.C Umavathi, J.P Kumar., and M. Shekar., (2010): Mixed convective flow of immiscible viscous fluids confined between a long vertical wavy wall and a parallel flat wall. *International Journal of Engineering, Science and Technology,* Vol. 2, No. 6, pp. 256-277.

[6] Mahadev M. Biradar., (2014): “Magneto convective flow and heat transfer of two immiscible fluids between vertical wavy wall and a parallel flat wall”, *Int. J. of Research in Engineering and Technology* , vol.3, Special Issue 3, pp. 777-790

[7] Nikodijevic, D., Zivojin Stamenkovic´, D. Milenkovic´, B. Blagojevi C. R. Savic and J. Nikodijevic, (2011): Flow and Heat Transfer of Two Immiscible Fluids in the Presence of Uniform Inclined Magnetic Field. *Hindawi Publishing Corporation, Mathematical Problems in Engineering,* Article ID 132302, pp.18.

[8] Umavathi, J.C., Liu, I.C. and Shaik Meera, D., (2008): Unsteady flow and heat transfer of three immiscible fluids, *Int. J. Appl.Mech. Eng.*, Vol. 13, pp. 1079-1100.

[9] Umavathi, J.C., Manjula, M.H., Pop, I and Liu, I. C., (2007): Flow and heat transfer of couple stress and viscous fluid in a vertical channel, *Int. J. Appl. Mech. Eng.*, Vol. 12, pp. 537-555.

[10] Vajravelu, K. and Sastri, K. S., (1978): Free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, *J. Fluid Mech.* Vol. 86, pp. 365-383.

[11] Verma, P.D. and B.S. Bhatt, (1973): Plane Couette flow of two immiscible incompressible fluids with suction at the stationary plate. *Proc. Indian Acad. Sci*., 78: 108.

[12] W. Abbas, S. M. El. Shinnawy, M. S.Emam and Mostafa A. M. Abdeen., (2017): Couette flow of two immiscible dusty fluids between two parallel plates with heat transfer. *Int. J. Heat Mass Transfer,* Vol. 06, pp. 334-343.