Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 15, Number 5 (2019), pp. 743-754

http://www.ripublication.com/gjpam.htm

On the Convex hull of γ_{vct}-sets in graphs

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Abstract

Let $G = (V, E)$ be an undirected, simple and connected graph. The *convex hull number of G with respect to γ_{vct}-sets*, denoted by $CH_{\gamma_{vct}}(G)$ is defined in [10]. The convex γ_{vct}-sets and the *hull γ_{vct}-sets* are also defined in [10]. In this paper, we study the convex hull of γ_{vct}-sets and hence the convex hull number with respect to γ_{vct}-sets for some standard regular graphs and corona graph.

Keywords: convex hull of γ_{vct}, convex hull number of G with respect to γ_{vct}-sets, convex γ_{vct}-sets, hull γ_{vct}-sets

1. INTRODUCTION

Hamid [5] introduced Independent transversal domination in graphs. Based on this concept, vertex covering transversal domination in graphs was introduced by Vasanthi and Subramanian [6]. The vertex covering transversal domination number of some standard graphs such as K_n, $K_{m,n}$, P_n, C_n, W_n and trees are dealt with in [6]. Also, the vertex covering transversal domination number of some regular graphs are analysed in [7]. The vertex covering transversal domatic number of graphs has been introduced and studied in [8]. Further studies on vertex covering transversal domination number and the vertex covering transversal dominating sets are carried out in [9]. Referring [1], we tried to integrate the concepts of convex sets and γ_{vct}-sets in graphs. As a consequence, we defined convex hull of γ_{vct}-sets, convex hull number with respect to γ_{vct}-sets, convex γ_{vct}-sets and hull γ_{vct}-sets. Here we try to analyse the convex hull of γ_{vct}-sets and hence the convex hull number with respect to γ_{vct}-sets in some standard regular graphs and corona graph.

A simple graph $G = (V, E)$ is said to be r-regular if each vertex of G is of degree r. Given a connected graph G and u, v are two vertices of G, the distance between u and v is the length of a shortest path between u and v, we denote it by $d(u, v)$. For a vertex v of a connected graph G, *the eccentricity e(v)* is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of G is called the *radius* and the maximum eccentricity is called the *diameter of* G and are denoted by $rad\ G$ and $diam\ G$ respectively. Two vertices u and v of a connected graph G are said to be antipodal if $d(u, v)$ = diam G. A set $I \subseteq V$ of vertices in G is called an independent set if no two vertices in I are adjacent. Also I is said to be a *maximum independent set* if there is no other independent set I' such that $|I'| > |I|$. The cardinality of a maximum independent set is called the *independence number* and is denoted by β_0(G). A set $C \subseteq V$ of vertices in G is called a *vertex covering set* (or simply *covering set*) if every edge of G is incident with at least one vertex in C. Also C is said to be a *minimum vertex covering set* if there is no other vertex covering set C' such that $|C'| < |C|$. The cardinality of a minimum vertex covering set is called the *vertex covering number* and is denoted by α_0(G). A set $D \subseteq V$ of vertices in a simple connected graph G is called a *dominating set* if every vertex in $V - D$ is adjacent to a vertex in D. A dominating set which intersects every minimum vertex covering set in G is called a *vertex covering transversal dominating set*. The minimum cardinality of a vertex covering transversal dominating set is called *vertex covering transversal domination number* of G and is denoted by γ_{vct}(G). A dominating set of minimum cardinality is denoted by γ-set and a vertex covering transversal dominating set of minimum cardinality is denoted by γ_{vct}-set. A shortest path between u and v is called *a u-v geodesic*. A set $C \subseteq V$ of vertices in G is called a *convex set* if $I(C)$ = C where $I(C)$ is the set of all vertices in the $u - v$ geodesic path of G for all u, $v \in C$. For any set $C \subseteq V$, the *convex hull of C* denoted by [C] is defined as the smallest convex subset of $V(G)$ containing C. For other graph theoretic terminologies, refer to [2], [3] and [4]. The following theorems proved in [9] are used in this paper.

Theorem 1.1. *Let G be a simple connected graph on n vertices.*
(i) If at least one γ_{vct}-set of G is convex, then $CH_{\gamma_{vct}}$(G) = γ_{vct}(G).
(ii) If all the γ_{vct}-sets of G are hull, then $CH_{\gamma_{vct}}$(G) = n.
(iii) If no γ_{vct}-set of G is convex or hull, then $\gamma_{vct}(G) < CH_{\gamma_{vct}}(G) < n$.

Theorem 1.2. *Let G be a simple connected graph and let S be a γ_{vct}-set of G. If $\langle S \rangle$ = K_2, then S is a convex γ_{vct}-set of G and $CH_{\gamma_{vct}}$(G) = 2.*

2. DEFINITIONS

Using the concepts of convex sets and the convex hull of a set in graphs, the convex hull of a γ_{vct}-set and the convex hull number with respect to γ_{vct}-sets in a graph are defined in [9]. Convex γ_{vct}-sets and hull γ_{vct}-sets are also defined accordingly.

Definition 2.1. *Let G = (V, E) be an undirected, simple and connected graph. Let S $\subseteq$ V be a minimum vertex covering transversal dominating set viz. a γ_{vct}-set. Then the convex hull of S is defined as the smallest convex set containing S and is denoted by [S].*

Definition 2.2. *The convex hull number of G with respect to γ_{vct}-sets, denoted by $CH_{\gamma_{vct}}$(G) is defined as $CH_{\gamma_{vct}}$(G) = min.$\{|C|$: C = [S] is the convex hull of γ_{vct}-set S$\}$ where the minimum is taken over all the γ_{vct}-sets of G.*

Definition 2.3. *If [S] = S, then S is called a convex γ_{vct}-set.*

Definition 2.4. *If [S] = $V(G)$, then S is called a hull γ_{vct}-set.*

Remark 2.5. *If S is a γ_{vct}-set of G, it is obvious that [S] $\supseteq$ S and so $CH_{\gamma_{vct}}$(G) $\geq$ γ_{vct}(G).*

3. CONVEX HULL OF γ_{vct}-SETS IN CUBIC REGULAR GRAPHS

In this section, the convex hull of γ_{vct}-sets and hence the convex hull number with respect to γ_{vct}-sets of some standard cubic regular graphs such as a particular Harary graph $H_{3,6}$, Peterson graph and the triangular prism graph are analysed.

Example 3.1. *$CH_{\gamma_{vct}}$($H_{3,6}$) = 2.*

By the definition of $H_{3,n}$, every vertex $v_i \in H_{3,n}$ is adjacent to the vertices v_{i+1}, v_{i-1} and v_{i+k} where $n = 2k$. Let $V(H_{3,6})$ = $\{v_0, v_1, v_2, v_3, v_4, v_5\}$.

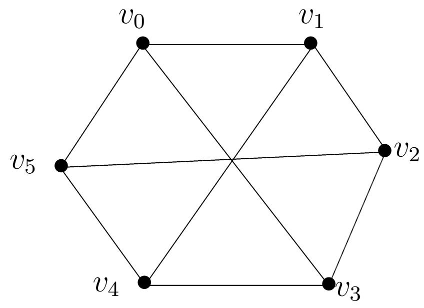

Figure 1: Harary graph $H_{3,6}$

Then $H_{3,6}$ is represented as shown in Figure 1. Obviously $S_1 = \{v_0, v_3\}$, $S_2 = \{v_1, v_4\}$ and $S_3 = \{v_2, v_5\}$ are γ_{vct}-sets of $H_{3,6}$. Also $\langle S_i \rangle = K_2$. Then by theorem 1.2, each S_i is a convex γ_{vct}-set of $H_{3,6}$ and $CH_{\gamma_{vct}}(H_{3,6}) = 2$.

Example 3.2. *Consider Peterson graph shown in Figure 2.*

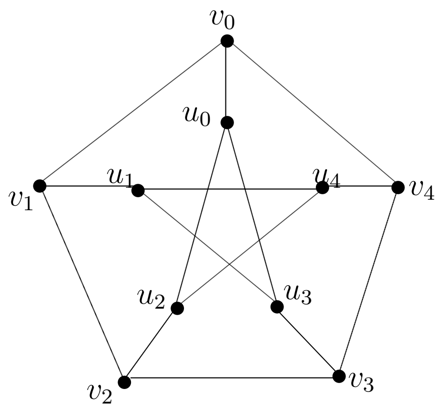

Figure 2: Peterson graph

Assuming that the graph G is labeled as shown in Figure 2, it is obvious that $I_i = \{v_{imod5}, v_{(i+3)mod5}, u_{(i+1)mod5}, u_{(i+2)mod5}\}$, $i = 0, 1, 2, 3, 4$ are β_0- sets of G. Then their complement $C_i = \{v_{(i+1)mod5}, v_{(i+2)mod5}, v_{(i+4)mod5}, u_{imod5}, u_{(i+3)mod5}, u_{(i+4)mod5}\}$, $i = 0, 1, 2, 3, 4$ are α_0-sets of G. Now $S_{i1} = \{v_{imod5}, u_{(i+2)mod5}, u_{(i+3)mod5}\}$, $i = 0, 1, 2, 3, 4$ are γ-sets intersecting each C_i and so are γ_{vct}-sets of G. Then the convex hull of S_{i1} for $i = 0, 1, 2, 3, 4$ is $[S_{i1}] = \{v_{imod5}, u_{imod5}, u_{(i+2)mod5}, u_{(i+3)mod5}\}$.

$S_{i2} = \{v_{imod5}, v_{(i+3)mod5}, u_{(i+4)mod5}\}$, $i = 0, 1, 2, 3, 4$ are also γ_{vct}-sets in G. Then $[S_{i2}]$ $= \{v_{imod5}, v_{(i+3)mod5}, v_{(i+4)mod5}, u_{(i+4)mod5}\}$, $i = 0, 1, 2, 3, 4$.
Hence $CH_{\gamma_{vct}}$(G) = 4.

Example 3.3. *Consider the triangular prism graph Y_3 shown in Figure 3. It is a cubic graph.*

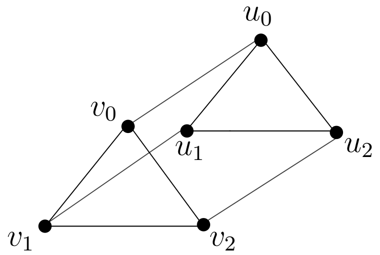

Figure 3: Triangular prism graph

Y_3 has 6 vertices and 9 edges. Assume that the graph Y_3 is labelled as shown in the diagram. It is clear that $\{u_{imod3}, v_{(i+1)mod3}\}$ and $\{u_{imod3}, v_{(i+2)mod3}\}$ for i = 0, 1, 2 are β_0-sets of Y_3. Then their complements $C_i = \{u_{(i+1)mod3}, u_{(i+2)mod3}, v_{imod3}, v_{(i+2)mod3}\}$ and $S_i = \{u_{(i+1)mod3}, u_{(i+2)mod3}, v_{imod3}, v_{(i+1)mod3}\}$ for i = 0, 1, 2 are α_0-sets of Y_3. Now each $D_i = \{u_{imod3}, v_{imod3}\}$, i = 0, 1, 2 is a γ-set for Y_3. Clearly it intersects C_i and S_i for each i = 0, 1, 2. Therefore each D_i is a γ_{vct}-set. Also the induced subgraph of each D_i is K_2. Hence by theorem 1.2, $CH_{\gamma_{vct}}(Y_3)$= 2.

4. CONVEX HULL OF γ_{vct}-SETS IN REGULAR GRAPHS

In this section, the convex hull of γ_{vct}-sets and hence the convex hull number with respect to γ_{vct}-sets of some standard regular graphs such as complete graph, particular hypercube and cycles of length 3, 4, 5 and 6 are provided. Also the convex hull number of a particular connected regular graph is also obtained.

Theorem 4.1. *If K_n, a complete graph on n vertices, then $CH_{\gamma_{vct}}(K_n)$ = 2.*

Proof: Let $V = \{v_1, v_2, v_3, v_4, ..., v_n\}$ be the vertex set of K_n.

Any two-element set $S_{i,j} = \{v_i, v_j\}$ of K_n is a γ_{vct}-set for all i, j = 1, 2, 3, ..., n, $i \neq j$. Since $\langle S_{i,j}\rangle = K_2$, $S_{i,j}$ is a convex γ_{vct}-set for every i, j = 1, 2, 3, ..., n, $i \neq j$ by theorem 1.2. Therefore $CH_{\gamma_{vct}}(K_n)$ = 2.

■

Definition 4.2. *The n-dimensional cube or hypercube Q_n is defined as follows. Q_n contains 2^n vertices. Each vertex in Q_n is represented by a n-tuple with 0's and 1's. Two vertices in Q_n are adjacent iff the n-tuples differ in exactly one position.*

Remark 4.3. *By the definition of Q_n, each vertex is adjacent to exactly n vertices which differ in exactly one position with it. So Q_n is n-regular.*

Remark 4.4. *Any vertex $v \in Q_n$ is the n-tuple binary number and its complement v^c is the n-tuple binary number obtained by replacing 0 by 1 and 1 by 0 in v. The weight of a 0,1 vertex is the number of 1's occuring in it. There are exactly 2^{n-1} vertices of odd weight and 2^{n-1} vertices of even weight. Each edge of Q_n consists of a vertex of even weight and a vertex of odd weight. Therefore the vertices of even weight form an independent set and do so the vertices of odd weight. Thus Q_n is bipartite.*

Lemma 4.5. *The length of the geodesic path joining any two vertices of the hypercube Q_n equals the number of positions in which the vertices differ between 0 and 1.*

Proof: Let u and v be any two vertices in Q_n. We claim that "'If k is the number of positions in which u and v differ between 0 and 1, then the length of the $u - v$ geodesic path in Q_n is k."

Let us prove this result by applying induction on k.

If k = 1, the number of positions in which u and v differ between 0 and 1 is one. Then by the definition of Q_n, u and v are adjacent and so the length of the $u - v$ geodesic path is 1.

Suppose k = 2. That is, the number of positions in which u and v differ between 0 and 1 is two. Then u and v are not adjacent in Q_n. Choose a vertex w which differs exactly in one position with both u and v. Then (u, w, v) is a $u - v$ geodesic path of length 2.

Assume that the result is true for k. Suppose $k + 1$ is the number of positions in which u and v differ between 0 and 1. Choose a vertex w in Q_n such that w is adjacent to v and w differs exactly in k positions with u. Then by induction principle, there exists a $u - w$ geodesic path P of length k. Thus the path P along with the edge wv forms a $u - v$ geodesic path of length $k + 1$.

■

Remark 4.6. *It is obvious that the maximum length of a geodesic path between any two vertices of Q_n is n.*

Theorem 4.7.

$$CH_{\gamma_{vct}}(Q_n) = \begin{cases} 2 & n = 2 \\ 2^n & n = 3, 4 \end{cases}$$

Proof: For n = 2, Q_2 is as shown in Figure 4.

Obviously, S_1 = $\{00,\ 10\}$, S_2 = $\{10,\ 11\}$, S_3 = $\{00,\ 01\}$ and S_4 = $\{01,\ 11\}$ are the γ_{vct}-sets of Q_2. It is obvious that $\langle S_i \rangle$ = K_2 for all i = 1, 2, 3, 4. By theorem 1.2, each S_i is a convex γ_{vct}-set of Q_2 and $CH_{\gamma_{vct}}(Q_2)$ = 2.

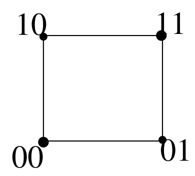

Figure 4: Q_2

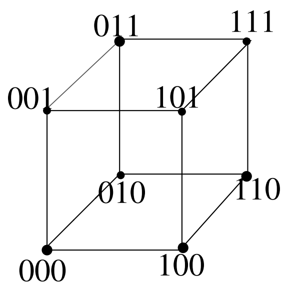

Figure 5: Q_3

For n = 3, Q_3 is the hypercube on 8 vertices which is 3-regular and is represented as shown in Figure 5.

The only two α_0- sets of Q_3 are C_1 = $\{000,\ 011,\ 110,\ 101\}$ and C_2 = $\{001,\ 010,\ 100,\ 111\}$. Then every two-element set of the form S = $\{v,\ v^c\}$ where $v \in C_1$ and $v^c \in C_2$ is a γ_{vct}-set. Obviously, the number of positions in which v and v^c differ between 0 and 1 is 3. So by lemma 4.5, the geodesic paths between v and v^c are of length 3. So the convex hull of S contains all the vertices of Q_3 since the geodesic paths of length 3 between v and v^c covers all the vertices of Q_3. Thus S is a hull γ_{vct}-set of Q_3. This implies that all the γ_{vct}-sets of Q_3 are hull and so by theorem 1.1, $CH_{\gamma_{vct}}(Q_3)$ = $V(Q_3)$ = 2^3.

If n = 4, the hypercube Q_4 contains 2^4 vertices and is 4-regular as shown in Figure 6.

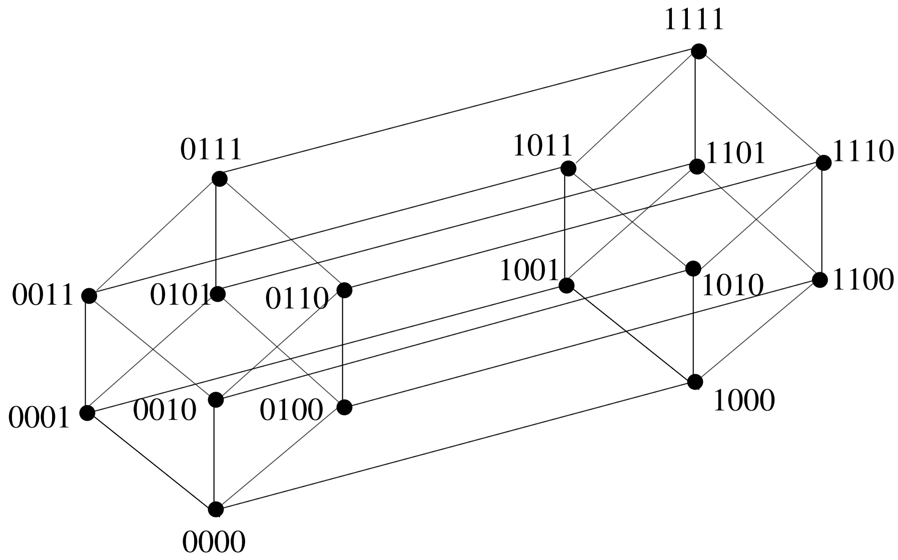

Figure 6: Q_4

Q_4 is bipartite with bipartitions C_1 = $\{$0000, 0011, 0110, 1100, 0101, 1010, 1001, 1111$\}$ and C_2 = $\{$0001, 0010, 0100, 1000, 0111, 1110, 1011, 1101$\}$. Also C_1 and C_2 are the only α_0-sets of Q_4. Any set containing two mutually complementary vertices from C_1 say 1100, 0011 and the other two mutually complementary vertices from C_2 say, 1000, 0111 form a γ_{vct}-set.

Thus S = $\{v_1, v_1^c, v_2, v_2^c\}$ where v_1,$v_1^c \in C_1$ and v_2,$v_2^c \in C_2$ is a γ_{vct}-set of Q_4. By lemma 4.5, the length of the geodesic path joining any two vertices of S equals the number of positions in which the vertices differ between 0 and 1. According to the choice of S, the geodesic paths of length 1, 2, 3 and 4 between the vertices of S covers all the vertices of Q_4. Therefore S is a hull γ_{vct}-set of Q_4. This implies that all the γ_{vct}-sets of Q_4 are hull. So by theorem 1.1, $CH_{\gamma_{vct}}(Q_4)$ = V(Q_4) = 2^4.

■

Theorem 4.8. *If C_n is a cycle on n vertices, then*

$$CH_{\gamma_{vct}}(C_n) = \begin{cases} 2 & n = 3, 4 \\ 3 & n = 5 \\ 4 & n = 6 \end{cases}$$

Proof: The result is obvious for n = 3, 4. When n = 5, the cycle C_5 on 5 vertices is as shown in Figure 7.

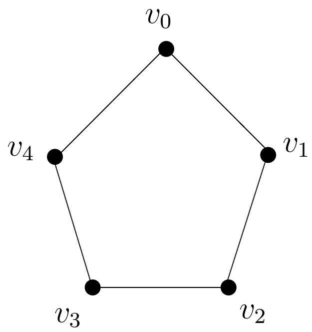

Figure 7: C_5

I_i = $\{v_{imod5}, v_{(i+2)mod5}\}$ for i= 0, 1, 2, 3, 4 are the maximum independent sets of C_5. Then their complements S_i = $\{v_{(i+1)mod5}, v_{(i+3)mod5}, v_{(i+4)mod5}\}$ for i= 0, 1, 2, 3, 4 are the minimum vertex covering sets of C_5. Any set containing three vertices taken from the vertex set of C_5 forms a dominating set which intersects each S_i for i = 0, 1, 2, 3, 4. Obviously each S_i is a minimum vertex covering transversal dominating set of C_5. In addition to these sets, T_j = $\{v_{jmod5}, v_{(j+1)mod5}, v_{(j+2)mod5}\}$ for j = 0, 1, 2, 3, 4 are the minimum vertex covering transversal dominating set of C_5.
Now the convex hull of S_i, $[S_i]$ = $\{v_0, v_1, v_2, v_3, v_4\}$ for each i = 0, 1, 2, 3, 4 and that of T_j, $[T_j]$ = T_j itself for every j = 0, 1, 2, 3, 4. Thus $CH_{\gamma_{vct}}(C_n)$ = 3.

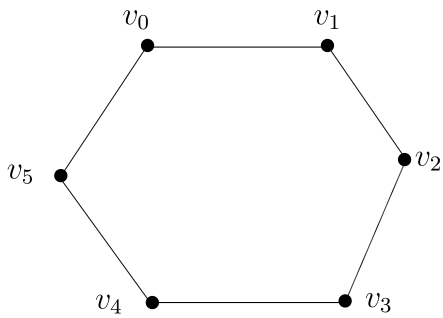

Figure 8: C_6

Now consider the cycle C_6 on 6 vertices shown in Figure 8.
Obviously $\{v_1, v_3, v_5\}$ and $\{v_0, v_2, v_4\}$ are the only α_0-sets of C_6. Also $\{v_0, v_3\}$, $\{v_1, v_4\}$ and $\{v_2, v_5\}$ are the γ_{vct}-sets as they intersect both the α_0-sets of C_6. Since each γ_{vct}-set of C_6 contains a pair of antipodal vertices, the length of any geodesic path between them is equal to the diameter of C_6. Thus the convex hull of each set contains 4 elements and so $CH_{\gamma_{vct}}(C_6)$ = 4. ■

Theorem 4.9. *If G is a connected regular graph of degree n−2 and O(G) = n where n is even, then $CH_{\gamma_{vct}}$(G) = 2.*

Proof: Let $V(G)$ = $\{v_1, v_2, v_3, v_4, ..., v_n\}$ be the vertex set of G. Choose any vertex $v_i \in V(G)$. Then $deg_G v_i = n - 2$ that is, v_i is adjacent to $n - 2$ vertices in G. Then there remains exactly one vertex, say v_j which is not adjacent to v_i. Therefore $S = \{v_i, v_j\}$ is an independent set of G. Also v_i is adjacent to $n - 2$ vertices in G except v_j. Hence no other vertex may be included in S. Therefore S is a maximum independent set of G.

Now let $v_k \in V - S$. Then v_k is adjacent to both v_i and v_j. Since v_i dominates every vertex in G except v_j and v_k dominates $n - 2$ vertices including v_j, it is obvious that $D_{ik} = \{v_i, v_k\}$ is a dominating set which intersects every minimum vertex covering set of G. So D_{ik} is a γ_{vct}-set in G for all i, k = 1, 2, ..., n, i $\neq$ k, k $\neq$ j. Obviously each v_i is adjacent to v_k. So $\langle D_{ik} \rangle = K_2$. Therefore by theorem 1.2, D_{ik} is a convex γ_{vct}-set of G and $CH_{\gamma_{vct}}$(G) = 2. ■

5. CONVEX HULL OF γ_{vct}-SETS IN CORONA GRAPH

The convex hull of γ_{vct}-sets and hence the convex hull number with respect to γ_{vct}-sets in a graph paves the way to filter the γ_{vct}-sets which are responsible for producing the convex hull number with respect to γ_{vct}-sets. This is best illustrated in the following example.

Example 5.1. *Consider the corona graph $C_4 \circ K_1$ formed from one copy of C_4 and $|V(C_4)|$ copies of K_1 as shown in Figure 9.*

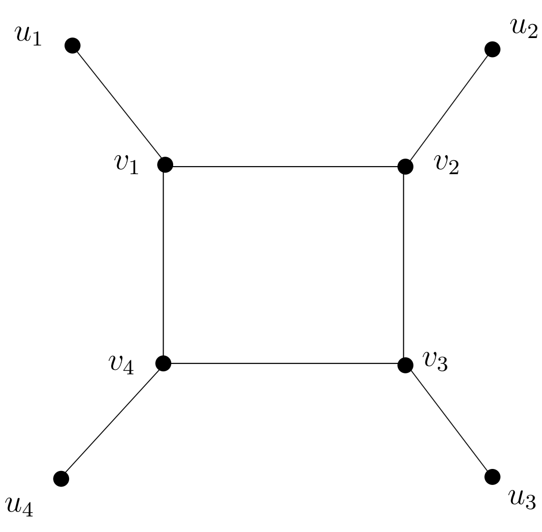

Figure 9: *Corona graph*

Obviously $\{u_1, u_2, u_3, u_4\}$, $\{u_1, u_2, u_3, v_4\}$, $\{u_1, u_2, u_4, v_3\}$, $\{u_1, u_3, u_4, v_2\}$, $\{u_2, u_3, u_4, v_1\}$, $\{u_1, u_3, v_2, v_4\}$ and $\{u_2, u_4, v_1, v_3\}$ are the β_0-sets of $C_4 \circ K_1$. So their complements $\{v_1, v_2, v_3, v_4\}$, $\{v_1, v_2, v_3, u_4\}$, $\{v_1, v_2, v_4, u_3\}$, $\{v_1, v_3, v_4, u_2\}$, $\{v_2, v_3, v_4, u_1\}$, $\{v_1, v_3, u_2, u_4\}$ and $\{v_2, v_4, u_1, u_3\}$ are the α_0-sets of $C_4 \circ K_1$.

It is obvious that all the β_0-sets and the α_0-sets of $C_4 \circ K_1$ are γ-sets of $C_4 \circ K_1$. Further $\{v_1, v_2, u_3, u_4\}$, $\{v_2, v_3, u_4, u_1\}$, $\{v_3, v_4, u_2, u_1\}$ and $\{v_4, v_1, u_2, u_3\}$ are also γ-sets of $C_4 \circ K_1$.

Among these γ-sets, $S_1 = \{v_1, v_2, v_3, v_4\}$, $S_2 = \{v_1, v_2, v_3, u_4\}$, $S_3 = \{v_1, v_2, v_4, u_3\}$, $S_4 = \{v_1, v_3, v_4, u_2\}$, $S_5 = \{v_2, v_3, v_4, u_1\}$, $S_6 = \{v_1, v_2, u_3, u_4\}$, $S_7 = \{v_2, v_3, u_4, u_1\}$, $S_8 = \{v_3, v_4, u_2, u_1\}$ and $S_9 = \{v_4, v_1, u_2, u_3\}$ are the γ_{vct}-sets of $C_4 \circ K_1$.

Also the convex hull of these γ_{vct}-sets are $[S_1] = S_1$, $[S_2] = \{v_1, v_2, v_3, v_4, u_4\}$, $[S_3] = \{v_1, v_2, v_3, v_4, u_3\}$, $[S_4] = \{v_1, v_2, v_3, v_4, u_2\}$, $[S_5] = \{v_1, v_2, v_3, v_4, u_1\}$, $[S_6] = \{v_1, v_2, v_3, v_4, u_3, u_4\}$, $[S_7] = \{v_1, v_2, v_3, v_4, u_4, u_1\}$, $[S_8] = \{v_1, v_2, v_3, v_4, u_2, u_1\}$ and $[S_9] = \{v_1, v_2, v_3, v_4, u_2, u_3\}$.

Therefore $CH_{\gamma_{vct}}(C_4 \circ K_1) = \min. \{4, 5, 6\} = 4$. This number corresponds to the one and only convex γ_{vct}-set $\{v_1, v_2, v_3, v_4\}$ of $C_4 \circ K_1$.

Remark 5.2. *The above example provokes an idea to concentrate on the family of γ_{vct}-sets that have their convex hulls of same cardinality which is equal to the convex hull number with respect to γ_{vct}-sets of a given graph.*

6. SCOPE

We observed that $\gamma_{vct} = \gamma$ in most of the graphs considered by us in [6, 7]. But even though $\gamma_{vct} = \gamma$, there are graphs in which γ-sets do not become γ_{vct}-sets as discussed in [8].

In general, the concept of γ-sets is applied for erecting towers in networking problems. In view of the analysis of convex hull number with respect to γ_{vct}-sets in a graph, it is observed that it provides us with the minimum number of nodes that are connected to each other by the shortest possible routes between them. Since γ_{vct}-sets are themselves γ-sets, they are dominating sets in G. Thus in networking problems, the convex hull of γ-sets or γ_{vct}-sets may be used to observe the shortest possible route between the nodes. This may be helpful in reaching the affected node in order to rectify the problem whenever there is a failure in the network.

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