Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 19, Number 2 (2023), pp. 523-546

http://www.ripublication.com/gjpam.htm

Optimal Control Strategies for Crime in Developing Countries

Bilali Mataru[1,2], Jeconia Okelo Abonyo[3] and David Malonza[4]

[1]*Pan African University Institute for Basic Sciences, Tech. and Innovation, Kenya.*
[2]*Muslim University of Morogoro, Tanzania*
[3]*Jomo Kenyatta University of Agriculture and Technology, Kenya.*
[4]*South Eastern Kenya University, Kenya.*
Email addresses: bilalmataru@gmail.com (Bilali Mataru), masenooj@gmail.com (Jeconia Okelo Abonyo), dmalo2004@gmail.com (David Malonza)

Abstract

The aim of this paper is to propose the control strategies for the crimes focusing on developing countries, we analyze the impact of unemployment insurance (unemployment benefit), free vocational training and new jobs creation as the control strategies for crime. The optimal control model is formulated from a deterministic mathematical model of crime dynamics. The Pontryagin Maximum principle was used to provide the necessary conditions of optimality to crime controls. The numerical results to optimal model are obtained by forward-backward sweep method (FBSM) and presented graphically. The Incremental Cost Effectiveness Ration (ICER) was used to assess the best cost effectiveness among the control strategies for crime and results found that, for single control measures, new jobs creation is the most cost-effective and for multiple control measures we found that free vocational training and new jobs creation is the best cost-effectiveness strategy for crime eradication.
Keywords: Unemployment, crime, Developing countries, mathematical model, optimal control.

1. INTRODUCTION

Developing countries are facing many challenges which are hindering social prosperity and also causing impediment to economic development. Crime is one of the most highlighted socioeconomic problems in which all governments of the different continents have been confronted with. Consequently, many innocent civilians have been endangered, maimed, quality of lives degraded and even lost their life due to criminal activities. Moreover, prolonged crimes result in a major human security threat and undermine domestic and foreign investments in many parts of the world [1]. Crime is considered as a human misconduct which goes against legal practices of daily human activities in any society. Mehran et al. [2] defined crime as an offence of breaching and violation of existing laws enforced by a government. From the criminal law point of view, crime is defined as any act against the law [3, 4].

Crime has been affecting thousands of human beings for generations socially and psychologically which puts global security in doubt, and comes with serious economic costs. According to [5, 6], cybercrimes cost the world over $945 billion in 2020 while affecting 89 million individuals and is projected to reach $10.5 trillion by the year 2025. Violence has cost the world economy an estimate of $14,758 billion in 2017, homicide was $3,650 billion and man power (military, police and judicial system) with a cost of $9,928 billion [7]. According to the report of United Nations Office on Drugs and Crime, around 468,000 deaths which occurs annually are due to homicide crime worldwide and approximately one third happens in Africa [8]. It is necessary to note that, though crime has been considered as a universal phenomenon and affects every country, but the strong dominance of criminal activities in some areas vary from one geographical location to another. For example, in developing countries in Africa, the prevalence of theft is about 25% and violence is about 13% . In Latin America, the prevalence is about 20% on theft, 11% on violence between 2006-2019. In Europe, 11% on theft, 4% on violence and while in North America, it is 14% on theft, 2% on violence within same period of 2006-2019 [9, 10]. The primary purpose of this study is to propose measures for combating unemployment related crimes with special attention on developing countries.

Developing countries are facing a lot of challenges, two of which include high unemployment rate and severe poverty among its people. According to the ILO report on employment in Africa, 34 million people are unemployed which gives an unemployment rate of 6.8% where 26.5% is from southern Africa (youth unemployment rate is 50.3% in southern African and 6% eastern Africa), 24.6% from western Africa and also around 286 million people are outside of labour force [11]. High rate of unemployment comes with many devastating consequences to any community. According to UN report on youth and unemployment, 40% of youth in Africa who are resorting to criminal activities do so because they have lost hope in securing decent jobs. Moreover, property crime becomes the most common offense committed during recession due to rapid increase in unemployed population, and therefore produce more career criminals. However, this crisis is reduced during the period of economic recovery [12]. According to [10], because of high rise in unemployment and poverty levels, Africa is the only continent which demonstrated increase in crime trends 2006-2019 by an average of 11.6% points with huge variation distribution across the region. Natarajan [13] argue that, most of terrifying criminal activities are taking place in developing countries and receive no proper attention compared to the developed countries which makes developing countries to be hotsport to criminals. Poor government quality of most developing country especially Africa that perpetuate corruption, internal conflicts, poor investment, and religions tensions cannot assure a security to its people instead they are perpetuating crime [14].

Several of factors have been mentioned in literatures of science and social science replete with possible causes for crime. Broadly, crime is influenced with the combination of several complex reasons including economic, social, psychological and political aspects. According to Guedes [15], high unemployment rate, extremely poverty level and inequality has a great impact of crime, and are main reasons of marine piracy crimes that spread in the gulf of Guinea which including Nigeria, Togo, Ghana, Cameroon, Gabon, Cote d'Ivoire and Sao Tome. In economic perspective, [16] outlined factors that exacerbating crimes include deeper economic hardship, unemployment, social inequality and homelessness which may result to psychological disturbance mostly to young population. Similarly, [17] studied the reason of poaching in Africa, and found that local unemployment and poverty is behind on the increase of poachers especially to young population and bring a major attention to international community. Crimes like pouching, marine piracy, kidnapping, illegal fishing are now days common become practices in developing countries while minimal consideration has taken. Generally, It is undeniable that crime can be caused by many reasons and can be experienced in various forms but, Unemployment, poverty, inequality, injustice, youth and lack of education are some of motivations in which people are involved in committing crime [10].

Many literature have tried to establish the connection between unemployment and crime but the actual connection remains complex and ambiguous because of unpredictable and variation of outcomes concerning their relationship from one country to another. There are significant numbers of literature that agree of the existing positive relationship between crime and unemployment and most especially in developing countries [18, 19]. Crisis resulting from unemployment and crime are more prevalent in countries dominated by income inequality, illiteracy, political instability and drug abuse. Furthermore, developing countries with higher unemployment rates like South Africa, Brazil, Mexico, Nigeria and many others demonstrate the rapid rising of criminal activities like drug trafficking, violence, robbery, cybercrime and property crimes among others [20–23]. Adeyemi et al. [24] argued that unemployment can lure people into committing crime because jobless individuals are most likely to commit criminal offences compared to the general population. These criminal offenses can be manifested in form of property crimes (robbery, burglary, theft), drug trafficking, armed robbery, kidnapping, drugs abuse, banditry, poaching, murder, assault and stalking among others. Richer [25] investigated the relationship between crime and unemployment in Croatia and found the positive relationship such that 3.3% increase in burglary triggered 11% growth in unemployment. According to World Bank Group in 2020, illegal activities attract unemployed young population which can be manifested everywhere: on land, air and at sea, hence fighting against them need modern technology equipment and well trained personnel (security officers) [26]. Eventually this investment is a huge challenge for low and middle income countries. Therefore taking preventive measures is more suitable to reduce crime prevalence.

Mathematical modeling plays a significant role in understanding, analyzing the dynamics of crimes and its determinants, moreover to suggest and analyzing the countermeasures to crimes and crime indicators. Furthermore, mathematical models are important in predicting on future trend regarding crimes and population of criminals. Opoku et al. [27] developed a crime model for controlling crimes committed during festivals suggesting detention, education and sucking as control measures to reduce crime below a threshold. Ugwuishiwu et al. [28] formulate and analyze a nonlinear mathematical model on criminality and victimization, proposing psychotherapy rehabilitation to victim of crime. Srivastav et al. [29] investigated on crime and corruption prediction and prevention focusing on the developing countries, while suggesting law enforcement in combating crimes. Soemarsono et al. [30] developed a model that analyze the role of unemployment to the trend of crime, moreover the analysis observe proportional between the increase of unemployment rate and crime. Amin [31] explore the regression analysis to develop a mathematical model of crime and unemployment including other factors like population density, literacy rate and per-capital income. Unemployment and crime demonstrate a +ve correlation which imply that the increase of unemployed population triggers to an increase of population of criminals. Sunder et al. [32] investigated the degree of which unemployment can influence the crime, by suggesting a nonlinear mathematical model and the result show that, as unemployment rate increases the number of criminals are also increase. Other studies of a mathematical analysis of crime can be found in [31, 33–35]. Most of existing models are suggesting non-preventive measures on combating crime (detention, imprisonment, strengthen of police forces etc). Less considerations/ efforts has been made on effective preventive and protective strategies which will expose people in productive activities which will protect them to criminal activities, this will be solving the root causes of unemployment-related crimes and could bring potential impact to the economy. Thus, this study provide an extension of a crime model for developing counties suggested in [36], to optimal control model, proposing unemployment insurance (unemployment benefit), free vocational training (job specific skills) and new jobs creation as control measures to crime which are preventive in nature, less expensive and also productive.

2. MODEL

The model formulation of crime due to unemployment such as property crimes (theft, burglary, shoplifting), violence among others, is modeled by the system of ordinary differential equations. We consider the dynamics of crimes proposed and described in [36], where the population is divided into five sub-population: Unemployed population $U(t)$, population exposed to crimes $S(t)$ (e.g. drug abusers, poorer, etc.), population of active criminals $C(t)$ (unemployed related crimes e.g., property crimes, poaching, pirates, smuggling, drugs trafficking etc.), population in vocational training $V(t)$, and population in regular and self employment denoted by $E(t)$. With the assumptions and formulation given in [36] together with parameters in table 1, the system model is given by

$$\begin{cases} \dot{U} = \tau - \beta UC + \pi C - (\mu + \alpha + \sigma + \phi)U \\ \dot{S} = \phi U - \beta(1-\theta)SC - (\omega + \mu)S \\ \dot{C} = \beta UC + \beta(1-\theta)SC - (\pi + \mu + \varepsilon + \delta + \varphi)C \\ \dot{V} = \omega S + \sigma U + \delta C - (\rho + \mu)V \\ \dot{E} = \varepsilon C + \alpha U + \rho V - \mu E \end{cases} \tag{2.1}$$

With the initial condition $U(0) = U_0 \geq 0$, $S(0) = S_0 \geq 0$, $C(0) = S_0 \geq 0$, $V(0) = V_0 \geq 0$ and $E(0) = E_0 \geq 0$ for all $t \in T$ and $U, S, C, V, E \in \Omega$. Where Ω is an invariant region, from (2.1), $\dot{U}$ represent the rate of change in unemployed population, $\dot{S}$ represent the rate of change in population of those who are exposed to crime, $\dot{C}$ is the rate of change of criminal population (unemployment related crimes), $\dot{V}$ is the rate of change of population in vocational training and $\dot{E}$ is the rate of change of employed population.

Parameters	Descriptions
u_1	control function - unemployment insurance (benefit)
u_2	control function - free vocational training
u_3	control function - new jobs creation
τ	unemployment recruitment
μ	rate at which individuals exist from a population
$\beta = kp$	transmission rate
k	contact rate
p	probability of contact lead to criminality behavior
π	rate at which criminals/crimes causes unemployment
ϕ	rate at which individual with the joining exposed class
φ	rate death due to criminal behavior
θ	efficacy rate (factor prevent individual of being a criminal)
δ	rate at which criminals joining vocational training after they finish sentences
α	rate at which unemployed individuals to get job
ρ	rate at which skilled individual get employment
σ	rate at which unemployed individuals to join vocational training

Table 1: State variables and parameter values for crime model

Remark 2.1. *Basic model properties of (2.1) which includes positivity of solution, invariant region, existence and uniqueness of model solution was given in [36]. Crime free equilibrium point was found to be locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$.*

3. OPTIMAL CONTROL MODEL

The study of dynamical system can be analyzed by various mathematical approaches in order to understand the dynamics and behavior of the intended physical situation, however optimal control strategy have been widely used in many field including sciences and social sciences in decision making relating to optimization which involve complex phenomena or systems [37]. The optimal control model for the dynamic of crime population can be seen in [27][33] [38, 39]. This study is interested optimal control strategies for crime in developing countries where unemployment and poverty level is too high. This model is based on previous study of deterministic crime model proposed in [36], the work considered a model for crime focusing the situation in developing countries line Africa and Latin America, where unemployment was considered to be a source of crimes related to financial hardship. In this study, we are proposing three preventive control $u_1(t)$,$u_2(t)$ and $u_3(t)$ which are time dependent. Control u_1 is targeting the unemployed population in sense that, the jobless individuals

to be provided with minimum payment which will enable them to meet a daily basic needs before obtaining a legal and decent work. The purpose of this control is reduce the risk of unemployed individuals to be engages in illegal activities. This control measure is expressed as $1 - u_1$ where $0 \leq u_1 \leq 1$, to define the impact of unemployment assurance to criminal population. When $u_1 = 1$ imply that, a measure is fully implemented and remove all the possibility of individual to be engaged in criminal activities because of being unemployed, at $u_1 = 0$ the unemployed assurance is not implemented.
The control u_2 is targeting unemployed and act as the rehabilitation process to criminals. The aim of this tactic is to provide optimal skill (job specific skills) which enable unemployed and a jobless criminal have a life skills for self-employment or regular-employment. Vocational training can encourage offender to change the way of life and engage in legal activities by utilizing available resource. Free vocational training can engage with skills for legitimate work. It is a fact that, lack of optimal skills hamper developing countries including Africa to benefit from gifted natural resource [40].
The control u_3 is intended to reduce the problem of unemployment to deter illicit activities. Thus, this tactic propose the government to create new jobs opportunities in order to tackle unemployment problem which will also impacted on criminal population. Moreover, the control measure u_3 is injected to motivate the enrollment rate to vocational training. Therefore, including control functions in (2.1), yield

$$\begin{cases} \dot{U} = \tau - (1-u_1)\beta UC + \pi C - (\mu + \alpha + \sigma + \phi)U \\ \dot{S} = \phi U - (1-u_1)\beta(1-\theta)SC - (\omega + \mu)S \\ \dot{C} = (1-u_1)\beta UC + (1-u_1)\beta(1-\theta)SC - \\ \qquad (\pi + \mu + \varphi)C - (u_2 + \varepsilon)C - (u_2 + \delta)C \\ \dot{V} = \omega S + \sigma U + (u_2 + \delta)C - (\rho + \mu)V \\ \dot{E} = (u_3 + \varepsilon)C + \alpha U + \rho V - \mu E \end{cases} \tag{3.1}$$

Where $0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1$ and $0 \leq u_3 \leq 1$. With IC $U_0 \geq 0, S_0 \geq 0, C_0 \geq 0, V_0 \geq 0$ and $E_0 \geq 0$.

The expression $(1-u_1)\beta UC$ and $(1-u_1)\beta(1-\theta)SC$ from the optimal system (3.1) are representing the control effort of unemployment insurance (benefit) in reducing the unemployment population to exposed S to criminal and the class of criminals C. $(u_2 + \delta)C$ Represent the effort of free vocational training as the rehabilitation process to criminal population by providing life skills. And the term $(u_3 + \varepsilon)C$ represent the effort of new jobs creation in enhancing the self-employment and regular employment in the fight against crime. The summarized list of parameter values and description can be seen in Table 1, and flowchart diagram is found in [36]. Thus model (3.1) is the same as model (2.1) when $u_i = 0$.

3.1. Crime Reproduction Number

We derive a formula for crime reproduction number (R_0) in terms of control functions and other parameters, in order to understand the relationship between R_0 and other control functions u_i. This relationship plays an important role on parameterize the model and planning the control schemes. The next generation matrix is used to obtain the crime reproduction number [41] , by considering the most affected class with crimes in (2.1) which is C compartment. The method work with two matrices f and v evaluate at crime equilibrium point $(0, y_0)$ defined as follows,

$$f = \frac{\partial F_i}{\partial C}(0, y_0), \quad v = \frac{\partial V_i}{\partial C}(0, y_0). \tag{3.2}$$

Where the decomposed F_i define the new criminal cases (transmission) in i class and V_i the progress of criminals (transition) to other classes i. Consider the crime compartment C then,

$$F = (1 - u_1)\beta UC + (1 - u_1)\beta(1 - \theta)SC, \tag{3.3}$$

$$V = (\pi + \mu + \varphi)C + (u_3 + \varepsilon)C + (u_2 + \delta)C. \tag{3.4}$$

The crime reproduction number formula is expressed as;

$$R_0 = \rho(fv^{-1}). \tag{3.5}$$

Then, Equations (3.3) and (3.4), corresponding to crime free equilibrium of (2.1), yield;

$$f = (1 - u_1)\beta U + (1 - u_1)\beta(1 - \theta)S, \tag{3.6}$$

$$v^{-1} = \frac{1}{(\pi + \mu + \varphi) + (u_3 + \varepsilon) + (u_2 + \delta)}. \tag{3.7}$$

From (3.5), (3.6) and (3.7), we get;

$$R_0 = \frac{\beta\tau(1 - u_1)\left((\omega + \mu) + \phi(1 - \theta)\right)}{(\omega + \mu)(\mu + \alpha + \sigma + \phi)\left((\pi + \mu + \varphi) + (u_3 + \varepsilon) + (u_2 + \delta)\right)}. \tag{3.8}$$

Remark 3.1. *Expression in (3.8) is a crime reproduction number R_0 for a system (2.1), representing the average number of secondary criminal cases produced by a new criminal individual during the entire period of criminality when introduced to unemployed population [42]. We can observe, that in order to reduce the value of R_0 we need to control the value of optimal function u_i. For this we should intensify the* *value to u_2 and u_3 while reducing the value of u_1 to its optimal values.*

Eliminating all crimes completely is a bit challenge due to many causative factors and complexity of human behavior, but unemployment-related crime can be reduced to attain a certain level of tolerance with optimal costs. To achieve the intended goal of minimizing the number of criminals into population, we propose the following minimizing objective functional J as used in [27].

$$J(C, u_1, u_2, u_3) = \int_0^{t_f} (M_3 C + \frac{W_1}{2} u_1^2 + \frac{W_2}{2} u_2^2 + \frac{W_3}{2} u_3^2) dt. \tag{3.9}$$

From the equation (3.9), the criminal population in C class is minimized by encouraging unemployed skills development and self-employment, and discourage active criminal to continue with illicit activities by providing them with unemployment benefits and jobs. Generally, a functional (3.9) in intending to minimize unemployment related crimes such as property crimes including theft, burglary and shoplifting, violence, moral crime such as prostitution, drug abuse and others. Where W_1, W_2 and W_3 represent the weight constant to balance the control functions and $M_3 > 0$ represent the balancing coefficient of crime situation. The set $u_i \in \mathcal{U}, \forall i = 1, 2, 3$ is a measurable set such that $0 \leq u_i \leq 1, \forall t \in [0, t_f]$. We note that, t_f is the final time of implementing the control. The aim is to obtain the pair (u_i^*, C^*) such that

$$J(C^*, u_i^*) = \min_{C, u_i \in \mathcal{D}} J(C, u_i). \tag{3.10}$$

Therefore C^* is called optimal trajectory and u_i^* is optimal control provided that they all exist.

4. OPTIMAL CONTROL ANALYSIS

4.1. Existence of an Optimal Control

The solution set of a model (2.1) is bounded for any initial condition, as studied in [36]. We study the existence of optimal control based on conditions suggested in [43].

Theorem 4.1. *Consider the model (2.1), the optimal control functions $\bar{u}^* = (u_1^*, u_2^*, u_3^*) \in \mathcal{U}$ correspond to corresponding to state variables $U_0 \geq 0, S_0 \geq 0, C_0 \geq 0, V_0 \geq 0$ and $E_0 \geq 0$ such that,*

$$J(u_1^*, u_2^*, u_3^*) = \min_{u_1, u_2, u_3 \in \mathcal{U}} J(u_1, u_2, u_3)$$

exist.

Proof. We prove the existence of u_1, u_2 and u_3 basing on the following conditions proposed by Fleming and Richel [43] theorem 4.1, therefore the following are sufficient conditions for an optimal control of (3.1) to exist.

1. The set $\mathcal{U} \neq \emptyset$ and $\Omega \neq \emptyset$.
2. The set $\mathcal{U} \neq \emptyset$ is closed and convex.
3. The boundedness of RHS of state equation by function in Ω and $\mathcal{U}$.
4. The integrand of cost functional is convex on $\mathcal{U}$.
5. We can find two constants $c_1, c_2 > 0$ and another constant $\beta > 1$ such that $I(C, u_1, u_2, u_3)$ of cost functional satisfy

$$I(C, u_1, u_2, u_3) \leq c_2 + c_1 \left(|u_1|^2 + |u_2|^2 + |u_3|^2\right)^{\beta/2} \tag{4.1}$$

We verify conditions (1)-(5) to obtain the existence of solution to model (2.1), for thus purpose we use similar result obtained in ([44] Thm. 9.2.1, pg 182) together with bounded control coefficients then condition 1 will follow. By the definition of control functions, then $\mathcal{U}$ is closed and convex, then condition 2 hold. The RHS if the state equation of (3.1) is bounded, since (2.1) is also bounded (see[36]) and the system is bi-linear with control variables. Further more, we can note that, the integral of cost functional is convex in u, since W_i are positive for all $i = 1, 2, 3$, which goes with condition 4. The fifth condition hold since,

$$C + \frac{W_1}{2}u_1^2 + \frac{W_2}{2}u_2^2 + \frac{W_3}{2}u_3^2 \leq c_2 + c_1 \left(|u_1|^2 + |u_2|^2 + |u_3|^2\right) \tag{4.2}$$

Where by c_2 depend on the upper bound on C , also $c_1 > 0$. Thus, the optimal controls are exist. □

4.2. Optimality System

Pontryagin's Maximum Principles (PMP) is the technique which provide necessary conditions for optimal solution u_1^*, u_2^* and u_3^*, the method based on formulation of Hamiltonian function (H) depend on adjoint variables λ_i which correspond with every states variable x_i, [37]. From (3.1) and (3.9), the Hamiltonian H is defined as

$$H = \left[M_3 C + \frac{W_1}{2}u_1^2 + \frac{W_2}{2}u_2^2 + \frac{W_3}{2}u_3^2\right] + \sum_{i=1}^{5} \lambda_i f_i. \tag{4.3}$$

Where f_i the right hand side of a system (2.1) for $i = 1, 2, 3, 4, 5$ and operator λ_i are adjoint variables. H is maximized with respect to controls u_i $\forall i = 1, 2, 3$ at optimal controls. From (4.3) gives

$$H = \left[M_3 C + \frac{W_1}{2}u_1^2 + \frac{W_2}{2}u_2^2 + \frac{W_3}{2}u_3^2\right] + \lambda_1(\tau - (1-u_1)\beta uc + \pi c -$$
$$(\mu+\alpha+\sigma+\phi)u) + \lambda_2(\phi u - (1-u_1)\beta(1-\theta)sc - (\omega+\mu)s) \quad (4.4)$$
$$\lambda_3((1-u_1)\beta uc + (1-u_1)\beta(1-\theta)sc - (\pi+\mu+\varphi)c - (u_3+\varepsilon)c -$$
$$(u_2+\delta)c) + \lambda_4(\omega s + \sigma u + (u_2+\delta)c - (\rho+\mu)v) + \lambda_5((u_3+\varepsilon)c + \alpha u + \rho v - \mu e).$$

Theorem 4.2. *If (4.4) is the Hamiltonian function of a model (3.1) and given optimal controls u_1^* ,u_2^* and u_3^* and optimal state variables U^*,S^*,C^*,V^* and E^* maximizing an objective function (3.9), there exist adjoint λ_i for $i = 1, 2, 3, 4, 5$ satisfy the co-state equations given by*

$$\begin{cases} \frac{d\lambda_1}{dt} = \lambda_1(1-u_1)\beta c + \lambda_1(\mu+\alpha+\sigma+\phi) - \lambda_2\phi - \lambda_3(1-u_1)\beta c - \lambda_4\sigma - \lambda_5\alpha, \\ \frac{d\lambda_2}{dt} = \lambda_2(1-u_1)\beta(1-\theta)c + \lambda_3(\omega+\mu) - \lambda_3(1-u_1)\beta(1-\theta)c - \lambda_4\omega, \\ \frac{d\lambda_3}{dt} = -M_3 + \lambda_1(1-u_1)\beta u - \lambda_1\pi + \lambda_2(1-u_1)\beta(1-\theta)s - \lambda_3(1-u_1)\beta u + \\ \qquad \lambda_3(\pi+\mu+\varphi) - \lambda_3(1-u_1)\beta(1-\theta)s + \lambda_3(u_3+\varepsilon) + \lambda_3(u_2+\delta) - \\ \qquad \lambda_4(u_2+\delta) - \lambda_5(u_3+\varepsilon), \\ \frac{d\lambda_4}{dt} = \lambda_4(\rho+\mu) - \lambda_5\rho, \\ \frac{d\lambda_5}{dt} = \lambda_5\mu. \end{cases} \quad (4.5)$$

With the transversality conditions is given by $\lambda_i(t_f) = 0$, $\forall i = 1, 2, 3, 4, 5$ moreover the optimal control parameters exist for all $u_i \in U$, are given by

$$\begin{cases} u_1^*(t) = \max\left\{0, \min\left(1, \frac{\lambda_3\beta(1-\theta)sc + \lambda_3\beta uc - \lambda_2\beta(1-\theta)sc - \lambda_1\beta uc}{W_1}\right)\right\}, \\ u_2^*(t) = \max\left\{0, min\left(1, \frac{(\lambda_2-\lambda_4)c}{W_2}\right)\right\}, \\ u_3^*(t) = \max\left\{0, min\left(1, \frac{(\lambda_3-\lambda_5)c}{W_3}\right)\right\}. \end{cases} \quad (4.6)$$

Proof. We considering the Pontryagin's Maximum Principles which gives the adjoint equations (4.5) and its transversality conditions. To obtain the adjoint system (4.5) relation (4.7) hold [45].

$$\frac{d\lambda_i}{dt} = -\frac{\partial H}{dx_i(t)}. \quad (4.7)$$

Equation (4.7), yield

$$
\begin{aligned}
\frac{dH}{du} &= -\lambda_1(1-u_1)\beta c - \lambda_1(\mu+\alpha+\sigma+\phi) + \lambda_2\phi + \lambda_3(1-u_1)\beta c + \lambda_4\sigma + \lambda_5\alpha \\
,\frac{dH}{ds} &= -\lambda_2(1-u_1)\beta(1-\theta)c - \lambda_2(\omega+\mu) - \lambda_3(1-u_1)\beta(1-\theta)c + \lambda_4\omega, \\
\frac{dH}{dc} &= M_3 - \lambda_1(1-u_1)\beta u + \lambda_1\pi - \lambda_2(1-u_1)\beta(1-\theta)s + \lambda_3(1-u_1)\beta u - \\
&\quad \lambda_3(\pi+\mu+\varphi) + \lambda_3(1-u_1)\beta(1-\theta)s - \lambda_3(u_3+\varepsilon) - \lambda_3(u_2+\delta) + \\
&\quad \lambda_4(u_2+\delta) + \lambda_5(u_3+\varepsilon), \\
\frac{dH}{dv} &= -\lambda_4(\rho+\mu) + \lambda_5\rho, \\
\frac{dH}{de} &= -\lambda_5\mu.
\end{aligned}
$$

$\lambda_i(t_f) = 0, i = 1,2,3,4,5$ is the terminal condition for adjoint variables, then again from equation (4.7) complete the proof of first part. Again, the proof of second part of the Theorem 2 we consider the derivative of Hamiltonian (4.4) with respect to every control variable u_1, u_2 and u_3, considering the optimality condition

$$\frac{\partial H}{du_i} = 0. \tag{4.8}$$

(4.8) gives optimal control (4.6) which complete the proof. □

Thus, the system (3.1) with initial conditions and adjoint system (4.5) with transversality conditions together give optimal system of (2.1), moreover numerical scheme is applied to determine optimal controls u_1^*, u_2^*, u_3^* and corresponding optimal state variables U^*, S^*, C^*, V^*, E^*.

5. NUMERICAL SIMULATION

Simulation of optimal control model is considered in this section, to illustrate the theoretical part in previous sections to obtain the numerical values of optimal controls. The solution of model obtained by solving (3.1) with its initial conditions, (4.5) and (4.6) with the terminal conditions. The forward and backward sweep method based on Runge-Kutta 4th order and implemented in MATLAB R2013 was used to solve for state variables, adjoint variables and then suitable result to obtain an optimal controls. Parameter values used for optimal model simulation are secondary data obtained from very recent studies and some are estimated based on real life situation in the developing countries, and summarized in Table 2 . The total of seven control strategies are considered in model simulation, in which includes unemployment assurance, free vocational training, new jobs creation, the combinations of two control measures and combination of all three measures together.

Parameters	Estimated value	Source
τ	5,000	[46]
μ	0.115	[27]
β	0.58	[27]
π	0.001	[36]
ϕ	0.1265	[27]
φ	0.004	[30]
θ	0.44	[30]
δ	0.015	Assumed
ε	0.5	varies
ω	0.015	[36]
α	9.6%	[38]
ρ	3.5%	Assumed
σ	0.35	[36]

Table 2: Parameter values for optimal control model simulation.

Strategy 1: New jobs creation, Strategy 2: Unemployment insurance (benefit), Strategy 3: Free vocational training, Strategy 4: Free vocational training and new jobs creation, Strategy 5: Free vocational training and unemployment insurance (benefit), Strategy 6: Unemployment insurance (benefit) and creation of new jobs, and Strategy 7: Unemployment insurance (benefit), free vocational training and new jobs creation.

5.1. Strategy 1: Optimal application of new jobs creation

The use of single control measure u_3 is to encourage legal work and discourage criminal activities, the availability of enough job opportunities will reduce the population of criminal. Figure 1(a) show that, without control measures, the population of criminals was growing logarithmic but due to the control measures we can see the number of criminals are decreasing and maintained at low level. Similar results can been to unemployed population in Figure 1(b). Without control the population of unemployed was increasing rapidly, but with control the population it can be seen unemployed population is falling to the minimum level. The diagram also demonstrate that, using a single control is not enough to get rid of all criminals in the population.

5.2. Strategy 2: Optimal application of unemployment insurance

Under this strategy, the unemployment insurance (benefit) u_1 is employed in optimization of the objective function J while other sets of control measures are not used. It can be seen in Figure 2(a), due to control measure the population of criminal decrease to crime free population in less than four years. Contrary to unemployed population, no significant change to the population under this strategy as it can be seen in Figure 2(b).

5.3. Strategy 3: Optimal application of free vocational training

Under this control tactic the use of free vocational training u_2 is implemented to optimize the cost function J. The result in figure 3(a) show that, a single control is able to reduce the population growth of criminals and similarly, it reduces the population of unemployed individuals to some level as it can be observed in figure 3(b) with control u_2 and with no control.

5.4. Strategy 4: Optimal application of free vocational training and new jobs creation

Under this scenario, new job creation control u_3 and free vocational training control u_2 are implemented simultaneously in 5 years to optimize a cost function J while u_3 was not be utilized. Figure 4(a) show the combination of these two measures reduces the number of criminals more effective compared to when they used separately.. The similar results can be seen in figure 4(b) the unemployed population decrease by 98% when the control strategy was implemented.

5.5. Strategy 5: Optimal application of free vocational training and unemployment insurance (benefit)

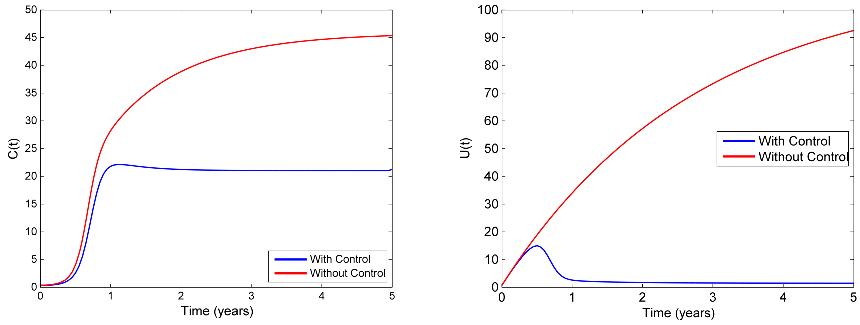
Under this scheme, two controls are implemented together, free vocational training u_2 and unemployment insurance u_1 to optimize cost function J while creation of the new job u_3 is set to be zero. Figure 5(a) shows that, the implementation of two control measures, the criminal population decreases rapidly, the crime free population was achieved in less than 2 years. Similarly, Figure 5(b) show the decrease of unemployed.

5.6. Strategy 6: Optimal application of unemployment insurance (benefit) and creation of new jobs

Under this scenario, unemployment insurance u_1 and free creation of new jobs u_3 are utilized simultaneously for optimization of the cost function J while free vocational training u_2 is not applied. Figure 6(a) demonstrate the population of active criminals is decreasing due to the effect of two control measures while the criminals increases when no strategy is applied. Figure 6(b) similarly show the effect of control strategy to the unemployed population, the population of unemployment is decreasing while they increase when no control used.

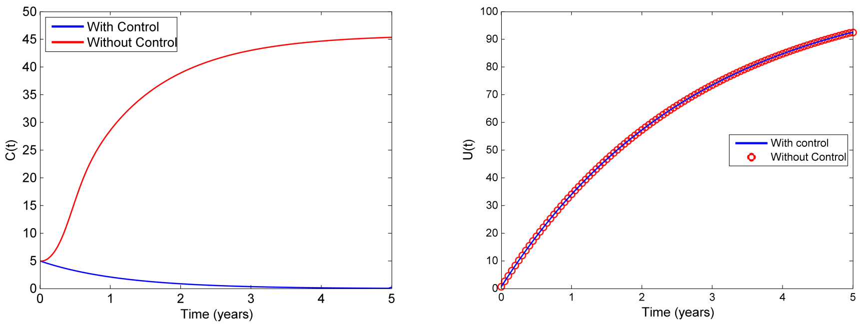
5.7. Strategy 7: Optimal application of unemployment insurance, free vocational training and new job creation

Under this scenario, unemployment insurance u_1, free vocational training u_2 and the creation of new jobs u_3 are all implemented to optimize the given objective cost J. Figure 7(a) exhibit the effectiveness of applying all the control strategies in which it is effectively eliminate all criminals in a short period of time compared to all other control strategies. The criminal free equilibrium was achieved in period of approximately one year period. Similarly, figure 7(b) shows a similar decreases to unemployed population with optimal control, while it is increasing when no control measures was applied.



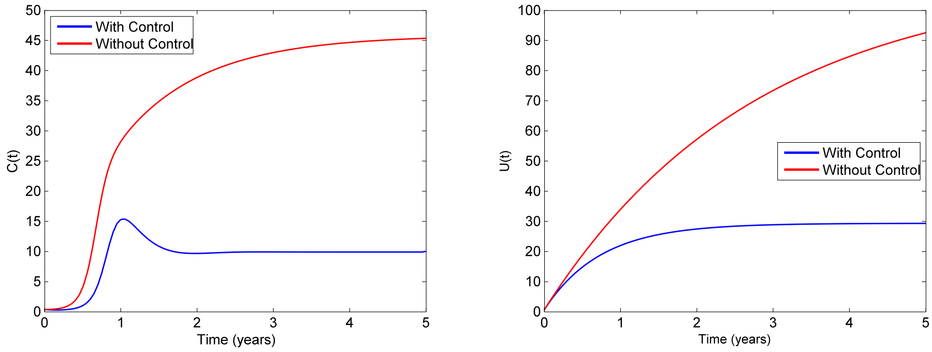
(a) $C(t)$ when $u_1 = 0$, $u_2 = 0$ and $u_3 \neq 0$ (b) $U(t)$ when $u_1 = 0$, $u_2 = 0$ and $u_3 \neq 0$

Figure 1: Criminal and Unemployment population dynamic under optimal number of new created jobs



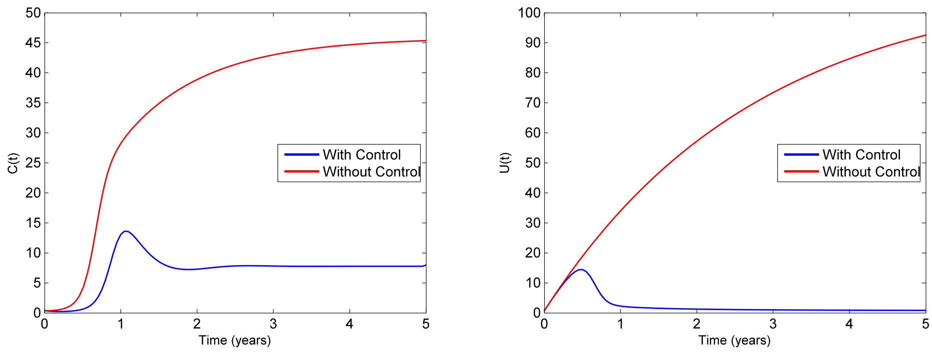
(a) $C(t)$ when $u_1 \neq 0$, $u_2 = 0$ and $u_3 = 0$ (b) $U(t)$ when $u_1 \neq 0$, $u_2 = 0$ and $u_3 = 0$

Figure 2: Criminal and Unemployment population dynamic under optimal number of unemployment insurance (benefit)



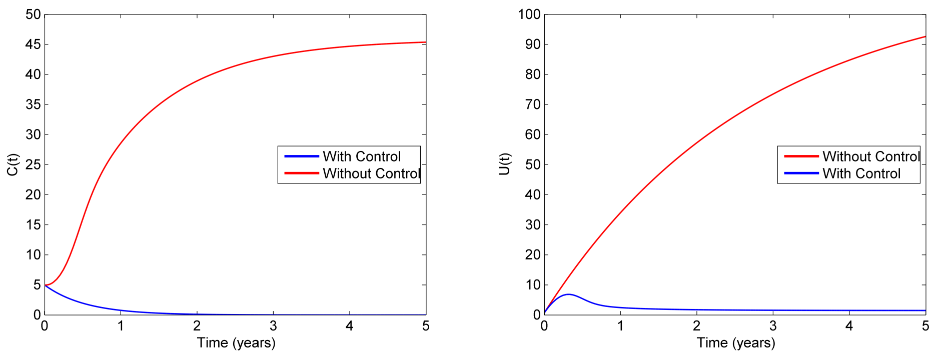
(a) $C(t)$ when $u_1 = 0$, $u_2 \neq 0$ and $u_3 = 0$ (b) $U(t)$ when $u_1 = 0$, $u_2 \neq 0$ and $u_3 = 0$

Figure 3: Criminal and Unemployment population dynamic under optimal number of free vocational training



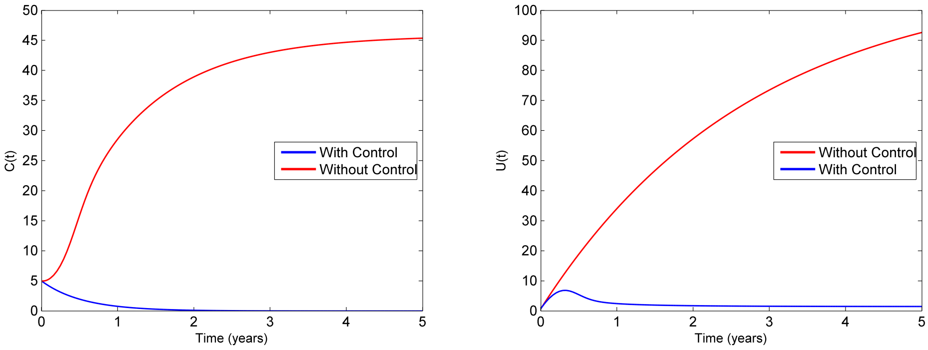
(a) $C(t)$ when $u_1 = 0$, $u_2 \neq 0$ and $u_3 \neq 0$ (b) $U(t)$ when $u_1 = 0$, $u_2 \neq 0$ and $u_3 \neq 0$

Figure 4: Criminal and Unemployment population dynamic under optimal number of free vocational training and new jobs creation



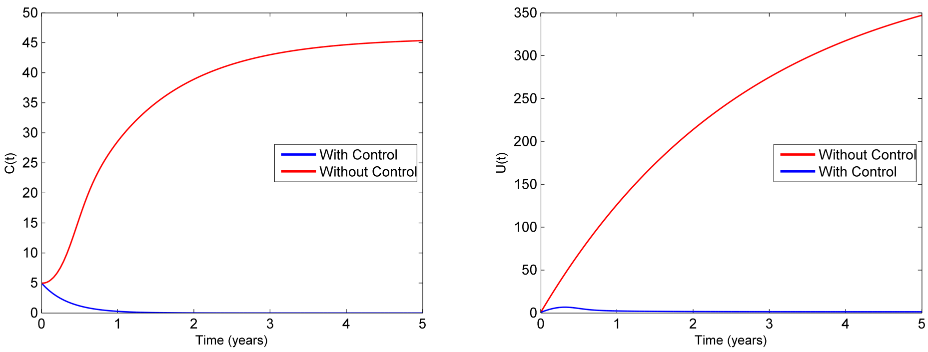
(a) $C(t)$ when $u_1 \neq 0$, $u_2 \neq 0$ and $u_3 = 0$ (b) $U(t)$ when $u_1 \neq 0$, $u_2 \neq 0$ and $u_3 = 0$

Figure 5: Criminal and Unemployment population dynamic under optimal number of free vocational training and unemployment insurance



(a) $C(t)$ when $u_1 \neq 0$, $u_2 = 0$ and $u_3 \neq 0$ (b) $U(t)$ when $u_1 \neq 0$, $u_2 = 0$ and $u_3 \neq 0$

Figure 6: Criminal and Unemployment population dynamic under optimal number of free unemployment insurance and new jobs creation



(a) $C(t)$ when $u_1 \neq 0$, $u_2 \neq 0$ and $u_3 \neq 0$ (b) $U(t)$ when $u_1 \neq 0$, $u_2 \neq 0$ and $u_3 \neq 0$

Figure 7: Criminal and Unemployment population dynamic under optimal number of free vocational training, unemployment insurance and new jobs creation

6. COST-EFFECTIVENESS ANALYSIS

The cost-effectiveness analysis for control strategies against crime is presented in this section. The analysis is aimed to determine the most effective strategy in combating crime at a low cost among seven optimal control strategies. For this, we employ the incremental cost effective ratio (ICER) to determine a strategy that will utilize the available scarce resources for crime eradication. This approach will enable us to compare between two competitive strategy (old strategy and a new strategy). If the new strategy is most cost effective, it will replace the older strategy otherwise the older strategy will be a dominant control measure. Incremental cost effective analysis can be seen in [27], [47–49]. The mathematical formulation of ICER is given as

$$ICER = \frac{Cost\ of\ strategy\ X - Cost\ of\ strayegy\ Y}{Effect\ of\ strategy\ X - Effect\ of\ strategy\ Y} \tag{6.1}$$

Where X and Y are two competitive strategies to be compared to one another in term of incremental cost and incremental effect (The total number of criminals averted for a given time interval is obtained by the difference of sum of criminal individuals without control and the sum of criminals with control). The control strategies that will be considered are in categories, single measure strategies (New jobs creation, unemployment insurance and free vocational training) and the combinations of multiple measures (free vocational training and new jobs creation, free vocational training and unemployment insurance, and unemployment insurance and new jobs creation, unemployment insurance, free vocational training and new jobs creation).

Strategies	Total Criminal Incidence Averted	Total cost	ICER
2	3866	4580.70	1.185
3	3860	4580.20	0.088
1	3867.3	4550.04	-0.808

Table 3: ICER for strategy 2, 3 and 1

Table 3 show the ICER of single measure strategies and the comparison between old strategy 2 and new strategy 3 show that, using strategy 3 could save around 0.088 units of cost over strategy 2. Likewise, compare with strategy 2, strategy 3 has small ICER and therefore strategy 3 will replace strategy 2.

Strategies	Total Criminal Incidence Averted	Total cost	ICER
3	3860	4580.20	0.088
1	3867.3	4550.05	-0.808

Table 4: ICER for strategy 3 and strategy 1

Table 4 show the ICER of strategy 3 and strategy 1 and the comparison show that, using strategy 1 could save 0.808 units of cost over the old strategy 3. Moreover, the ICER of strategy 1 available to fourth quadrant of the cost effectiveness plane which imply that it is more effective and cost saving, Therefore, strategy 1 is dominant, and will replace strategy 3. Table 5 show the ICER of multiple measures strategies, we comparing strategy 4 and strategy 5, we find that, using strategy 4 could save around 0.721 units of cost over strategy 5. Likewise ICER of strategy 4 is less that strategy 5, therefore strategy 4 will be chosen over strategy 5.

Strategies	Total Criminal Incidence Averted	Total cost	ICER
4	5000.5	5367.50	0.741
5	5563	6540.70	2.157
6	5563.5	6580.70	6.154
7	5682.2	6890.80	2.751

Table 5: ICER for strategy 4, 5, 6 and 7

Strategies	Total Criminal Incidence Averted	Total cost	ICER
4	5000.5	5367.50	0.741
6	5593.5	6580.70	2.132
7	5582.2	6890.80	2.751

Table 6: ICER for strategy 4, 6 and 7

Table 6 show the ICER for three remained strategies, it illustrate that strategy 4 is cost saving by 0.741 units of cost over strategy 6, moreover strategy 4 has a small ICER compared to strategy 6. Thus strategy 4 will continue dominate in the fight against crimes.

Strategies	Total Criminal Incidence Averted	Total cost	ICER
4	5000.5	5367.50	0.741
7	5682.2	6890.80	2.235

Table 7: ICER for strategy 4 and 7

Table 7 show the ICER of remaining two strategies 4 and 6,it demonstrate that strategy 4 saves 0.741 units of cost over tactic 7. The lower ICER of strategy 4 demonstrate that 7 is strongly dominated. Thus, For the above ICER analysis we conclusion that, for a single control strategy for crime eradication the new jobs creation has a smallest ICER. For the combination of two strategies, free vocational training and new jobs creation has a least ICER. Hence, the recommended strategy among all seven is strategy number 4 because is most cost effectiveness.

7. CONCLUSION

In this paper, we derive optimal control model for crime that caused by unemployment challenges focusing in developing countries. Control strategies proposed are unemployment insurance (benefit), free vocational training and new jobs creation are suggested to eliminate crimes. The necessary condition for optimal model was derived by Pontryagin Maximum principle and analyzed.

The Incremental cost effectiveness ratio (ICER) was used to analyze the cost-effectiveness of all control measures in order to obtain the best strategy which is most effective in utilizing limited resources in combating crime at a minimum cost. The analysis shows that, For single measure control strategies, the unemployment assurance was able to eliminate criminal population by 100% within 3 years with the total cost of 4580.70 units while the total number of averted criminals was 3866 units. This strategy is effective but more expensive for developing countries compare to other strategy, moreover it not effective in reducing an unemployed population. Free vocational training reduces criminal population by 77.7% and unemployed population by 67.4% while costs 4580.20 units, the strategy is effective to both criminal and unemployed population but more expensive. Also new jobs creation strategy costs 4550.04 units with 78% reduction of criminal population and reducing unemployed population by 67.4%, this control strategy found to be most cost-effective among single control measures. For cases of strategies with combinations of two control measures; the cost of using free vocational training and unemployment insurance (benefit) is 5367.50 unit, with 84% reducing the population of criminals and 99% in reducing of an unemployed population, and this strategy found to be cost-effectiveness compare to other forms of tactic, free vocational training and unemployment insurance (benefit) which costs 6540.70 units to reduce 100% criminal population and unemployed population by 66.7%. And also unemployment assurance and new jobs creation which costs 6580.70 units to reduce 100% criminal population and 97.8% unemployed population. Finally, we consider the cost of using all three strategies at once; unemployment insurance (benefit), new jobs creation and free vocational training which costs 6890.80 units to reduce criminal population by 100% within a year and unemployed population by 97.8%, this strategy is most effective because it eradicate crime in less than a year, but the cost of using this strategy is more effective but more expensive.

Basing on the ICER analysis of the model, we conclude that, free vocational training and new jobs creation is most cost-effectiveness strategy for crime eradication in developing countries. Also, any control strategy which include unemployment insurance benefit, come out to be effective but more expensive for developing countries, since paying unemployed people without producing is a disadvantage to economy and the nation in general. Moreover unemployment benefit program might also be source of irresponsibility to the beneficiaries for example job search effect [50, 51]. For developing countries especially African the tactic of free vocational training and creation of new jobs opportunities have many advantages, because Africa is one among continents that are facing the lack or inadequate optimal skills which are relevant to the available opportunities and resources [52]. Therefore, in provision of optimal vocational training, the need of figuring out what skills that are most needed should not be ignored. This strategy will reduce the production of skilled personnel who are less potential in boosting economywide productivity advantage. Technical optimal skills (job specific skill) is necessary for catalyzing production in formal and non-form sectors and trigger the economic transformation, reduce unemployment-related crimes through self-employment. Moreover, vocational skills (job specific skills) can be acquired by everyone with and out side education systems.

ACKNOWLEDGMENTS

: The Authors are so grateful and appreciating the African Union for providing the financial support through Pan African University Scholarship in every step of this work.

Conflicts of Interest: The authors declare that there is no conflict of interest regarding the publication of this article.

REFERENCES

[1] Keith Blackburn, Kyriakos C Neanidis, and Maria Paola Rana. A theory of organized crime, corruption and economic growth. *Economic Theory Bulletin*, 5:227–245, 2017.

[2] Ramin Mehran, Alexis Oyama, and Mubarak Shah. Abnormal crowd behavior detection using social force model. In *2009 IEEE conference on computer vision and pattern recognition*, pages 935–942. IEEE, 2009.

[3] Sanford H Kadish. Excusing crime. In *The Structure and Limits of Criminal Law*, pages 503–536. Routledge, 2017.

[4] Kevin Jon Heller. International crime. In *Concepts for International Law*, pages 524–539. Edward Elgar Publishing, 2019.

[5] Olha Kovalchuk, Mykola Shynkaryk, and Mariia Masonkova. Econometric models for estimating the financial effect of cybercrimes. In *2021 11th International Conference on Advanced Computer Information Technologies (ACIT)*, pages 381–384. IEEE, 2021.

[6] Md Haris Uddin Sharif and Mehmood Ali Mohammed. A literature review of financial losses statistics for cyber security and future trend. *World Journal of Advanced Research and Reviews*, 15(1):138–156, 2022.

[7] Mohib Iqbal, Harrison Bardwell, and David Hammond. Estimating the global economic cost of violence: Methodology improvement and estimate updates. *Defence and Peace Economics*, 32(4):403–426, 2021.

[8] UNODC. The global study on homicide. Technical report, United Nations Office on Drugs and Crime, 2019.

[9] Randriamihanta Jery Elis, Yideng Liu, et al. Human rights in the context of criminal justice: a study of urban crime. *Open Journal of Political Science*, 8(03):305, 2018.

[10] Jan van Dijk, Paul Nieuwbeerta, and Jacqueline Joudo Larsen. Global crime patterns: An analysis of survey data from 166 countries around the world, 2006–2019. *Journal of Quantitative Criminology*, pages 1–36, 2021.

[11] Ken Chamuva Shawa, Pamphile Sossa, and Ni O'Higgins. Report on employment in africa (re-africa): tackling the youth employment challenge, 2020.

[12] Richard Rosenfeld. Crime and the great recession: introduction to the special issue, 2014.

[13] Mangai Natarajan. Crime in developing countries: the contribution of crime science, 2016.

[14] Muhammad Waqar Saleemi and Rafi Amir-ud Din. How does quality of governance influence occurrence of crime? a longitudinal analysis of asian countries. 2019.

[15] Henrique Portela Guedes. Maritime piracy in the gulf of guniea. *JANUS. NET e-journal of International Relations*, 11:112–119, 2020.

[16] Christopher Chimaobi Onyeneke and Aly H Karam. An exploratory study of crime: examining lived experiences of crime through socioeconomic, demographic, and physical characteristics. *Urban Science*, 6(3):43, 2022.

[17] Severin Hauenstein, Mrigesh Kshatriya, Julian Blanc, Carsten F Dormann, and Colin M Beale. African elephant poaching rates correlate with local poverty, national corruption and global ivory price. *Nature Communications*, 10(1):2242, 2019.

[18] Fredrik Lundqvist. Unemployment and crime, 2018.

[19] Daniel Francois Meyer. An analysis of the short and long-run effects of economic growth on employment in south africa. *International Journal of Economics and Finance Studies*, 9(1):177–193, 2017.

[20] Brian Tavonga Mazorodze. Youth unemployment and murder crimes in kwazulu-natal, south africa. *Cogent economics & finance*, 8(1):1799480, 2020.

[21] NG Tshabalala. Crime and unemployment in south africa; revisiting an established causality: Evidence from the kwazulu natal province. *Mediterranean Journal of Social Sciences*, 5(15):519, 2014.

[22] Okechukwu Odinaka Ajaegbu. Rising youth unemployment and violent crime in nigeria. *American Journal of Social Issues and Humanities*, 2(5):315–321, 2012.

[23] Ebele M Onwuka, Kelechi Enyinna Ugwu, and Ejike Daniel Chukwuma. Implications of youth unemployment and violent crime on the economic growth (a case study of anambra state, nigeria). *Australian Journal of Commerce Study*, 2020.

[24] Rasheed A Adeyemi, James Mayaki, Temesgen T Zewotir, and Shaun Ramroop. Demography and crime: A spatial analysis of geographical patterns and risk factors of crimes in nigeria. *Spatial Statistics*, 41:100485, 2021.

[25] Vedran Recher. Unemployment and property crime: evidence from croatia. *Crime, Law and Social Change*, 73:357–376, 2020.

[26] Wendy V Cunningham. *Youth at risk in Latin America and the Caribbean: Understanding the causes, realizing the potential*. World Bank Publications, 2008.

[27] Nicholas Kwasi-Do Ohene Opoku, Georg Bader, and Edem Fiatsonu. Controlling crime with its associated cost during festive periods using mathematical techniques. *Chaos, Solitons & Fractals*, 145:110801, 2021.

[28] Chikodili Helen Ugwuishiwu, DS Sarki, and Godwin Christopher Ezike Mbah. Nonlinear analysis of the dynamics of criminality and victimisation: a mathematical model with case generation and forwarding. *Journal of Applied Mathematics*, 2019, 2019.

[29] Akhil Kumar Srivastav, S Athithan, and Mini Ghosh. Modeling and analysis of crime prediction and prevention. *Social Network Analysis and Mining*, 10(1):1–21, 2020.

[30] AR Soemarsono, I Fitria, K Nugraheni, and N Hanifa. Analysis of mathematical model on impact of unemployment growth to crime rates. In *Journal of Physics: Conference Series*, volume 1726, page 012003. IOP Publishing, 2021.

[31] Ruhul Amin. Mathematical model of crime and unemployment. *International Journal of Engineering Research & Technology*, 8(9):33–41, 2019.

[32] Shyam Sundar, Agraj Tripathi, and Ram Naresh. Does unemployment induce crime in society? a mathematical study. *American Journal of Applied Mathematics and Statistics*, 6(2):44–53, 2018.

[33] Oluwasegun M Ibrahim, Daniel Okuonghae, and Monday NO Ikhile. Optimal control model for criminal gang population in a limited-resource setting. *International journal of dynamics and control*, pages 1–16, 2022.

[34] Jongo Park and Pilwon Kim. Mathematical analysis of crime dynamics in and out of prisons. *Mathematical Methods in the Applied Sciences*, 44(1):650–667, 2021.

[35] Meskerem Abebaw Mebratie and Mohammed Yiha Dawed. Mathematical model analysis of crime dynamics incorporating media coverage and police force. *J. Math. Comput. Sci.*, 11(1):125–148, 2020.

[36] Bilali Mataru, Okelo Jeconiah Abonyo, David Malonza, et al. Mathematical model for crimes in developing countries with some control strategies. *Journal of Applied Mathematics*, 2023, 2023.

[37] Suzanne Lenhart and John T Workman. *Optimal control applied to biological models*. Chapman and Hall/CRC, 2007.

[38] Steady Mushayabasa. Modeling optimal intervention strategies for property

crime. *International Journal of Dynamics and Control*, 5:832–841, 2017.

[39] JO Akanni, FO Akinpelu, S Olaniyi, AT Oladipo, and AW Ogunsola. Modelling financial crime population dynamics: optimal control and cost-effectiveness analysis. *International Journal of Dynamics and Control*, 8(2):531–544, 2020.

[40] Yaw Ansu and Jee-Peng Tan. Skills development for economic growth in sub-saharan africa. *World Bank*, 2008.

[41] Carlos Castillo-Chavez and Baojun Song. Dynamical models of tuberculosis and their applications. *Mathematical Biosciences & Engineering*, 1(2):361, 2004.

[42] Haiyun Zhao, Zhilan Feng, and Carlos Castillo-Chavez. The dynamics of poverty and crime. *Journal of Shanghai Normal University (Natural Sciences· Mathematics)*, 43(5):486–495, 2014.

[43] WH Fleming, RW Rishel, GI Marchuk, AV Balakrishnan, AA Borovkov, VL Makarov, AM Rubinov, RS Liptser, AN Shiryayev, NN Krassovsky, et al. Applications of mathematics. *Deterministic and Stochastic Optimal Control*, 1975.

[44] Dahlard L Lukes. Differential equations: classical to controlled. 1982.

[45] LS Pontryagin, VG Boltyanskii, RV Gamkrelidze, and EF Mishchenko. The maximum principle. *The Mathematical Theory of Optimal Processes. New York: John Wiley and Sons*, 1962.

[46] AK Misra and Arvind K Singh. A mathematical model for unemployment. *Nonlinear Analysis: Real World Applications*, 12(1):128–136, 2011.

[47] Joshua Kiddy K Asamoah, Eric Okyere, Afeez Abidemi, Stephen E Moore, Gui-Quan Sun, Zhen Jin, Edward Acheampong, and Joseph Frank Gordon. Optimal control and comprehensive cost-effectiveness analysis for covid-19. *Results in Physics*, 33:105177, 2022.

[48] A Abidemi and JO Akanni. Dynamics of illicit drug use and banditry population with optimal control strategies and cost-effectiveness analysis. *Computational and Applied Mathematics*, 41(1):53, 2022.

[49] Afeez Abidemi. Optimal cost-effective control of drug abuse by students: insight from mathematical modeling. *Modeling Earth Systems and Environment*, pages 1–19, 2022.

[50] Kangoh Lee. Unemployment insurance, mobile capital, output, and distributive effects. *Economic Modelling*, 115:105949, 2022.

[51] Josef Zweimüller. Unemployment insurance and the labor market. *Labour Economics*, 53:1–14, 2018.

[52] Omar Arias, David K Evans, and Indhira Santos. *The skills balancing act in Sub-Saharan Africa: Investing in skills for productivity, inclusivity, and adaptability*. World Bank Publications, 2019.