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General Mixed Quasi Equilibrium Variational Inequalities

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Abstract

In this paper, we introduce a new class of equilibrium problems, which is called the general mixed quasi equilibrium variational inequalities. Using auxiliary principle technique to suggest and analyze an iterative scheme for solving general mixed quasi equilibrium variational inequalities. The convergence of the proposed method requires partially relaxed jointly strongly monotone and skew symmetry. Our results represent an improvement and refinement of previously known results. Since the general mixed quasi equilibrium variational inequalities as special cases, results proved in this paper continue to hold for these problems.

Keywords: Equilibrium problem; mixed variational inequality; Auxiliary principle technique; Convergence
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1. INTRODUCTION

It is well known that the variational inequality theory, first proposed and examined by (Stampacchia [1]), offers us a cohesive, novel and all-encompassing

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framework for examining a broad range of issues that come up in the fields of applied sciences, network analysis, finance, economics, elasticity and optimization and transportation. Using the new and innovative methods, variational inequalities have been expanded and generalized in a number of ways. Noor and Oettli [2] studied and investigated a family of trifunction-related variational inequalities. They used the Fan–Glicksberg–Hoffman Lemma to discuss the existence and uniqueness of the trifunction variational inequalities. Regarding the use of trifunction variational inequalities and various methods for resolving them (Noor, [3]; Noor and Noor, [4]; Noor and Oettli, [2] and the references therein).

We may explore a broad class of issues that arise in finance, economics, network analysis, transportation, elasticity, and optimization using the unified, natural, creative, and general framework that equilibrium problems theory offers. Theoretical developments and applications of this theory have proliferated in all fields of the pure and applied sciences, as seen in [5, 6, 7, 8, 9, 10, 11, 12]. Using fresh and creative approaches, equilibrium issues have been expanded and generalized in several ways. encouraged and inspired by the ongoing studies and endeavors in this intriguing field.

We present and examine a novel class of trifunction variational inequalities known as the new general mixed quasi equilibrium trifunction variational inequality containing the nonlinear variable $\varphi(.,.)$. This research and activity in this intriguing field serve as our inspiration and motivation. As special instances, this class comprises many classes of trifunctions, bifunctions, and classical variational inequalities. It is a very general and unifying class.

A number of numerical methods, such as projection, resolvent, and auxiliary principles, have been created and examined recently with the purpose of resolving variational inequalities. It is important to note that the trifunction hemivariational inequalities cannot be solved using projection-type approaches or their invariant forms. Glowinski et al. [13] developed the auxiliary principle approach, which is typically used to get around this limitation. Numerous approaches to resolving trifunction variational inequalities and associated optimization issues have been proposed and examined using this technique. It has been demonstrated that this strategy may provide a significant number of numerical techniques as special instances (Noor [14, 15, 16, 17, 18, 19, 20, 4, 21, 22, 3, 23]; Noor and Noor [24, 25]; Noor et al [26, 27]).

In this paper, we again use the auxiliary principle technique to propose and examine an iterative scheme aimed at solving these general mixed quasi equilibrium variational inequalities. The convergence of the method we propose necessitates partially relaxed jointly strongly monotone and skew symmetry conditions. Our findings signify an enhancement and refinement of results that were previously established. Given that the general mixed quasi equilibrium variational inequalities serve as special cases, the results demonstrated in this paper remain applicable to these specific problems.

2. PRELIMINARIES

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle ., . \rangle$ and $\|.\|$ respectively. Let $C(H)$ be a family of all nonempty compact subsets of H. Let $T : H \to C(H)$ be a multivalued operator. Let K be a nonempty closed convex set in H. Let $\varphi(.,.) : H \times H \to R\bigcup\{+\infty\}$ be a continuous bifunction. For a given trifunction $F(.,.,.) : K \times K \times K \to C(H)$, we consider the problem of finding $u \in K, v \in T(u)$ such that

$$F(g(u), Tu, g(v)) + \phi(g(v), g(v)) - \phi(g(u), g(v)) \geq 0, \quad \forall g(u) \in K \tag{1}$$

which is called the New general mixed quasi-equilibrium trifunction variational inequalities.

If $F(g(u), Tu, g(v)) \equiv \langle Tu, g(v) - g(u) \rangle$, then problem (1) is equivalent to finding $g(u) \in K$ such that

$$\langle Tu, g(v) - g(u) \rangle + \phi(g(v), g(v)) - \phi(g(u), g(v)) \geq 0, \quad \forall g(u) \in K \tag{2}$$

which is known as the general mixed quasi equilibrium variational inequality
If $g \equiv$ I in (2), then

$$\langle Tu, v - u \rangle + \phi(v, v) - \phi(u, v) \geq 0, \quad \forall u \in K \tag{3}$$

which is known as the general mixed quasi-variational inequality; see [9, 10, 11, 12, 28, 33, 34, 35, 36, 37] for applications and numerical results. We remark that if K(v) is a closed convex-valued set in H and

$$\phi(v, v) = \begin{cases} 0 & \text{if } v \in K(v) \\ +\infty & \text{otherwise,.} \end{cases}$$

is the indicator function of K(v), then the problem (2) is equivalent to finding $v \in K(v)$ such that

$$F(g(u), Tu, g(v)) \geq 0 \tag{4}$$

is known as the quasi-equilibrium problem and If $K(u) = K$, which is a convex set in H, we derive the original generalized equilibrium problem that was introduced and examined by Noor and Oettli in 1994 [2]. In summary, with an appropriate selection of the operator and the spaces, one can derive various known and novel classes of variational inequalities and equilibrium problems as specific instances of problem (1). This indicates that problem (1) is quite general, adaptable, and serves as a unifying framework. Moreover, it is widely recognized that a broad range of obstacle, unilateral, contact, free, moving, and equilibrium problems that emerge in mathematics, engineering, economics, and finance can be analyzed within the unified and general context of problems (1)–(4) and their specific cases, as referenced in [5, 6, 7, 8, 9, 10, 11, 12, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38].

We also need the following concepts and results.

Lemma 1. $\forall u, v \in H$

$$2\langle u, v\rangle = ||u+v||^2 - ||u||^2 - ||v||^2. \tag{5}$$

Definition 1. . The trifunction $F(.,.,.) : K \times K \times K \to R$ with respect to the operator T is said to be partially relaxed jointly strongly monotone, if there exists a constant $\alpha > 0$ such that

$$F(g(u), Tu, g(v)) + F(g(v), Tv, g(z)) \leq \rho||g(u) - g(z)||^2 \quad \forall g(u), g(v), g(z) \in K$$

Definition 2. . The trifunction $F(.,.,.) : K \times K \times K \to R$ with respect to the operator T is said to be jointly monotone, if

$$F(g(u), Tu, g(v)) + F(g(v), Tv, g(u)) \leq 0 \quad \forall g(u), g(v), g(z) \in K$$

Definition 3. . The trifunction $F(.,.,.) : K \times K \times K \to R$ with respect to the operator T is said to be jointly pseudomonotone, if

$$\begin{aligned} F(g(u), Tu, g(v)) + \phi(g(v), g(v)) - \phi(g(u), g(v)) \geq 0 \Rightarrow \\ - F(g(v), Tv, g(v)) + \phi(g(v), g(v)) - \phi(g(u), g(v)) \geq 0 \\ \forall g(u), g(v), g(z) \in K \end{aligned}$$

Definition 4. The bifunction $\phi(.,.)$ is said to be skew-symmetric, if,

$$\phi(g(u), g(u)) - \phi(g(u), g(v)) - \phi(g(v), g(u)) + \phi(g(v), g(v)) \geq 0, \quad \forall g(u), g(v) \in H$$

3. MAIN RESULT

We now use the auxiliary principle technique to suggest and analyze an iterative scheme for solving problem (1).

For a given $u \in K$, consider the problem of finding a unique $w \in H$ satisfying the auxiliary equilibrium problem such that

$$\begin{aligned}\rho F(g(u), Tu, g(v)) + \langle g(w) - g(u), g(v) - g(w)\rangle + \rho\phi(g(v), g(v)) \\ - \rho\phi(g(u), g(v)) \geq 0 \quad g(v) \in K\end{aligned} \tag{6}$$

where $\rho > 0$ is a constant.

We note that if w = u, then clearly w is a solution of the general mixed equilibrium problem (1). This observation enables us to suggest the following iterative method for solving (1).

Algorithm 1. For a given $u_0 \in H$, compute the approximate solution u_{n+1} by the iterative scheme

$$\begin{aligned}\rho F(g(w_n), Tw_n, g(v)) + \langle g(u_{n+1}) - g(w_n), g(v) - g(u_{n+1})\rangle - \rho\phi(g(u), g(u_{n+1})) \\ + \rho\phi(g(u_{n+1}), g(u_{n+1})) \geq 0 \qquad g(v) \in H\end{aligned} \tag{7}$$

$$\begin{aligned}\beta F(g(y_n), Ty_n, g(v)) + \langle g(w_n) - g(y_n), g(v) - g(w_n)\rangle - \beta\phi(g(u), g(w_n)) \\ + \beta\phi(g(w_n), g(w_n)) \geq 0 \qquad g(v) \in H\end{aligned} \tag{8}$$

$$\begin{aligned}\gamma F(g(u_n), Tu_n, g(v)) + \langle g(y_n) - g(u_n), g(v) - g(y_n)\rangle - \gamma\phi(g(u), g(y_n)) \\ + \gamma\phi(g(y_n), g(y_n)) \geq 0 \qquad g(v) \in H\end{aligned} \tag{9}$$

where $\rho > 0, \beta > 0$ and $\gamma > 0$ are constants.

If $F(u, Tu, v) \equiv F(u, v)$, then Algorithm 1 reduces to the following one.

Algorithm 2. For a given $u_0 \in H$, compute the approximate solution u_{n+1} by the iterative schemes

$$\begin{aligned}\rho F(g(w_n), g(v)) + \langle g(u_{n+1}) - g(w_n), g(v) - g(u_{n+1})\rangle - \rho\phi(g(u), g(u_{n+1})) \\ + \rho\phi(g(u_{n+1}), g(n_{n+1})) \geq 0 \qquad g(v) \in H\end{aligned} \tag{10}$$

$$\begin{aligned}\beta F(g(y_n), g(v)) + \langle g(w_n) - g(y_n), g(v) - g(w_n)\rangle - \beta\phi(g(u), g(w_n)) \\ + \beta\phi(g(w_n), g(w_n)) \geq 0 \qquad g(v) \in H\end{aligned} \tag{11}$$

$$\begin{aligned}\gamma F(g(u_n), g(v)) + \langle g(y_n) - g(u_n), g(v) - g(y_n)\rangle - \gamma\phi(g(u), g(y_n)) \\ + \gamma\phi(g(y_n), g(y_n)) \geq 0 \qquad g(v) \in H\end{aligned} \tag{12}$$

where $\rho > 0, \beta > 0$ and $\gamma > 0$ are constants.

If $F(u, v) = \langle Tu, v - u\rangle$, then Algorithm 2 reduces to the following one.

Algorithm 3. For a given $u_0 \in H$, compute the approximate solution u_{n+1} by the iterative schemes

$$\begin{aligned}\langle \rho Tg(w_n) + g(u_{n+1}) - g(w_n), g(v) - g(u_{n+1})\rangle - \rho\phi(g(u), g(u_{n+1}))\\ +\rho\phi(g(u_{n+1}), g(n_{n+1})) \geq 0 \qquad g(v) \in H\end{aligned}$$

$$\begin{aligned}\langle \beta Tg(y_n) + g(w_n) - g(y_n), g(v) - g(w_n)\rangle - \beta\phi(g(u), g(w_n))\\ +\beta\phi(g(w_n), g(w_n)) \geq 0 \qquad g(v) \in H\end{aligned}$$

$$\begin{aligned}\langle \gamma Tg(u_n) + g(y_n) - g(u_n), g(v) - g(y_n)\rangle - \gamma\phi(g(u), g(y_n))\\ +\gamma\phi(g(y_n), g(y_n)) \geq 0 \qquad g(v) \in H\end{aligned}$$

If the function $\phi(v, u) = \phi(v), \forall u \in H$, is the indicator function of a closed convex set K in H, then Algorithm 1 reduces to the following method for solving equilibrium (4).

Algorithm 4. For a given $u_0 \in H$, compute the approximate solution u_{n+1} by the iterative schemes

$$\rho F(g(w_n), Tw_n, g(v)) + \langle g(u_{n+1}) - g(w_n), g(v) - g(u_{n+1})\rangle \geq 0 \quad g(v) \in H$$

$$\beta F(g(y_n), Ty_n, g(v)) + \langle g(w_n) - g(y_n), g(v) - g(w_n)\rangle \geq 0 \qquad g(v) \in H$$

$$\gamma F(g(u_n), Tu_n, g(v)) + \langle g(y_n) - g(u_n), g(v) - g(y_n)\rangle \geq 0 \qquad g(v) \in H$$

For a suitable choice of the operators and the space H, one can obtain various new and known methods for solving variational inequalities and complementarity problems. For the convergence analysis of Algorithm 1, we need the following result.

Theorem 2. Let $v \in K$ be a solution of (1) and v_{n+1} be the approximate solution obtained from Algorithm 1. If $F(.,.,.) : K \times K \times K \to R$ and T is partially relaxed strongly monotone with constant $\alpha > 0$, and the bifunction $\phi(.,.)$ is skew-symmetric, then:

$$||v_{n+1} - v||^2 \leq (1 + 2\rho\alpha)||w_n - v||^2 - ||v_{n+1} - w_n||^2 \tag{13}$$

$$||w_n - v||^2 \leq (1 + 2\beta\alpha)||y_n - v||^2 - ||w_n - y_n||^2 \tag{14}$$

$$||y_n - v||^2 \leq (1 + 2\gamma\alpha)||v_n - v||^2 - ||y_n - v_n||^2 \tag{15}$$

Proof. Let $\bar{v} \in K$ be a solution of (1). Then

$$\begin{aligned}\rho F(g(\bar{u}), T\bar{u}, g(v)) + \rho\phi(g(v), g(v)) - \rho\phi(g(\bar{u}), g(v)\\ \geq 0 \qquad v \in K\end{aligned} \tag{16}$$

$$\beta F(g(\bar{u}), T\bar{u}, g(v)) + \beta\phi(g(v), g(v)) - \beta\phi(g(\bar{u}), g(v)) \geq 0 \qquad v \in K \tag{17}$$

$$\gamma F(g(\bar{u}), T\bar{u}, g(v)) + \gamma\phi(g(v), g(v)) - \gamma\phi(g(\bar{u}), g(v)) \geq 0 \qquad v \in K \tag{18}$$

where $\rho > 0, \beta > 0, \gamma > 0$ are constants.
Now taking $\bar{u} = v_{n+1}$ in (16) and $u = v$ in (7), then we have

$$\rho F(g(v_{n+1}), T\bar{u}, g(v)) + \rho\phi(g(v), g(v)) - \rho\phi(g(v_{n+1}), g(v)) \geq 0 \qquad v \in K \tag{19}$$

$$\rho F(g(w_n), Tw_n, g(v)) + \langle g(v_{n+1}) - g(w_n), g(v) - g(v_{n+1})\rangle + \rho\phi(g(v_{n+1}), g(v_{n+1})) - \rho\phi(g(v), g(v_{n+1})) \geq 0 \tag{20}$$

Adding (18) and (19), we have

$$\begin{aligned}\langle g(v_{n+1}) - g(w_n), g(v) - g(v_{n+1})\rangle &\geq -\rho\{F(g(w_n), Tw_n, g(v)) + F(g(v_{n+1}), T\bar{u}, g(v))\}\\ &\quad -\rho\{\phi(g(v), g(v)) - \phi(g(v), g(v_{n+1})) - \phi(g(v_{n+1}), g(v) + \phi(g(v_{n+1}), g(v_{n+1}))\}\\ &\geq -\alpha\rho|||g(w_n) - g(v)|||^2\end{aligned} \tag{21}$$

taking $g \equiv I$ in above we have

$$\langle v_{n+1} - w_n, v - v_{n+1}\rangle \geq -\alpha\rho|||w_n - v|||^2 \tag{22}$$

where we have used the fact that F(.,.,.) is partially relaxed jointly strongly monotone with constant $\alpha > 0$ and the skew symmetry of the bifunction $\phi(.,.)$.

putting $u = v - v_{n+1}$ and $v = v_{n+1} - w_n$, in (5) we get

$$2\langle v_{n+1} - w_n, v - v_{n+1}\rangle = |||v - w_n|||^2 - |||v - v_{n+1}|||^2 - |||v_{n+1} - w_n|||^2. \tag{23}$$

combining (22) and (23), we have

$$|||v_{n+1} - v\| \leq (1 + 2\alpha\rho)|||w_n - v|||^2 - |||v_{n+1} - w_n|||^2,$$

which is the required (13).

Taking $\bar{u} = w_n$ in (17) and u = v in (8)

$$\beta F(g(w_n), T\bar{u}, g(v)) + \beta\phi(g(v), g(v)) - \beta\phi(g(w_n), g(v)) \geq 0 \tag{24}$$

$$\beta F(g(y_n), Ty_n, g(v)) + \langle g(w_n) - g(y_n), g(v) - g(w_n)\rangle - \beta\phi(g(v), g(w_n)) + \beta\phi(g(w_n), g(w_n)) \geq 0 \tag{25}$$

Adding (25) and (24), we have

$$\begin{aligned}\langle g(w_n)-g(y_n), g(v) - g(w_n)\rangle &\geq \beta\{F(g(y_n), Ty_n, g(v)) + F(g(w_n), T\bar{u}, g(v))\} \\ &\quad -\beta\{\phi(g(v), g(v)) - \phi(g(v), g(w_n)) - \phi(g(w_n), g(v)) + \phi(g(w_n), g(w_n))\} \\ &\quad \geq -\beta\alpha|||g(y_n) - g(v)|||^2\end{aligned} \tag{26}$$

taking g = I in above we have

$$\langle w_n - y_n, v - w_n\rangle \geq -\beta\alpha|||y_n - v|||^2 \tag{27}$$

Since $F(.,.,.)$ is partially relaxed strongly monotone operator with constant $\alpha > 0$ and $\phi(.,.)$ is skew - symmetric.

Now taking $v = w_n - y_n$ and $u = v - w_n$ in (5), can be written as

$$2\langle w_n - y_n, v - w_n\rangle = |||v - y_n|||^2 - |||v - w_n|||^2 - |||w_n - y_n|||^2. \tag{28}$$

combining (27) and (28), we have

$$|||w_n - v|| \ \leq (1 + 2\beta\alpha)|||y_n - v|||^2 - |||w_n - y_n|||^2,$$

which is the required (14).

Similarly by taking $\bar{u} = v_{n+1}$ in (18) and $u = v$ in (9) and using the jointly partially relaxed strongly monotonicity of $F(.,.)$ and T; and skew symmetry of bifunction $\phi(.,.)$, we have

$$\gamma F(g(v_{n+1}), T\bar{u}, g(v)) + \gamma\phi(g(v), g(v)) - \gamma\phi(g(v_{n+1}), g(v)) \geq 0 \tag{29}$$

$$\begin{aligned}\gamma F(g(v_n), Tu_n, g(v)) + \langle g(y_n) - g(v_n), g(v) - g(y_n)\rangle - \gamma\phi(g(v), g(y_n)) \\ +\gamma\phi(g(y_n), g(y_n)) \geq 0\end{aligned} \tag{30}$$

Adding (29) and (30), we have

$$\begin{aligned}\langle g(y_n)-g(v_n), g(v) - g(y_n)\rangle &\geq \gamma\{F(g(v_n), Tu_n, g(v)) + F(g(v_{n+1}), T\bar{u}, g(v))\} \\ &\quad -\gamma\{\phi(g(v), g(v)) - \phi(g(v_{n+1}), g(v)) - \phi(g(v), g(y_n)) + \phi(g(y_n), g(y_n))\} \\ &\quad \geq -\gamma\alpha|||g(v_n) - g(v)|||^2\end{aligned} \tag{31}$$

taking g = I in above, we have

$$\langle y_n - v_n, v - y_n\rangle \geq -\gamma\alpha|||v_n - v|||^2 \tag{32}$$

letting $v = y_n - v_n$ and $u = v - y_n$ in (5), and combining the resultant with (32), we have

$$|||y_n - v\| \leq (1 + 2\gamma\alpha)|||y_n - v|||^2 - |||y_n - v_n|||^2,$$

which is required result (15) □

Theorem 3. Let H be a finite dimensional subspace, and let $0 < \rho < \frac{1}{2\alpha}$, $0 < \beta < \frac{1}{2\alpha}$, $0 < \gamma < \frac{1}{2\alpha}$. Let u_{n+1} be the approximate solution obtained from algorithm 1 and $v \in H$ be a solution of (1), then $\lim_{n\to\infty} v_n = v$.

Proof. Let v be the solution of (1). Since $0 < \rho < \frac{1}{2\alpha}$, $0 < \beta < \frac{1}{2\alpha}$, $0 < \gamma < \frac{1}{2\alpha}$, from (13), (14) and (15), we see that the sequences $\{v - w_n\}$, $\{v - y_n\}$ and $\{v - v_n\}$ are nonincreasing and consequently it follows that the sequences $\{v_n\}$, $\{y_n\}$ and $\{w_n\}$ are bounded.
furthermore, we have

$$\sum_{n=0}^{\infty} ||v_{n+1} - w_n||^2 \leq (1 + 2\rho\alpha)||w_0 - v||^2 - ||v_0 - v||^2$$

$$\sum_{n=0}^{\infty} ||w_n - y_n||^2 \leq (1 + 2\beta\alpha)||y_0 - v|| - ||w_0 - v||^2$$

$$\sum_{n=0}^{\infty} ||y_n - v_n||^2 \leq (1 + 2\gamma\alpha)||v_0 - v|| - ||y_0 - v||^2$$

which imply that

$$\lim_{n\to\infty} ||v_{n+1} - w_n|| = 0$$

$$\lim_{n\to\infty} ||w_n - y_n|| = 0$$

$$\lim_{n\to\infty} ||y_n - v_n|| = 0$$

Thus

$$\lim_{n\to\infty} ||v_{n+1} - v_n|| \leq \lim_{n\to\infty} ||v_{n+1} - w_n|| + \lim_{n\to\infty} ||w_n - y_n|| + \lim_{n\to\infty} ||y_n - v_n|| = 0 \quad (33)$$

Let $\hat{v}$ be the cluster point of $\{v_n\}$ and the subsequence $\{v_{n_j}\}$ converge to $\hat{v} \in H$. Replacing w_n and y_n by $\{u_{n_j}\}$ in (7), (8) and (9), taking the limit $n_j \to \infty$ and using (33), we have

$$F(g(\hat{u}), T(\hat{u}), g(v)) + \rho\phi(g(v), g(v)) - \rho\phi(g(u), g(v)) \geq 0 \quad g(v) \in H$$

which implies that $\hat{v}$ solves the general mixed quasi equilibrium problem (1) and

$$||v_{n+1} - \bar{v}||^2 \leq ||v_n - \bar{v}||^2$$

Thus it follows from the above inequality that the sequence $\{u_n\}$ has exactly one cluster point $\hat{v}$ and

$$\lim_{n\to\infty} v_n = v.$$

the require result. $\square$

4. CONCLUSION

In this research, we present general mixed quasi equilibrium variational inequalities, a new class of equilibrium problems. analyzing and proposing an iterative method for resolving general mixed quasi equilibrium variational inequalities using the auxiliary principle technique. The partially relaxed jointly strongly monotone and skew symmetry are necessary for the suggested approach to converge. Our findings constitute an advancement and enhancement of previously established findings. This paper's solutions are still valid for these kinds of problems since the general mixed quasi equilibrium variational inequalities are special examples.

REFERENCES

[1] G.Stampacchia, Formes bilineaires coercivities sur les ensembles coercivities sur les ensembles convexes. C. R. Acad. Sci. Paris 258 (1964), 4413–4416.

[2] M. A. Noor, W. Oettli, On general nonlinear complementarity problems and quasi-equilibria. Le Matematiche (Catania) 49 (1994), 313–331.

[3] M. A. Noor, Multivalued mixed quasi bifunction variational inequalities. J. Math. Anal. 1 (2010d), 1–7

[4] M. A. Noor, An extragradient algorithm for solving the general nonconvex variational inequalities. Appl. Math. Lett. 23 (2010a), 917–921.

[5] E. Blum, W. Oettli, From optimization and variational inequalities to equilibrium problems, Mathematics Students 63 (1994) 123–145.

[6] M.A. Noor, W. Oettli, On general nonlinear complementarity problems and quasi-equilibria, Le Matematiche 49 (1994) 313–331.

[7] M.A. Noor, K.I. Noor, On equilibrium problems, Applied Mathematics-Enotes (2004) 125–132.

[8] M.A. Noor, Multivalued general equilibrium problems, Journal of Mathematical Analysis and Applications 283 (2003) 140–149.

[9] F. Giannessi, A. Maugeri, P.M. Pardalos, Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models, Kluwer Academics Publishers, Dordrecht, Holland, 2001.

[10] F. Giannessi, A. Maugeri, Variational Inequalities and Network Equilibrium Problems, Plenum Press, New York, NY, 1995.

[11] F. Flores-Bazan, Existence theorems for generalized noncoercive equilibrium problems: The quasi-convex case, SIAM Journal on Optimization 11 (2000) 675–690.

[12] U. Mosco, Implicit Variational Problems and Quasi Variational Inequalities, Lecture Notes in Mathematics, vol. 543, Springer-Verlag, Berlin, 1976, pp. 83–126.

[13] R. Glowinski, J. Lions, R. Tremolieres, Numerical Analysis of Variational Inequalities. North-Holland, Amsterdam, Holland (1981).

[14] M. A. Noor, Set-valued mixed quasi variational inequalities and implicit resolvent equations. Math. Comput. Model. 29 (1999), 1–11.

[15] M. A. Noor, New approximation schemes for general variational inequalities. J. Math. Anal. Appl. 251 (2000), 217–229.

[16] M. A. Noor, Auxiliary principle technique for equilibrium problems. J. Optim. Theory Appl. 122 (2004a), 371–386.

[17] M. A. Noor, Some developments in general variational inequalities. Appl. Math. Comput. 152 (2004b), 199–277.

[18] M. A. Noor, Fundamentals of mixed quasi variational inequalities. Inter. J. Pure Appl. Math. 15 (2004c), 137–258.

[19] M. A. Noor, Fundamentals of equilibrium problems. Math. Inequal. Appl. 9 (2006), 529–566.

[20] M. A. Noor, . Implicit iterative methods for nonconvex variational inequalities. J. Optim. Theory Appl. 143 (2009), 619–624.

[21] M. A. Noor, On an implicit method for nonconvex variational inequalities. J. Optim. Theory Appl. 147 (2010b).

[22] M. A. Noor, Auxiliary principle technique for solving general mixed variational inequalities. J. Adv. Math. Stud. 3 (2010c) (2).

[23] M. A. Noor, New implicit method for general nonconvex variational inequalities. Bull. Math. Anal. Appl. 3 (2010e), 7–14.

[24] M. A. Noor, K. I. Noor, Iterative schemes for trifunction hemivariational inequalities. Optim. Lett. 4 (2010a).

[25] M. A. Noor, K. I. Noor, New system of general nonconvex variational inequalities. Appl. Math. E-Notes 10 (2010b), 76– 85.

[26] M. A. Noor, K. I. Noor, Th.M. Rassias, Some aspects of variational inequalities. J. Comput. Appl. Math. 47 (1993), 285–312.

[27] M. A. Noor, K. I. Noor, Z.Y. Huang, Bifunction hemivariational inequalities. J. Appl. Math. Comput (2010).

[28] R. Glowinski, J.L. Lions, R. Tremolieres, Numerical Analysis of Variational Inequalities, North-Holland, Amsterdam, Holland, 1981.

[29] M.A. Noor, Proximal methods for mixed quasi variational inequalities, Journal of Optimization Theory and Applications 115 (2002) 453–459.

[30] M.A. Noor, Some developments in general variational inequalities, Applied Mathematics and Computation 152 (2004) 199–277.

[31] B. Martinet, Regularization d'Inequations Variationnelles par Approximation Successive, Revue d'Automatique, Informatique, et Recherche Operationnelle, Serie Rouge, vol. 3, pp. 154–159, 1970.

[32] R.T. Rockafellar, Monotone operators and proximal point algorithm, SIAM Journal on Control and Optimization 14 (1976) 877–898.

[33] M. A. Noor, Nonlinear variational inequalities in elastostatics, International Journal of Engineering Sciences 26 (1988) 1043–1053.

[34] M. Patriksson, Nonlinear Programming and Variational Inequality Models: A Unified Approach, Kluwer Academic Publishers, Dordrecht, Holland, 1999.

[35] M. A. Noor, K.I. Noor, T.M. Rassias, Some aspects of variational inequalities, Journal of Computational and Applied Mathematics 47 (1993) 285–312.

[36] M. A. Noor, Set-valued mixed quasi variational inequalities and implicit resolvent equations, Mathematical and Computer Modelling 29 (1999) 1–11.

[37] M. A. Noor, Mixed quasi variational inequalities, Applied Mathematics and Computations 146 (2003) 553–578.

[38] D.L. Zhu, P. Marcotte, Cocoercivity and its role in the convergence of iterative schemes for solving variational inequalities, SIAM Journal on Optimization 6 (1996) 714–726.