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An Efficient Analytical Fractional Integral Transform Technique with Some Properties and Applications

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Abstract

In this work, we propose a new analytical fractional integral transform technique for the first time in the fractional calculus environment. The fractional derivatives are considered with reference to modified Riemann-Liouville derivatives. Some properties including derivative theorem and integral theorem are presented for this transform. Certain elementary and useful functions are produced analytically to examine the efficaciousness and simplicity of this technique. The proposed technique is an efficient technique for obtaining the exact solutions of linear fractional steady heat transfer problem, fractional initial value problem and fractional kinetic equation. The present technique performs extremely well in terms of efficiency and simplicity.

Keywords: Fractional Integral transform; Fractional partial differential equation; Mittag-Leffler function; Fractional derivative and fractional integral; Analytical solution.

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1. INTRODUCTION

Integral transforms have been tremendously applied by the investigators to solve ordinary and partial differential equations since their origination. This technique has also been used as a mathematical tool for solving many real world problems arising in different sectors of science and engineering specifically in the field of information

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technology, communication systems, fluid flow, heat-diffusion theory, fluid mechanics, viscoelasticity, biological sciences, chemistry, physics, applied mathematics and finance [1, 2, 3, 4, 5, 6].

The Laplace transform of the function $f(t)$ is given as follows[1]

$$L[f(t)] = \hat{f}(\varpi) = \int_0^\infty f(t)e^{-\varpi t}dt \tag{1}$$

provided the above integral converges for some s, and L denotes the Laplace tranform operator.

In the last two decades many Laplace-type integral transforms were introduced by the inventors such as Sumudu [7], Aboodh [8], Natural [9], Elzaki [10, 11], Kamal [12], G-transform [13, 14], Mahand [15], Pourreza [16], Sawi transforms [17] and new general integral transform [18].

A new Laplace-type integral transform (IT) was proposed by Yang in 2016 for finding the analytical solution for the steady heat-transfer problem, which is given as follows [19]

$$Y[\psi(t)] = \hat{\psi}(\varpi) = \int_0^\infty \psi(t)\exp\left(-\frac{t}{\varpi}\right)dt \tag{2}$$

provided the above integral converges for some ϖ, where $t \geq 0$, $\varpi \neq 0$ and Y denotes the new integral transform operator.

Although the integral transform techniques are highly recognized by the inventors using different sectors of mathematical physics and engineering sciences for more than a century, but some traditional mathematical problems could not be handled by standard integral transforms, therefore we need a fractionalization of transforms.

In the recent years, extension of the transforms have been a dominant area of investigation because It has been established through investigation that fractionalization of the transform increases its efficiency and productivity, so the transforms becomes more competent and powerful [20, 21, 22].

In last two decades, multiple definitions of fractional order integral transforms were experimented by the investigators using different mathematical aspects such as fractional Fourier [23], fractional Hankel [24], fractional Laplace [25], fractional Sumudu [26], fractional Wavelet [27], and fractional Mellin [28] transforms. In fact due to their out standing performance they have been much explored and become more prominent and efficacious tools as compare to their classical counterparts.

With this context, the main objective of this paper is focused on:

1. Introduce a new definition of fractional order integral transform.
2. Some algebraic and operational properties are presented.
3. Few examples are given for this transform, which are frequently used in many

applications of mathematical science and engineering.
4 This novel technique has been employed for finding the analytical solutions for the fractional steady heat transfer problem, fractional initial value problem and fractional kinetic equation.
This paper is organized as follows: In Section 2, a new fractional order integral transform is presented including some of its properties. In the Section 3, some examples are given for this transform. Section 4 contains exact solutions of fractional steady-heat transfer problem and fractional initial value problem using this technique. Finally , Section 5 concludes the article.

2. A NEW FRACTIONAL ORDER INTEGRAL TRANSFORM AND SOME OF ITS PROPERTIES

In this section, we propose a new fractional order integral transform, and some of its important properties including derivative theorem and integral theorem have been derived analytically.
Let $\varphi(x)$ be a real valued and integrable function defined on the interval $(0,+\infty)$, then a new fractional order integral transform of this function, is denoted by $\hat{\varphi}_\alpha(\eta)$ and is defined by the following integral

$$\Lambda_\alpha[\varphi(x)] = \hat{\varphi}_\alpha(\eta) = \int_0^{+\infty} E_\alpha\left(-\frac{x^\alpha}{\eta^\alpha}\right)\varphi(x)(dx)^\alpha, 0<\alpha\leq 1 \qquad (3)$$

provided that the integral (3) converges for some η, where $x \geq 0$, $\eta \neq 0$, Λ_α denotes a new FrIT operator, and $E_\alpha(z)$ denotes the Mittag-Leffler function [29], and it is given by

$$E_\alpha(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+1)}, \alpha \in C, z \in C \quad and \quad Re(\alpha) > 0 \qquad (4)$$

We notice that a new fractional order integral transform (3) is reduces to a new integral transform (2) when $\alpha = 1$. Therefore it is clear that, this new transform (3) is a fractionalization of (2).
Theorem 1. If $\Lambda_\alpha[\varphi_k(x)]$ is a new fractional order intrgral transform of the function $\varphi_k(x)$, then

$$\Lambda_\alpha[\sum_{k=1}^{n} b_k\varphi_k(t)] = \sum_{k=1}^{n} b_k\Lambda_\alpha[\varphi_k(t)]$$

where $k \in N$ and $b_k, k = 1,2,...,n$ are any arbitrary constants.
Proof: Using Eq. (3), we have the result.

Theorem 2. If $\Lambda_\alpha[\varphi(x)] = \hat{\varphi}_\alpha(\eta)$, then

$$\Lambda_\alpha[\varphi(bx)] = \frac{1}{b^\alpha}\hat{\varphi}_\alpha(b\eta)$$

where b is an any arbitrary constant. Provided that $b > 0$
Proof: Using Eq. (3), and substituting $bx = v$, we have the result.
Theorem 3. If $\Lambda_\alpha[\varphi(x)] = \hat{\varphi}_\alpha(\eta)$, then

$$\Lambda_\alpha[\varphi^\alpha(x)] = \frac{1}{\eta^\alpha}\hat{\varphi}_\alpha(\eta) - \Gamma(\alpha+1)\varphi(0), \tag{5}$$

provided that $\eta > 0$
where $\varphi^\alpha(x)$ denotes the Modified fractional Riemann-Liouville derivative of the function $\varphi(x)$, which is proposed by Jumarie in 2006 [30].
Proof: Using Eq. (3), we have

$$\Lambda_\alpha[\varphi^\alpha(x)] = \int_0^{+\infty} E_\alpha\left(-\frac{x^\alpha}{\eta^\alpha}\right)\varphi^\alpha(x)(dx)^\alpha \tag{6}$$

Taking fractional integration by parts formula in right hand side of above equation, we obtain the desired result.
Theorem 4. If $\Lambda_\alpha[\varphi(x)] = \hat{\varphi}_\alpha(\eta)$, then

$$\Lambda_\alpha\left[\int_0^x \varphi(t)(dt)^\alpha\right] = \Gamma(\alpha+1)\eta^\alpha\hat{\varphi}_\alpha(\eta) \tag{7}$$

where $(dt)^\alpha$ denotes the fractional integral, which is given as [31]

$$\int_0^u g(x)(dx)^\alpha = \alpha\int_0^u (u-x)^{\alpha-1}g(x)dx \tag{8}$$

Proof: Using Eq. (5), we have

$$\Lambda_\alpha\left[\int_0^x \varphi(t)(dt)^\alpha\right] = \eta^\alpha\Lambda_\alpha\left[\frac{d^\alpha}{dt^\alpha}\int_0^x \varphi(t)(dt)^\alpha\right] \tag{9}$$

Therefore, we have the result.
Theorem 5. If ${}_0D_t^{-\alpha}$ denotes the Riemann-Liouville fractional integral operator [32], $Re(\alpha) > 0$, then

$$\Lambda_\alpha[{}_0D_t^{-\alpha}\varphi(t)] = \eta^\alpha\hat{\varphi}_\alpha(\eta) \tag{10}$$

where

$$ {}_0D_t^{-\alpha}\varphi(t)(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-x)^{\alpha-1}\varphi(x)dx, for \quad t>0 \quad and \quad Re(\alpha)>0. \tag{11}$$

Proof: Using a new fractional order integral transform, we have

$$\Lambda_\alpha[{}_0D_t^{-\alpha}\varphi(t)]=\Lambda_\alpha\left[\frac{1}{\Gamma(\alpha)}\int_0^t (t-x)^{\alpha-1}\varphi(x)dx\right] \tag{12}$$

Using Eq. (7), we have the result.

Theorem 6. If $\varphi(x)=E_\alpha(-cx^\alpha)$, where c is any arbitrary constant, then

$$\Lambda_\alpha[E_\alpha(-cx^\alpha)]=\frac{\Gamma(\alpha+1)\eta^\alpha}{1+c\eta^\alpha} \tag{13}$$

Proof: With the help of Eq.(3), we have

$$\Lambda_\alpha[E_\alpha(-cx^\alpha)]=\int_0^{+\infty} E_\alpha\left[-x^\alpha\left(\frac{1}{\eta^\alpha}+c\right)\right](dx)^\alpha \tag{14}$$

Using the following result in above equation, we have the desired result

$$\Gamma_\alpha(u)=\frac{1}{\Gamma(\alpha+1)}\int_0^\infty E_\alpha(-t^\alpha)t^{(u-1)\alpha}(dt)^\alpha \tag{15}$$

Remark: If we substitute $c=0$ in above equation, then we have

$$\Lambda_\alpha[1]=\Gamma(\alpha+1)\eta^\alpha \tag{16}$$

3. EXAMPLES

In this section, we obtain a new fractional order integral transform for some elementary functions. These functions are frequently used in many applications of mathematical science and engineering.

Ex.1: We consider a following functions

$$\varphi(x)=x^{\alpha k}, 0<\alpha\leq 1 \quad and \quad k\geq 0 \tag{17}$$

then a new fractional order integral transform of above function is given by

$$\Lambda_\alpha[\varphi(x)]=(\Gamma(\alpha+1))^{(k+1)}(k!)\eta^{(k+1)\alpha} \tag{18}$$

Proof: Using the definition of a new fractional order integral transform, we have

$$\Lambda_\alpha[x^{\alpha k}] = \int_0^{+\infty} E_\alpha\left(-\frac{x^\alpha}{\eta^\alpha}\right) x^{\alpha k}(dx)^\alpha \tag{19}$$

With the help of Eq.(15), we have

$$\Lambda_\alpha[\varphi(x)] = (\Gamma(\alpha+1))^{(k+1)}(k!)\eta^{(k+1)\alpha} \tag{20}$$

Ex.2: We consider a following functions

$$\varphi(x) = \exp(ax^\alpha), a > 0. \tag{21}$$

then, we have

$$\Lambda_\alpha[\exp(ax^\alpha)] = \Gamma(\alpha+1)\eta^\alpha \sum_{k=0}^{\infty} (a\Gamma(\alpha+1)\eta^\alpha)^k \tag{22}$$

Proof: Using (3), we have

$$\Lambda_\alpha[\exp(ax^\alpha)] = \int_0^{+\infty} E_\alpha\left(-\frac{x^\alpha}{\eta^\alpha}\right) \exp(ax^\alpha)(dx)^\alpha \tag{23}$$

Using (18) in above equation, we have the required result.

Ex.3: We consider a following function

$$\varphi(x) = \sin(ax^\alpha), a > 0 \tag{24}$$

then a new fractional order integral transform of above function is given by

$$\Lambda_\alpha[\sin(ax^\alpha)] = (\Gamma(\alpha+1))^2\eta^{2\alpha} \sum_{k=0}^{\infty} (-1)^k a^{2k+1}(\Gamma(\alpha+1))^{2k}\eta^{2k\alpha} \tag{25}$$

Proof: Using (3) and (18), we have the desired result.

Similarly we can derived the following important results using (3) and (18)

Ex.4: If

$$\varphi(x) = \cos(ax^\alpha), a \geq 0$$

then

$$\Lambda_\alpha[\cos(ax^\alpha)] = (\Gamma(\alpha+1))\eta^\alpha \sum_{k=0}^{\infty} (-1)^k a^{2k}(\Gamma(\alpha+1))^{2k}\eta^{2k\alpha}$$

Ex.5: If

$$\varphi(x) = \sinh(ax^{\alpha}), a \geq 0$$

then

$$\Lambda_{\alpha}[\sinh(ax^{\alpha})] = (\Gamma(\alpha+1))^{2}\eta^{2\alpha}\sum_{k=0}^{\infty} a^{2k+1}(\Gamma(\alpha+1))^{2k}\eta^{2k\alpha}$$

Ex.6: If

$$\varphi(x) = \cosh(ax^{\alpha}), a \geq 0$$

then

$$\Lambda_{\alpha}[\cosh(ax^{\alpha})] = (\Gamma(\alpha+1))\eta^{\alpha}\sum_{k=0}^{\infty} a^{2k}(\Gamma(\alpha+1))^{2k}\eta^{2k\alpha}$$

4. APPLICATIONS

In this section, we have obtained analytical solutions for the fractional steady-heat transfer and fractional initial value problems using a new fractional order integral transform.

4.1. Fractional steady-heat transfer problem

We consider the fractional steady-heat transfer equation in the form of [34]

$$-hMT(x) = \rho V c_p T^{\alpha}(x), x \in (0,\infty) 0 < \alpha \leq 1 \tag{26}$$

with the condition

$$T(0) = \gamma, \quad when \quad x = 0 \tag{27}$$

where h- the convection heat transfer coefficient, M-the surface area of the material, V-the volume, c_p- the specific heat of the mater and $T(x)$ is the temperature.
Applying a new fractional order integral transform in Eq. (26) with respect to x , we have

$$-hM\hat{T_{\alpha}}(\eta) = \rho V c_p\left(\frac{T_{\alpha}(\eta)}{\eta^{\alpha}} - \Gamma(\alpha+1)T(0)\right) \tag{28}$$

Using the Eq. (27), we have

$$\hat{T_{\alpha}}(\eta) = \frac{\Gamma(\alpha+1)\gamma\eta^{\alpha}}{\left(1+\frac{hM\eta^{\alpha}}{\rho V c_p}\right)} \tag{29}$$

Applying inverse operator of a new fractional order integral transform in above equation, we have

$$T(x) = \gamma E_{\alpha}\left(-\frac{hMx^{\alpha}}{\rho V c_p}\right), x \in (0, \infty) \tag{30}$$

which is the exact solution in terms of Mittag-Leffler function of fractional steady-heat transfer problem and this solution is in agreement with the exact solution of steady-heat transfer problem by using (2) when $\alpha = 1$ [19].

4.2. Fractional initial value problem

We consider a fractional initial value problem in the form of [35]

$$\frac{d^{\alpha}y(x)}{dx^{\alpha}} = Ay(x), x \in (0, \infty) \tag{31}$$

where $y : [0, \infty) \to R^n$ and $A \in R^{n \times n}$
with the initial condition

$$y(0) = y_0 \tag{32}$$

Taking a new fractional order integral transform on both sides in Eq.(31) with respect to x, we have

$$\left[\frac{\hat{y}_{\alpha}(\eta)}{\eta^{\alpha}} - \Gamma(\alpha+1)y(0)\right] = A\hat{y}_{\alpha}(\eta) \tag{33}$$

Using the initial condition (32) in above equation, we have

$$\hat{y}_{\alpha}(\eta) = \frac{\Gamma(\alpha+1)y_0\eta^{\alpha}}{1 - A\eta^{\alpha}} \tag{34}$$

Applying inverse operator of a new fractional order integral transform in above equation, we have obtained the exact solution in terms of Mittag-Leffler function of fractional initial value problem

$$y(x) = y_0 E_{\alpha}(Ax^{\alpha}), x \in (0, +\infty) \tag{35}$$

4.3. Fractional Kinetic Equation

Considered the following fractional kinetic equation [36]

$$N(t) - N_0 f(t) = -c^{\nu}{}_0D_t^{-\nu}N(t), for \quad Re(\nu) > 0 \tag{36}$$

where $N(t)$ denotes the number density of a given species at time t, c is a positive constant and ${}_0D_t^{-\nu}$ denotes the Riemann-Liouville fractional integral operator, which is defined by Eq.(11)
We apply a new fractional order integral transform on both side of equation (36) and using Eq. (10) we have

$$\hat{N}_\nu(\eta) - N_0\hat{f}_\nu(\eta) = -c^\nu\eta^\nu\hat{N}_\nu(\eta) \tag{37}$$

Therefore, we have

$$\hat{N}_\nu(\eta) = \frac{N_0}{1+c^\nu\eta^\nu}\hat{f}_\nu(\eta), provided\,(|c\eta| < 1) \tag{38}$$

Applying inversion operator of a new fractional order integral transform of both sides of above equation, we have

$$N(t) = N_0{\Lambda_\alpha}^{-1}\left[\frac{\hat{f}_\nu(\eta)}{1+c^\nu\eta^\nu}\right] \tag{39}$$

provided inverse operator exists and this is the required general solution of given equation.
Special Cases: Substitutiong $f(t) = 1$ in above equation, then we have

$$N(t) = N_0\Gamma(\alpha+1){\Lambda_\alpha}^{-1}\left[\frac{\eta^\nu}{1+c^\nu\eta^\nu}\right] \tag{40}$$

Using the equation (13), we have

$$N(t) = N_0E_\alpha(-c^\alpha t^\alpha) \tag{41}$$

which is the required analytical solution in exact form for the fractional kinetic equation.

5. CONCLUSION

In this work, we introduce a new fractional order integral transform in fractional calculus environment for the first time and this transform is different form existing transforms. Several properties and examples have been presented to examine the efficacy and simplicity of the proposed technique. An efficient technique used to solve fractional heat-steady transfer problem, fractional initial value problem and fractional kinetic equation. We see that this analytic technique gives exact solution, therefore this technique will be very efficient and powerful for solving the various physical problems of mathematical sciences and engineering. We sincerely hope that the produced results will yield numerous applications in diverse fields of sciences and engineering.

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