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Fixed Point Theorems via Generalised $\phi - WF$-contraction with Application to Beam Bending Problems

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Abstract

This paper, we present a fixed point theorems in b-metric spaces using ϕ-altering distance functions and generalized Wardowski-type contractions[22]. The results are applied to a fourth-order boundary value problem arising from Euler–Bernoulli beam theory. By reformulating the differential equation as an integral equation using Green's function, we establish the existence and uniqueness of the solution. A numerical example with graphical illustration is provided to verify the theoretical results, highlighting the practical relevance of fixed point theory in structural engineering.

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Key Words and Phrases : Fixed point theorem, b-metric space $\phi - alteringfunction$, Wardowski-type Contractions.

1. INTRODUCTION

Fixed point theory is a relevant area of research, mainly due to its applicability to various fields, specially in the study of the properties of the solutions to differential equations. Banach Contraction Principle is the most celebrated result, and it

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provides not only the existence, but also the uniqueness and an iterative process to approximate the fixed point. Another relevant extension was given by Alber and Guerre-Delabriere[1] for weakly contractive maps in Hilbert Spaces, see [1]. Rhoades [20] continued the study on the topic and also Dutta and Chaudhury [7] provided a generalization. The starting point in our study will be the result by Gubran, Alfaqih and Imdad included in [8], where the following terminology was used.Some other interesting concepts to deal with weak contractions are, for instance, the notion of implicit contractive function given in 1997 by Popa [16], which was later discussed and considered by other authors, as we can see, for instance, in the research works [2, 3, 10, 11, 17].

Another interesting work is that by Wardowski [22], who gave in 2012 an extension of contraction mapping principle for the case of the new concept of F-contractions, notion that fits adequately in the sense that the corresponding mappings exhibit uniqueness of fixed point in the context of complete metric spaces.

In 2022 Rosana Rodríguez-López et.al. [21] introduced a new class of mixed contractions which allow to revised and generalized some results of R. Gubran, W. M. Alfaqih and M. Imdad[8]. And they also provided an example corresponding to this class of mappings. Further, presented an application to the solvability of a two-point boundary value problem for second order differential equations.

2. PRELIMINARIES

We begin by introducing some concepts needed, where we denote $\mathbb{R}_+ = [0, \infty)$. The notion of F-contraction introduced by Wardowski[22] is presented in Definition 1. For this definition, the author considered the class of functions F consisting of all functions $F : \mathbb{R}_+ \to \mathbb{R}$ satisfying the conditions $(F1)$–$(F3)$ specified below:

1. (F1): F is strictly increasing.
2. (F2): For every sequence $\{s_n\}$ of positive real numbers, we have
$$\lim_{n\to\infty} s_n = 0 \iff \lim_{n\to\infty} F(s_n) = -\infty.$$
3. (F3): There exists $k \in (0, 1)$ such that $\lim_{s\to 0^+} s^k F(s) = 0$.

As mentioned in [8], the functions F given, respectively, by $F(s) = \ln(s)$, $F(s) = s + \ln(s)$ and $F(s) = \frac{-1}{\sqrt{s}}$, belong to the family f.

Definition 2.1. *[22] A self-mapping f on a metric space (X, d) is an $F-contraction$ if there exists $\tau \geq 0$ such that, for all $x, y \in X$ with $d(fx, fy) > 0$, we have*

$$\tau + F(d(fx, fy)) \leq F(d(x, y)), \tag{2.1}$$

where $F : \mathbb{R}_+ \to \mathbb{R}$ is a mapping satisfying conditions $(F1)$–$(F3)$. More recently, in [8], Gubran et al. introduced a new class of contractions, called WF-contractions,presented as a mixed type of weak and F-contractions, but different from both concepts, and defined as explained in Definition (2.2)*. For this concept, they considered two families of functions: $\mathbb{G}$ The family $\mathbb{G}$ given by all functions $G : [0, \infty) \to [0, \infty)$ satisfying the following two properties:*

1. *(G1): G is strictly increasing.*
2. *(G2): There exists $k \in (0, 1)$ such that $\lim_{s \to 0^+} s^k G(s) = 0$. $\triangle$ The family $\triangle$ given by all functions $\delta : [0, \infty) \to [0, \infty)$ satisfying the property:*
3. *(G3): $\delta(t) > 0$ for all $t > 0$ and for every strictly decreasing sequence $\{s_n\}$ of positive real numbers, we have*

$$\lim_{n \to \infty} \delta(s_n) = 0 \ \Rightarrow \ \lim_{n \to \infty} s_n = 0.$$

Definition 2.2. *[8] A self-mapping f on a metric space (X, d) is said to be a $WF - contraction$ if there exist two functions $G, \delta : [0, \infty) \to [0, \infty)$ such that, for all $x, y \in X$ with $d(fx, fy) > 0$, we have*

$$\delta(d(x, y)) + G(d(fx, fy)) \leq G(d(x, y)), \tag{2.2}$$

where $G \in \mathbb{G}$ and $\delta \in \triangle$.

Definition 2.3. *[21] A self-mapping f on a metric space (X, d) is a Generalized WF-contraction if there exist two functions $G, \delta : [0, \infty) \to [0, \infty)$ such that, for all $x, y \in X$ with $d(fx, fy) > 0$, we have*

$$\delta(d(x, y)) + G(d(fx, fy)) \leq G\left(\max\left\{d(x, y), d(x, fx), d(y, fy)\right\}\right), \tag{2.3}$$

where G satisfies condition $(G1)$, and δ satisfies the conditions $(G2)$ and $(G3)$, that is, $G \in \widetilde{G}$ and $\delta \in \widetilde{\triangle}$.

Lemma 1. *[[8], Lemma 1] Let (X, d) be a metric space and $\{t_n\}$ be a sequence of positive real numbers such that*

$$\delta(t_n) + G(t_{n+1}) \leq G(t_n),$$

for all n, where $G \in \mathbb{G}$ and $\delta \in \triangle$. Then, the sequence $\{t_n\}$ is strictly decreasing and

$$\sum_{i=1}^{\infty} \delta(t_i) < \infty.$$

In this section, we introduce the concept of a generalized WF-contraction[21] and prove fixed point results in $\phi - WF$-contraction with application to beam bending problems and uniqueness of the theorem and establish a fixed point theorem valid for b- metric spaces.

3. MAIN RESULT

Theorem 3.1. *Let (X, d) be a complete b-metric space with constant $s \geq 1$ and let $f : X \to X$ be a continuous mapping satisfies the Generalized WF- contraction condition.*

$$\delta(d(fx, fy)) + G(d(fx, fy)) \leq G\left(\max\left\{d(x, y),\ d(x, fx),\ d(y, fy)\right\}\right). \tag{3.1}$$

for some $\delta \in \widetilde{\triangle}$, $G \in \widetilde{G}$ where $\delta(t) > 0$ for $t > 0$ and G is strictly increasing. Then f has a unique fixed point in X.

Proof. We start by selecting an arbitrary element $x_0 \in X$, and defining the sequence $\{x_n\}$ in X by recurrence, as follows $x_{n+1} = fx_n$, for all $n \in \mathbb{N}_0$, In case that $x_n = x_{n+1}$ for some $n \in \mathbb{N}_0$, then we have proved the existence of a fixed point for f . Therefore, we assume that $d(x_n, x_{n+1}) > 0$ for every $n \in \mathbb{N}_0$. Similar to [8], we denote by $t_n = d(x_n, x_{n+1})$, $n \in \mathbb{N}_0$, so that $\{t_n\}$ is a sequence of positive real numbers. Now, choosing $x = x_n$ and $y = x_{n+1}$ in the inequality (2.3), we get

$$\delta(t_n) + G(t_{n+1}) \leq G(\max\{t_n, t_{n+1}\}. \tag{3.2}$$

By the positiveness of δ on $(0, +\infty)$ and the strictly increasing character of G, we get that

$$G(t_{n+1}) < G(\max\{t_n, t_{n+1}\}) = \max\{G(t_n), G(t_{n+1})\},$$

so that

$$\max\{G(t_n), G(t_{n+1})\} = G(t_n),$$

and (3.2) is reduced to

$$\delta(t_n) + G(t_{n+1}) \leq G(t_n),$$

for every $n \in \mathbb{N}_0$. Hence, similar to [8], by applying Lemma 1, we deduce the strictly decreasing character of the sequence $\{t_n\}$ and the convergence of the series

$\sum_{i=0}^{\infty} \delta(t_i) < \infty$,thus, the general term is convergent to zero, that is, $\lim_{n\to\infty} \delta(t_n) = 0$, which implies that

$$\lim_{n\to\infty} t_n = 0. \tag{3.3}$$

Let $m > n$. Then using b-triangle inequality

$$\begin{aligned}(d(x_m, x_n)) &\leq s(d(x_m, x_{m-1}) + d(x_{m-1}, x_{m-2}) + \cdots + d(x_{n+1}, x_n)) \\ &= s \sum_{i=n}^{m-1} d(x_i, x_{i+1}) \\ &= s \sum_{i=n}^{m-1} t_i.\end{aligned}$$

Now using $\lim_{n\to\infty} t_n = 0$ as $n \to \infty$ due to convergence of $\sum_{i=0}^{\infty} t_i < \infty$. This proves that $\{x_n\}$ is a Cauchy sequence.
By the completeness of X, there exists $x \in X$ which is its limit, and using the continuity of f , we have

$$x = \lim_{n\to\infty} x_{n+1} = f\left(\lim_{n\to\infty} x_n\right) = f(x).$$

If $x \neq y$ are fixed point of f then using contradiction condition $\delta(d(x, y)) \leq 0$ but $\delta > 0$ for $d(x, y) > 0$. Since the uniqueness was justified, the proof is finished □

Theorem 3.2. *Let (X, d) be a complete b-metric space with constant $s \geq 1$ and let $f : X \to X$ be a continuous function. Suppose there exist function $\phi \in \Phi$, $\delta \in \widetilde{\triangle}$ and $G \in \widetilde{G}$ such that $x, y \in X$.*

$$\phi(\delta(d(fx, fy)) + G(d(fx, fy)) \leq \phi(G\left(\max\left\{d(x, y),\ d(x, fx),\ d(y, fy)\right\}\right). \tag{3.4}$$

Then f has a unique fixed point in X.

Proof. We start by selecting an arbitrary element $x_0 \in X$, and defining the sequence $\{x_n\}$ in X by recurrence, as follows $x_{n+1} = fx_n$, for all $n \in \mathbb{N}_0$, in case that $x_n = x_{n+1}$ for some $n \in \mathbb{N}_0$, then we have proved the existence of a fixed point for f. Therefore, we assume that $d(x_n, x_{n+1}) > 0$ for every $n \in \mathbb{N}_0$. Similar to [8], we denote by $t_n = d(x_n, x_{n+1})$, $n \in \mathbb{N}_0$, so that $\{t_n\}$ is a sequence of positive real numbers. Now, choosing $x = x_n$ and $y = x_{n+1}$ in the inequality (2.3), we get

$$\phi(\delta(t_n)) + G(t_{n+1}) \leq \phi(G(\max\{t_n, t_{n+1}\}).$$

Since ϕ is strictly increasing. By the positiveness of δ on $(0, +\infty)$ and the strictly increasing character of G, we get that

$$G(t_{n+1}) < G(\max\{\phi(t_n, t_{n+1})\} = max\phi(G(t_n), G(t_{n+1})),$$

so that

$$\max(\phi\{G(t_n), G(t_{n+1})\}) = \phi(G(t_n)),$$

and (3.2) is reduced to

$$\phi(\delta(t_n)) + G(t_{n+1}) \leq \phi(G(t_n)),$$

for every $n \in \mathbb{N}_0$. Hence, similar to [8] , by applying Lemma 1, we deduce the strictly decreasing character of the sequence $\{x_n\}$ and the convergence of the series $\sum_{i=0}^{\infty} \delta(t_i) < \infty$,thus, the general term is convergent to zero, that is,$\lim_{n\to\infty} \delta(t_n) = 0$, which implies that

$$\lim_{n\to\infty} t_n = 0. \tag{3.3}$$

Let $m > n$. Then using b-triangle inequality

$$\begin{aligned}(d(x_m, x_n)) &\leq s(d(x_m, x_{m-1}) + d(x_{m-1}, x_{m-2}) + \cdots + d(x_{n+1}, x_n))\\ &= s\sum_{i=n}^{m-1} d(x_i, x_{i+1})\\ &= s\sum_{i=n}^{m-1} t_i.\end{aligned}$$

Now using $\lim_{n\to\infty} t_n = 0$ as $n \to \infty$ due to convergence of $\sum_{i=0}^{\infty} t_i < \infty$. This proves that $\{x_n\}$ is a Cauchy sequence.
By the completeness of X, there exists $x \in X$ which is its limit, and using the continuity of f , we have

$$x = \lim_{n\to\infty} x_{n+1} = f\left(\lim_{n\to\infty} x_n\right) = f(x).$$

Assume $x \neq y$ are fixed points. Then $\phi(\delta(d(x, y) + G(d(x, y)) \leq \phi(G(max(d(x, y), 0, 0)) = \phi(G(d(x, y)),$
$\delta(d(x, y) \leq 0$. Which is contradiction, since $\delta > 0$ on $(0, \infty)$ the uniqueness was justified, the proof is finished □

Example 3.3. *Let $X = [0, \infty)$ and $f : X \to X$ be a self mapping on X, defined by $d(x, y) = |x - y|^2$ with constant $s = 2$, $f(x) = \frac{x}{2}$ continuous on X and $\phi(t) = \sqrt{2}$, $\delta(t) = \frac{t}{2}$, $G(t) = t$.So (X, d) is a complete b-metric space.*

Solution: $f(x) = \frac{x}{2}$ and $f(y) = \frac{y}{2}$.

$$So\ \ d(fx, fy) = \frac{|x-y|^2}{2} = \frac{d(x, y)}{4}, \quad \delta(d(fx, fy)) = \frac{d(x, y)}{8},\ \ G(d(fx, fy)) = \frac{d(x, y)}{4}.$$

$$
\begin{aligned}
\phi(\delta(d(fx,fy)) + G(d(fx,fy)) &\leq \phi(G\left(\max\left\{d(x,y),\ d(x,fx),\ d(y,fy)\right\}\right)\\
\phi(\frac{d(x,y)}{8} + \frac{d(x,y)}{4}) &\leq \phi(G(max(|x-y|^2, |x-\frac{x}{2}|^2, |y-\frac{y}{2}|^2)\\
\phi(3d\frac{d(x,y)}{8} &\leq \phi(|x-y|^2)\\
\sqrt{\frac{3|x-y|^2}{8}} &\leq \sqrt{|}x-y|^2.\\
Therefore \qquad \sqrt{\frac{3|x-y|^2}{8}} &\leq |x-y|.
\end{aligned}
$$

Hence the (3.1) is verified by this example.

Example 3.4. *Let $X = [0, \infty)$ and $f : X \to X$ be a self mapping on X, defined by $d(x,y) = (|x-y|^2)$ with constant $s = 2$, $f(x) = \frac{x}{2}$ continuous on X and $\phi(t) = ln(1+x)$, $\delta(t) = \frac{t}{2}$, $G(t) = t$.So (X, d) is a complete b-metric space.*

Solution: $f(x) = \frac{x}{2}$ and $f(y) = \frac{y}{2}$.

$$So\ \ d(fx,fy) = \frac{|x-y|^2}{4} = \frac{d(x,y)}{4}, \quad \delta(d(fx,fy)) = \frac{d(x,y)}{8},\ \ G(d(fx,fy)) = \frac{d(x,y)}{4}.$$

$$
\begin{aligned}
\phi(\delta(d(fx,fy)) + G(d(fx,fy)) &\leq \phi(G\left(\max\left\{d(x,y),\ d(x,fx),\ d(y,fy)\right\}\right)\\
\phi(\frac{d(x,y)}{8} + \frac{d(x,y)}{4}) &\leq \phi(G(max(|x-y|^2, |x-\frac{x}{2}|^2, |y-\frac{y}{2}|^2)\\
\phi(3\frac{d(x,y)}{8} &\leq \phi(|x-y|^2)\\
ln(1 + \frac{3|x-y|^2}{8}) &\leq ln(1+|x-y|^2).
\end{aligned}
$$

Hence the (3.4) is verified by this example.

4. APPLICATION

Beam bending problems arise naturally in structural mechanics, where the deflection of a beam under an applied load is studied. The classical Euler–Bernoulli beam theory leads to a fourth-order differential equation describing the deflection curve. These equations are essential in analyzing the stability and deformation of mechanical structures such as bridges, buildings, aircraft wings, and machine parts. Beam bending theory finds widespread application in civil, mechanical, aerospace, and biomedical engineering for the safe and efficient design of load-bearing components. In this section, we investigate the existence and uniqueness of solutions to a boundary value problem associated with beam bending using fixed point theory in a b-metric space with a WF-contraction. We present an example of a two-point boundary value problem for differential equations of fourth order, and we apply the new results in order to deduce the existence of a unique solution.

$$\begin{cases} EIu^4(x) = g(x, u(x)), & x \in (0,1), \\ u(x) = u(1) = 0, & u^{\cdot}(0) = u^{\cdot}(1) = 0. \end{cases} \tag{4.1}$$

The general solution to the fourth order boundary value problem (BVP) with clamped ends can be written as

$$u(x) = \int_0^1 G(x,s)g(s,u(s))\,ds.$$

Let the mapping $g : (0,1) \times \mathbb{R} \to \mathbb{R}$ is a continuous function. For the above-mentioned boundary value problem (4.1), the Green's function is defined as:

$$K(x,s) = \begin{cases} \frac{1}{6}(1-x)^2 s^2 (3x-1), & 0 \le s < x \le 1, \\ \frac{1}{6}(1-s)^2 x^2 (3x-1), & 0 \le x \le s \le 1. \end{cases}$$

Define the operator f is

$$f(x) = \int_0^1 G(x,s), g(s,u(s))\,ds$$

Considering the space $C(0,1)$ consisting of all continuous real functions defined on $(0,1)$, it is well-known that $(C(0,1), d)$ is a complete metric space $s = 2$. if we consider the supremum distance

$$d(u,v) = \max_{t \in (0,1)} \left\{ |u(s) - v(s)|^2 \right\}, \quad u, v \in C(0,1). \tag{4.2}$$

In the following, we use as the supremum norm. By applying the results proved, we can deduce the existence of a unique solution for problem (4.1).

Theorem 4.1. *Suppose that there exists $\delta \in \widetilde{\triangle} \bigcup \hat{\triangle}$ such that, for all $x, s \in (0,1)$ and non-identical $u, v \in C(0,1)$, we have*

$$|K(x,s)\,[g(s,u(s)) - g(s,v(s))]| \le \max\{|u(s)-v(s)|^2 \left| \int_0^1 K(x,s)g(s,u(s))ds \right|, \left| \int_0^1 K(x,s)g(s,v(s))\,ds \right| \} - \delta(d(u,v)).$$

Then, problem (4.1) *has a unique solution $x^* \in C^2$.*

Proof. It is well known that $x \in C^2$ is a solution to (4.1) if and only if $x \in C$ is a solution to

$$x(t) = \int_0^1 K(x,s)g(s,u(x))\,ds, \quad \text{for all } t \in (0,1). \tag{4.3}$$

Hence, we can define a mapping $f : C(0,1) \to C(0,1)$ by

$$fx(t) = \int_0^1 K(x,s)g(s,u(s))\,ds, \quad \text{for all } t \in (0,1). \tag{4.4}$$

This definition of f clearly allows to affirm that the fixed points of f in $C(0,1)$ are the continuous solutions to (4.3), and, therefore, the solutions to the boundary value problem (4.1). Next, we check the validity of the conditions in theorem (3.1). It is obvious that f is a continuous mapping. Let $x, y \in C(0,1)$ with $d(fx, fy) > 0$, that is, x not coincident with y, then we obtain, for $t \in (0.1)$, that

$$\begin{aligned} |fu(s) - fv(s)| &= \left| \int_0^1 K(x,s)g(s,u(s))\,ds - \int_0^1 K(x,s)g(s,v(s))\,ds \right| \\ &\leq \int_0^1 |K(x,s)\,[g(s,u(s)) - g(s,v(s))]|\,ds \end{aligned}$$

$$\begin{aligned} |K(x,s)[g(s,u(s)) - g(s,v(s))| &\leq \max\{|u(s) - v(s)|^2\, | \int_0^1 K(x,s)g(s,u(s))\,ds|, \\ &\left| \int_0^1 K(x,s)g(s,v(s))\,ds \right|\} - \delta(d(u,v)), \\ &\leq \max\{d(u,v), d(x,Tu), d(v,Tv)\} - \delta(d(u,v)), \end{aligned}$$

therefore,

$$d(fu, fv) \leq \max\{d(u,v), d(u,fu), d(v,fv)\} - \delta(d(u,v)).$$

This proves that condition (2.3) is satisfied for G chosen as the identity mapping. By applying theorem (3.1), there exists a unique solution to problem (4.1). □

5. CONCLUSION

From our explorations we reach a conclusion that the fixed point theorems via Generalised $\phi{-}WF{-}contraction$ with Application to Beam Bending Problems metric space defined on the classical Euler–Bernoulli beam theory leads to a fourth-order differential equation describing the deflection curve. b-metric space satisfying and have a unique common fixed point. Our explorations and result obtained were supported by the suitable example which provides new path for empiricists in the concerned field.

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