

Inventory Optimization for Weibull Deteriorating Items with Mixed Polynomial Demand Patterns and Time-Dependent Backlogging

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Abstract

This study develops an Economic Order Quantity (EOQ) model for deteriorating items incorporating a mixed-demand structure, Weibull deterioration, and time-dependent partial backlogging. The demand rate follows a cubic (third-degree polynomial) pattern during the non-shortage period to represent complex and dynamic market behaviour, while a linear demand pattern is assumed during the shortage period to capture customer responses under limited stock conditions. The deterioration of items is characterized by a two-parameter Weibull distribution, reflecting a realistic and flexible decay pattern over time. Additionally, shortages are permitted and partially backlogged, with the backlogging rate dependent on the waiting time until the next replenishment. The total cost function, including ordering, holding, shortage, and deterioration costs, is formulated, and optimal inventory policies are derived to minimize the overall system cost. A numerical example is provided to illustrate the applicability and effectiveness of the model, followed by a sensitivity analysis that demonstrates the influence of key parameters on the optimal solution. The proposed model provides valuable insights for inventory managers handling deteriorating products under varying demand conditions and time-sensitive backlogging behaviour.

Keywords: Weibull deterioration, cubic demand, linear demand and Linearly Time-Dependent Partial Backordering.

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1. INTRODUCTION

Inventory management plays a crucial role in the smooth operation and profitability of businesses dealing with physical goods. One of the major challenges in inventory systems is handling items that deteriorate over time, such as perishable food, pharmaceuticals, and volatile chemicals. To address this, researchers have developed various Economic Order Quantity (EOQ) models incorporating deterioration. Among these, models assuming a Weibull distribution for item deterioration are particularly notable due to their flexibility in modelling different rates of decay. This research develops an Economic Order Quantity (EOQ) model for inventory items whose deterioration over time is characterized by a two-parameter Weibull distribution, effectively capturing both increasing and decreasing failure rates. Unlike classical EOQ models with constant or linearly time-dependent demand, we consider a cubic demand pattern, which better represents real-world situations where demand may increase, decrease, or vary non-linearly with time. Incorporating a cubic demand function adds complexity to the system but also enhances its applicability to industries experiencing dynamic market behavior. The model also assumes a constant holding cost, simplifying the inventory carrying cost structure. Moreover, the model permits stock outs and treats unmet demand as only partially back ordered; specifically, the proportion of customers willing to place a back order diminishes in a linear fashion as the anticipated wait for the next replenishment grows. This mirrors real world conditions, where buyers tolerate brief delays but become progressively less inclined to wait when lead times lengthen. This research aims to develop and analyse an Economic Order Quantity (EOQ) model tailored to inventory systems where items deteriorate over time, demand varies cubically with time, holding costs remain constant, and shortages are partially backordered with a backordering rate linearly dependent on the waiting time for the next replenishment. The primary goal is to identify the optimal ordering policy that minimizes the total inventory cost, encompassing ordering, holding, deterioration, and shortage costs. To validate the model's applicability and understand its behaviour under varying conditions, numerical examples are presented, and sensitivity analyses are conducted to assess the impact of key parameters on the optimal solution.

2. LITERATURE REVIEW

Over the past decade, inventory models addressing deteriorating items have garnered increasing attention due to their critical role in managing perishable and time-sensitive goods. Industries such as food, pharmaceuticals, and healthcare face unique challenges in inventory management, as products like fresh produce, medications, and medical

supplies are susceptible to spoilage or obsolescence over time. Effective inventory control strategies are essential to balance product availability with the minimization of waste and associated costs. Consequently, researchers have developed various models to optimize ordering policies, considering factors like deterioration rates, demand variability, and storage constraints, to enhance efficiency in supply chain operations. The incorporation of time-dependent demand patterns, deterioration distributions and various cost considerations has led to a diverse body of research that extends classical EOQ theory. Rangarajan and Karthikeyan (2017) developed EOQ models for deteriorating items with cubic demand under inflation and permissible delay in payments, considering both instantaneous and non-instantaneous deterioration. Their study emphasizes the impact of financial factors and complex demand patterns on inventory decisions [1]. Sharma and Sharma (2016) proposed a deterministic inventory model featuring a cubic demand function and constant deterioration over an infinite time horizon. Their inclusion of salvage value provides practical relevance for industries that can recover value from unsold inventory [2]. In a related study, Sharma and Chaudhary (2013) addressed deterioration governed by the Weibull distribution with time-dependent demand and shortages. Their model highlights the flexibility of the Weibull distribution in capturing various deterioration behaviours [3]. Similarly, Krishnaraj and Ramasamy (2013) analysed an inventory model incorporating stock-dependent demand and Weibull-distributed deterioration, focusing on systems without shortages [4]. Sahoo and Paul (2021) extended the study of cubic demand to include deteriorating items governed by a cubic deterioration pattern and Weibull demand, but excluded shortages. Their work contributes to understanding the interplay between nonlinear deterioration and demand [5]. In another study, Sahoo, Paul, and Sahoo (2021) presented an EOQ model with a three-parameter Weibull deterioration and cubic demand, further enriching the literature by analysing models without shortages and salvage value [7]. Mandal (2020) developed an EOQ model for items that can both deteriorate and ameliorate over time, considering cubic demand and shortages. The dual nature of item behaviour and the inclusion of salvage value make this model particularly versatile [6]. In a subsequent study, Mandal (2020) examined time-varying deteriorating items with cubic demand and shortages, reinforcing the role of variable deterioration rates in inventory optimization [8]. Later, Mandal (2021) extended his previous work to include permissible delays in payments, highlighting financial flexibility in managing deteriorating inventories [9]. Lastly, Sharma, Sharma, and Ramani (2012) explored an EOQ model where items follow a Weibull deterioration pattern, with power-pattern demand, shortages, and time-dependent holding costs. Their integration of multiple real-world factors underscores the complexity and applicability of modern inventory models [10]. In summary, the reviewed literature illustrates a growing interest in EOQ models that

account for more realistic assumptions, such as cubic or nonlinear demand, Weibull-type deterioration, partial backlogging, and flexible cost structures. However, there remains a research gap in models that simultaneously address Weibull deterioration, cubic demand, constant holding costs, and linearly time-dependent partial backlogging. The present study aims to fill this gap by developing a comprehensive EOQ model incorporating all these features.

3. ASSUMPTIONS AND NOTATIONS

3.1. Assumptions

- The inventory model considers a single product managed over a predetermined, finite planning horizon denoted by T .
- Replenishment occurs instantaneously upon placing an order, implying no delay between ordering and receiving the inventory.
- $D(t) = a + bt + ct^2 + dt^3$; $a \geq 0, b \neq 0, c \neq 0, d \neq 0$.
- The deterioration rate of items over time $t > 0$ is characterized by a two-parameter Weibull distribution, expressed as $\theta(t) = \alpha\beta t^{(\beta-1)}$, where $\alpha(0 < \alpha \ll 1)$ serves as the scale parameter and $\beta > 0$ as the shape parameter.
- During the stockout phase, the demand rate is modelled as a linear function of time. $D(t) = p + qt, p > 0, q > 0$.
- The model permits shortages, and during the stock-out period, the backordering rate is a variable function of the waiting time until the next replenishment. Specifically, the backordering rate is expressed as $B(t) = e^{(-\delta(T-t))}$, where δ is a backlogging parameter satisfying $0 < \delta < 1$ and $(T - t)$ represents the waiting time interval from t_1 to T .
- Holding cost is constant.

3.2. Notations

- $I_1(t)$ – Stock level at time ‘ t ’, $0 \leq t \leq t_1$.
- $I_2(t)$ – Stock level at time ‘ t ’, $t_1 \leq t \leq T$.
- Q – Order quantity per cycle.

- Q_1 – Opening stock level.
- T – Duration of the inventory cycle.
- t_1 – The time span over which the inventory level is sufficient to meet demand without incurring any stockouts
- C_o – Ordering price
- C_p – Purchasing price
- C_h – Holding price
- C_d – Deterioration price
- C_S – Shortage price used for backordered things
- C_L – Lost sales price
- TC – Total price

4. MATHEMATICAL FORMULATION OF THE MODEL

At the commencement of each inventory cycle, an initial order quantity Q is placed. Assuming an initial inventory level of Q_1 , the inventory depletes over the time interval $[0, t_1]$ due to both demand and deterioration. During this period, the inventory level decreases to zero. Following this, in the subsequent period $[t_1, T]$, shortages occur as demand continues and these shortages are partially backordered.

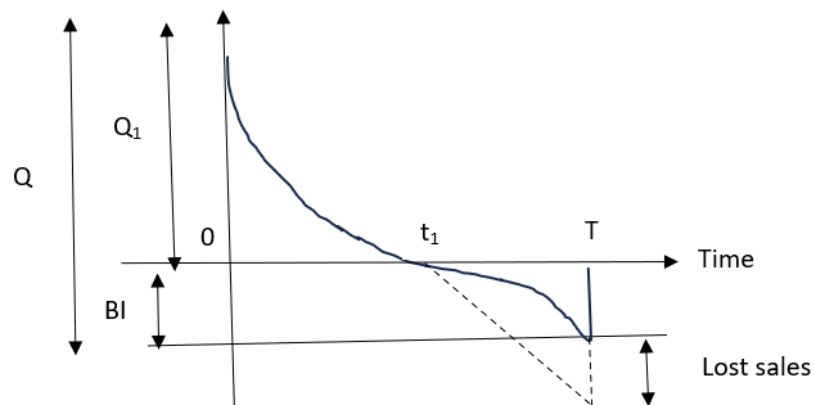


Figure 1: Representation of the inventory system

The inventory level's rate of change can be expressed as

$$\frac{dI_1(t)}{dt} + \alpha\beta t^{(\beta-1)} I_1(t) = -(a + bt + ct^2 + dt^3), 0 \leq t \leq t_1 \quad (1)$$

Subject to the initial condition $I_1(0) = Q_1$ and the terminal condition $I_1(t_1) = 0$. The solution to equation (1) is

$$I_1(t) = \frac{1}{1 + \alpha t^\beta} \left\{ a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) \right. \\ \left. + \frac{a\alpha}{\beta + 1}(t_1^{\beta+1} - t^{\beta+1}) + \frac{b\alpha}{\beta + 2}(t_1^{\beta+2} - t^{\beta+2}) \right. \\ \left. + \frac{c\alpha}{\beta + 3}(t_1^{\beta+3} - t^{\beta+3}) + \frac{d\alpha}{\beta + 4}(t_1^{\beta+4} - t^{\beta+4}) \right\} \quad (2)$$

$$Q_1 = a t_1 + \frac{b t_1^2}{2} + \frac{c t_1^3}{3} + \frac{d t_1^4}{4} + \frac{a\alpha t_1^{\beta+1}}{\beta + 1} + \frac{b\alpha t_1^{\beta+2}}{\beta + 2} \\ + \frac{c\alpha t_1^{\beta+3}}{\beta + 3} + \frac{d\alpha t_1^{\beta+4}}{\beta + 4} \quad (3)$$

Between times t_1 and T , the system experiences shortages, with demand being only partially backordered. Let $I_2(t)$ represent the inventory level at time t within this interval. The behavior of the inventory during this period is governed by the following differential equation

$$\frac{dI_2(t)}{dt} = -e^{-\delta(T-t)} (p + qt), \quad t_1 \leq t \leq T. \quad (4)$$

At time $t = t_1$, the inventory level is zero, which serves as the boundary condition: $I_2(t_1) = 0$. The solution to equation (4) is

$$I_2(t) = (t_1 - t) \left[p(1 - \delta T) + \frac{p\delta + q - q\delta T}{2} (t_1 + t) + \frac{q}{3} (t_1^2 + tt_1 + t^2) \right]. \quad (5)$$

The peak level of backordered inventory, denoted as BI, occurs at the end of the cycle, specifically at time $t = T$.

$$BI = -I_2(T) = -(t_1 - T) \left[p(1 - \delta T) + \frac{p\delta + q - q\delta T}{2} (t_1 + T) \right. \\ \left. + \frac{q}{3} (t_1^2 + t_1 T + T^2) \right]. \quad (6)$$

Consequently, the total quantity ordered over the entire time horizon $[0, T]$ is: $Q = Q_1 + BI$

$$Q = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} - (t_1 - T) \left[p(1 - \delta T) + \frac{(p\delta + q - q\delta T)}{2} (t_1 + T) + \frac{q}{3} (t_1^2 + t_1 T + T^2) \right] \tag{7}$$

Ordering Cost (OC) = C_0

Purchase Cost (PC) = $C_p Q$

$$PC = C_p \left\{ at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} - (t_1 - T) \left[p(1 - \delta T) + \frac{(p\delta + q - q\delta T)}{2} (t_1 + T) + \frac{q}{3} (t_1^2 + t_1 T + T^2) \right] \right\} \tag{8}$$

Holding Cost

$$HC = \int_0^{t_1} C_h I_1(t) dt$$

$$HC = C_h \left\{ t_1^2 \left(\frac{a}{2} + \frac{bt_1}{3} + \frac{ct_1^2}{4} + \frac{dt_1^3}{5} \right) + \frac{\beta\alpha t_1^{\beta+2}}{\beta+1} \left(\frac{a}{\beta+2} + \frac{bt_1}{\beta+3} + \frac{ct_1^2}{\beta+4} + \frac{dt_1^3}{\beta+5} \right) - \frac{\alpha^2 t_1^{2\beta+2}}{\beta+1} \left(\frac{a}{2\beta+2} + \frac{bt_1}{2\beta+3} + \frac{ct_1^2}{2\beta+4} + \frac{dt_1^3}{2\beta+5} \right) \right\} \tag{9}$$

Deterioration Cost

$$DC = C_d \left\{ Q_1 - \int_0^{t_1} D(t) dt \right\}$$

$$DC = C_d \left\{ \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} \right\} \tag{10}$$

Shortage Cost

$$SC = \{-C_s\} \int_{t_1}^T I_2(t) dt$$

$$\begin{aligned}
SC = -C_s \left\{ p(1 - \delta T) \left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} \right) \right. \\
\left. + \frac{1}{2} (p\delta + q - q\delta T) \left(t_1^2 T - \frac{T^3}{3} - \frac{2t_1^3}{3} \right) \right. \\
\left. + \frac{1}{3} q\delta \left(t_1^3 T - \frac{T^4}{4} - \frac{3t_1^4}{4} \right) \right\} \quad (11)
\end{aligned}$$

$$\text{Lost Sales Cost } LSC = C_L \int_{t_1}^T [1 - e^{-\delta(T-t)}] (p + qt) dt$$

$$LSC = C_L \delta \left(\frac{pT^2}{2} + \frac{qT^3}{6} - pt_1 T + \frac{pt_1^2}{2} - \frac{qTt_1^2}{2} + \frac{qt_1^3}{3} \right) \quad (12)$$

TC = Ordering cost+Holding cost+Purchase cost+Deterioration cost+Shortage cost+Lost sales cost

$$\begin{aligned}
TC = C_o + C_h \left\{ t_1^2 \left(\frac{a}{2} + \frac{bt_1}{3} + \frac{ct_1^2}{4} + \frac{dt_1^3}{5} \right) + \frac{\beta \alpha t_1^{\beta+2}}{\beta+1} \left(\frac{a}{\beta+2} + \frac{bt_1}{\beta+3} + \frac{ct_1^2}{\beta+4} + \frac{dt_1^3}{\beta+5} \right) \right. \\
\left. - \frac{\alpha^2 t_1^{2\beta+2}}{\beta+1} \left(\frac{a}{2\beta+2} + \frac{bt_1}{2\beta+3} + \frac{ct_1^2}{2\beta+4} + \frac{dt_1^3}{2\beta+5} \right) \right\} \\
+ C_p \left\{ at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} \right. \\
\left. - (t_1 - T) \left[p(1 - \delta T) + \frac{(p\delta + q - q\delta T)}{2} (t_1 + T) + \frac{q}{3} (t_1^2 + Tt_1 + T^2) \right] \right\} \\
+ C_d \left\{ \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} \right\} \\
- C_s \left\{ p(1 - \delta T) \left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} \right) + \frac{1}{2} (p\delta + q - q\delta T) \left(t_1^2 T - \frac{T^3}{3} - \frac{2t_1^3}{3} \right) \right. \\
\left. + \frac{1}{3} q\delta \left(t_1^3 T - \frac{T^4}{4} - \frac{3t_1^4}{4} \right) \right\} \\
+ C_L \delta \left\{ \frac{pT^2}{2} + \frac{qT^3}{6} - pTt_1 + \frac{pt_1^2}{2} - \frac{qTt_1^2}{2} + \frac{qt_1^3}{3} \right\}. \quad (13)
\end{aligned}$$

Our objective is to minimize the total cost.

The necessary condition is $\frac{\partial TC}{\partial t_1} = 0$ and $\frac{\partial^2 TC}{\partial t_1^2} > 0$ for all $t_1 > 0$.

We get

$$\begin{aligned}
 \frac{\partial TC}{\partial t_1} = & C_p \left\{ a + bt_1 + ct_1^2 + dt_1^3 + a\alpha t_1^\beta + b\alpha t_1^{\beta+1} + c\alpha t_1^{\beta+2} + d\alpha t_1^{\beta+3} \right. \\
 & \left. - p(1 - \delta T) - (p\delta + q - q\delta T)t_1 - q\delta t_1^2 \right\} \\
 & + C_h \left\{ at_1 + bt_1^2 + ct_1^3 + dt_1^4 + a\alpha t_1^{\beta+1} + b\alpha t_1^{\beta+2} + c\alpha t_1^{\beta+3} + d\alpha t_1^{\beta+4} \right. \\
 & - \frac{a\alpha t_1^{\beta+1}}{\beta + 1} - \frac{b\alpha t_1^{\beta+2}}{\beta + 1} - \frac{c\alpha t_1^{\beta+3}}{\beta + 1} - \frac{d\alpha t_1^{\beta+4}}{\beta + 1} - \frac{a\alpha^2 t_1^{2\beta+1}}{\beta + 1} \\
 & \left. - \frac{b\alpha^2 t_1^{2\beta+2}}{\beta + 1} - \frac{c\alpha^2 t_1^{2\beta+3}}{\beta + 1} - \frac{d\alpha^2 t_1^{2\beta+4}}{\beta + 1} \right\} \\
 & - C_s \left\{ p(1 - \delta T)(T - t_1) + (p\delta + q - q\delta T)(t_1 T - t_1^2) + q\delta(Tt_1^2 - t_1^3) \right\} \\
 & + C_d \left\{ a\alpha t_1^\beta + b\alpha t_1^{\beta+1} + c\alpha t_1^{\beta+2} + d\alpha t_1^{\beta+3} \right\} + C_L \delta \{-pT + pt_1 - qTt_1 + qt_1^2\}
 \end{aligned} \tag{14}$$

and

$$\begin{aligned}
 \frac{\partial^2 TC}{\partial t_1^2} = & C_p \left\{ b + 2ct_1 + 3dt_1^2 + a\alpha\beta t_1^{\beta-1} + b\alpha(\beta + 1)t_1^\beta + c\alpha(\beta + 2)t_1^{\beta+1} \right. \\
 & \left. + d\alpha(\beta + 3)t_1^{\beta+2} - p\delta - q + q\delta T - 2q\delta t_1 \right\} \\
 & + C_h \left\{ a + 2bt_1 + 3ct_1^2 + 4dt_1^3 + a\alpha(\beta + 1)t_1^\beta + b\alpha(\beta + 2)t_1^{\beta+1} \right. \\
 & + c\alpha(\beta + 3)t_1^{\beta+2} + d\alpha(\beta + 4)t_1^{\beta+3} \\
 & - a\alpha t_1^\beta - \frac{b\alpha(\beta + 2)t_1^{\beta+1}}{\beta + 1} - \frac{c\alpha(\beta + 3)t_1^{\beta+2}}{\beta + 1} - \frac{d\alpha(\beta + 4)t_1^{\beta+3}}{\beta + 1} \\
 & \left. - \frac{a\alpha^2(2\beta + 1)t_1^{2\beta}}{\beta + 1} - 2b\alpha^2 t_1^{2\beta+1} - \frac{c\alpha^2(2\beta + 3)t_1^{2\beta+2}}{\beta + 1} - \frac{d\alpha^2(2\beta + 4)t_1^{2\beta+3}}{\beta + 1} \right\} \\
 & - C_s \left\{ -p(1 - \delta T) + (p\delta + q - q\delta T)(T - 2t_1) + q\delta(2Tt_1 - 3t_1^2) \right\} \\
 & + C_d \left\{ a\alpha\beta t_1^{\beta-1} + b\alpha(\beta + 1)t_1^\beta + c\alpha(\beta + 2)t_1^{\beta+1} + d\alpha(\beta + 3)t_1^{\beta+2} \right\} \\
 & + C_L \delta \{p - qT + 2qt_1\}
 \end{aligned} \tag{15}$$

5. NUMERICAL EXAMPLE

A numerical example is presented to demonstrate the practical applicability and effectiveness of the proposed inventory model. By substituting realistic parameter values into the model equations, key decision variables such as the optimal order quantity, the time before shortages occur, and the total cost are computed. The results provide a clear understanding of how the model behaves under specific conditions and verify its accuracy in balancing costs related to ordering, holding, and shortages. This example serves as a validation of the theoretical model, showing how it can be implemented to optimize inventory decisions in real-world scenarios.

Let's examine an inventory system characterized by the following parameters, each expressed in appropriate units.

$$a = 100, b = 15, c = 4, d = 2, p = 250, q = 30, \alpha = 0.2, \beta = 2, \delta = 0.001, T = 2, C_n = 1, C_p = 8, C_d = 0.1, C_s = 2.5, C_L = 0.3, C_0 = 20.$$

Then we get $t_1 = 1.5596$, $Q = 214.3753$ and $TC = 1899.6$

6. SENSITIVITY ANALYSIS

Sensitivity analysis is a crucial tool in inventory management used to study how changes in key parameters affect the overall performance of the model. It helps determine the robustness and reliability of the optimal solutions obtained for order quantity, total cost, and time-related variables. By systematically varying one parameter at a time while keeping others constant, decision-makers can identify which factors have the most significant impact on the system. This analysis provides valuable insights for managers to make informed decisions under uncertain conditions and to adjust inventory policies efficiently. In summary, sensitivity analysis enhances the practical applicability of the model by revealing how sensitive the results are to fluctuations in model parameters.

Table 1: Variation in α

α	t_1	Q	TC
0.1	1.7044	224.6939	1935.2
0.2	1.5596	214.3753	1899.6
0.3	1.4557	206.2329	1875.8
0.4	1.3750	199.4318	1857.9
0.5	1.3094	193.5620	1843.5

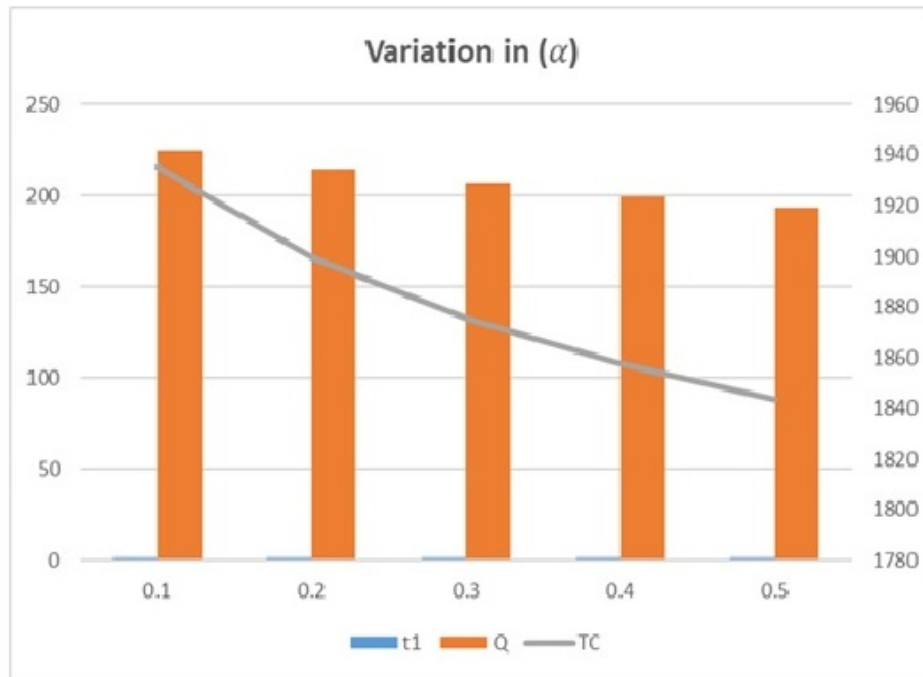


Table 1 illustrates the impact of varying the parameter α on the system's performance measures t_1, Q, TC . As the value of α increases from 0.1 to 0.5, there is a noticeable decline in the values of t_1, Q, TC . This indicates that a higher deterioration or sensitivity factor (α) leads to a reduction in both the optimal order quantity and the time before shortages occur. Consequently, the total cost also decreases slightly, suggesting that increasing α results in a more frequent replenishment cycle with smaller order quantities, thereby minimizing overall cost. Thus, α plays a significant role in controlling inventory decisions and cost efficiency.

Table 2: Variation in p

p	t_1	Q	TC
230	1.4822	198.6323	1791.5
240	1.5218	206.5760	1845.7
250	1.5596	214.3753	1899.6
260	1.5957	222.0413	1953.3
270	1.6304	229.5841	2006.8

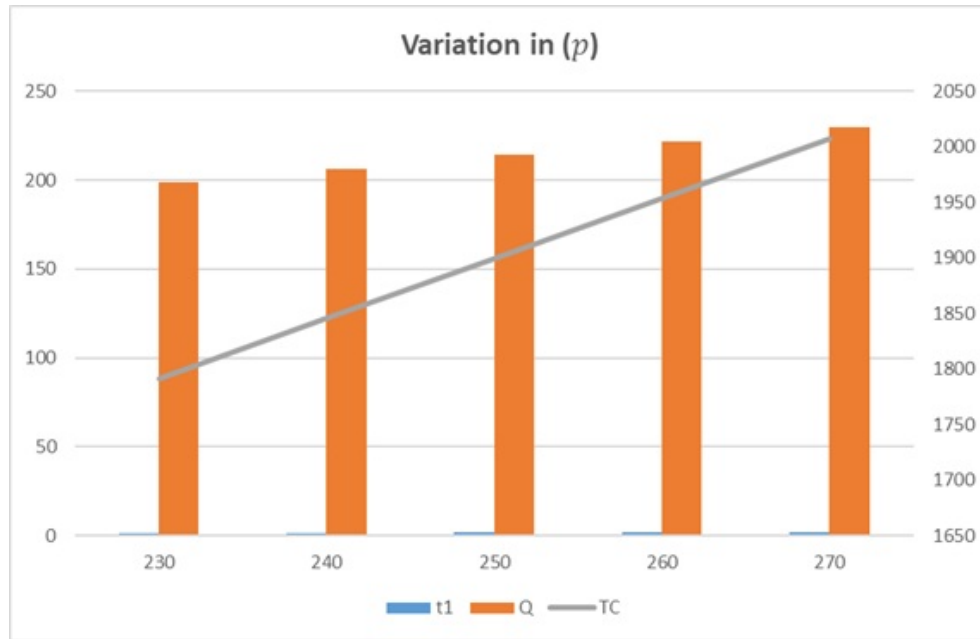


Table 2 demonstrates the effect of varying the parameter p on the key inventory measures t_1 , Q and TC . As the value of p increases from 230 to 270, the values of t_1 , Q and TC all show a consistent upward trend. This implies that when the selling price or demand-influencing factor p rises, the optimal order quantity and the time before shortages occur also increase. Consequently, the total cost rises due to higher inventory levels and longer holding durations. In essence, an increase in p stimulates higher demand, requiring larger replenishment quantities and leading to higher overall inventory costs. Therefore, p plays a crucial role in determining the balance between profitability and cost efficiency in the inventory system.

Table 3: Variation in q

q	t_1	Q	TC
20	1.6044	223.8956	1991.6
25	1.5816	219.0255	1943.5
30	1.5596	214.3753	1899.6
35	1.5382	209.9311	1859.7
40	1.5174	205.6803	1823.4

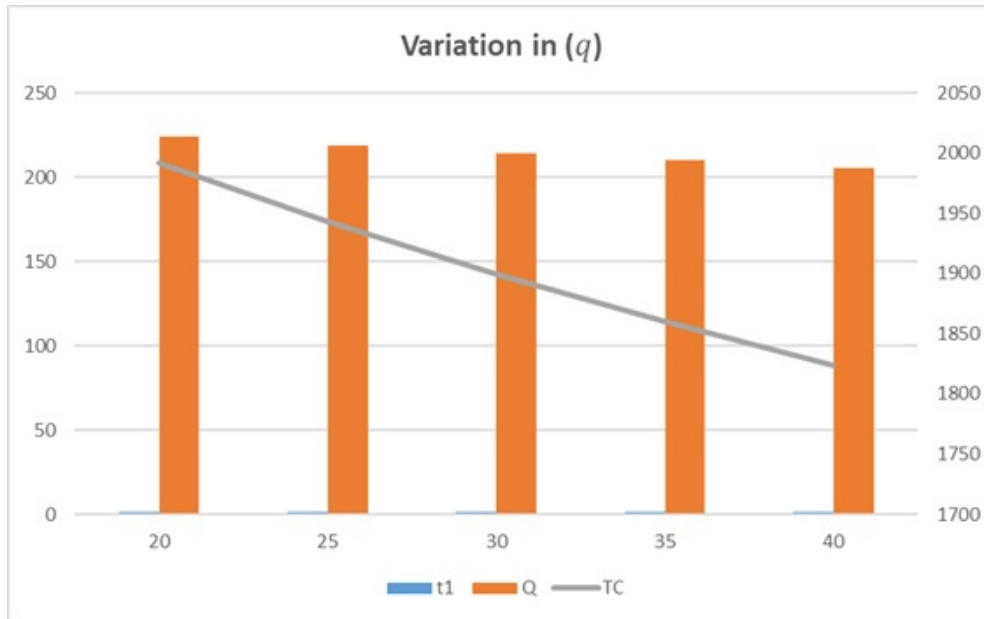


Table 3 presents the influence of the parameter q on the inventory system’s performance indicators t_1 , Q and TC . As q increases from 20 to 40, all three values t_1 , Q and TC decrease gradually. This trend indicates that higher values of q lead to a reduction in the optimal order quantity, a shorter period before shortages occur, and a lower total cost. The decrease in total cost suggests that increasing q improves system efficiency, possibly due to better demand management or reduced replenishment requirements. Thus, the parameter q significantly affects the ordering strategy and cost behavior, where higher q values contribute to minimizing overall inventory-related expenses.

Table 4: Variation in a

a	t_1	Q	TC
80	1.6745	199.6852	1722.5
90	1.6165	207.5473	1815.1
100	1.5596	214.3753	1899.6
110	1.5037	220.2161	1976.5
120	1.4487	225.1122	2046.0

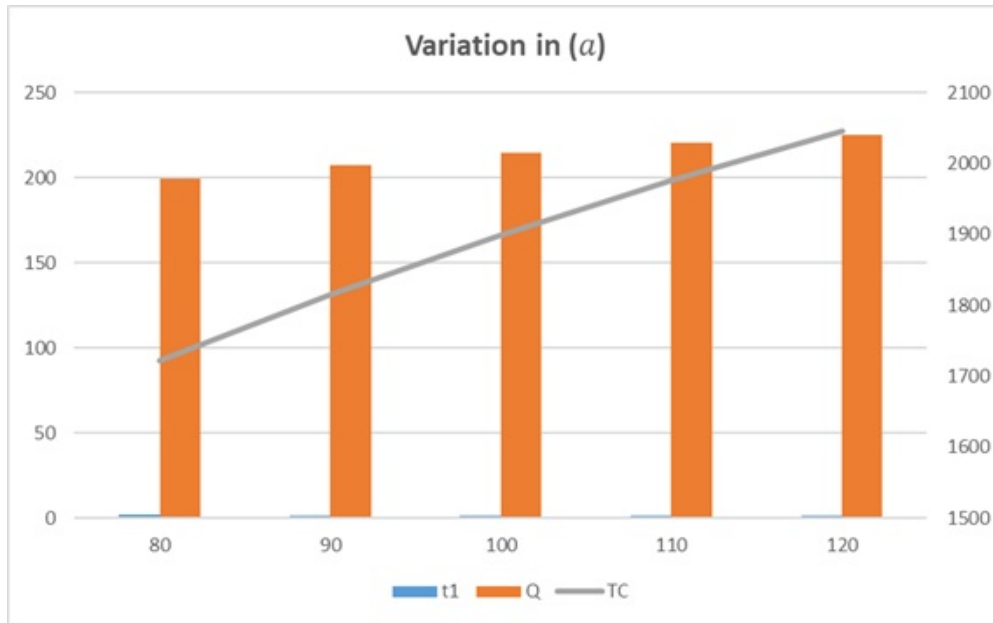


Table 4 illustrates the effect of varying the parameter a on the system's decision variables t_1 , Q and TC . As the value of a increases from 80 to 120, the order quantity Q and total cost TC rise steadily, while the time before shortages occur (t_1) gradually decreases. This indicates that a higher value of a , which typically represents the base demand level, leads to greater overall demand, requiring larger replenishment quantities to meet customer needs. Consequently, the total cost increases due to higher purchasing and holding costs. Meanwhile, the reduction in t_1 signifies that the available stock is depleted more quickly under higher demand conditions. Therefore, the parameter a significantly influences inventory decisions, reflecting the direct relationship between demand intensity, order quantity and cost.

Table 5: Variation in b

b	t_1	Q	TC
13	1.5774	215.0245	1896.8
14	1.5684	214.7016	1898.2
15	1.5596	214.3753	1899.6
16	1.5508	214.0457	1900.9
17	1.5422	213.7130	1902.2

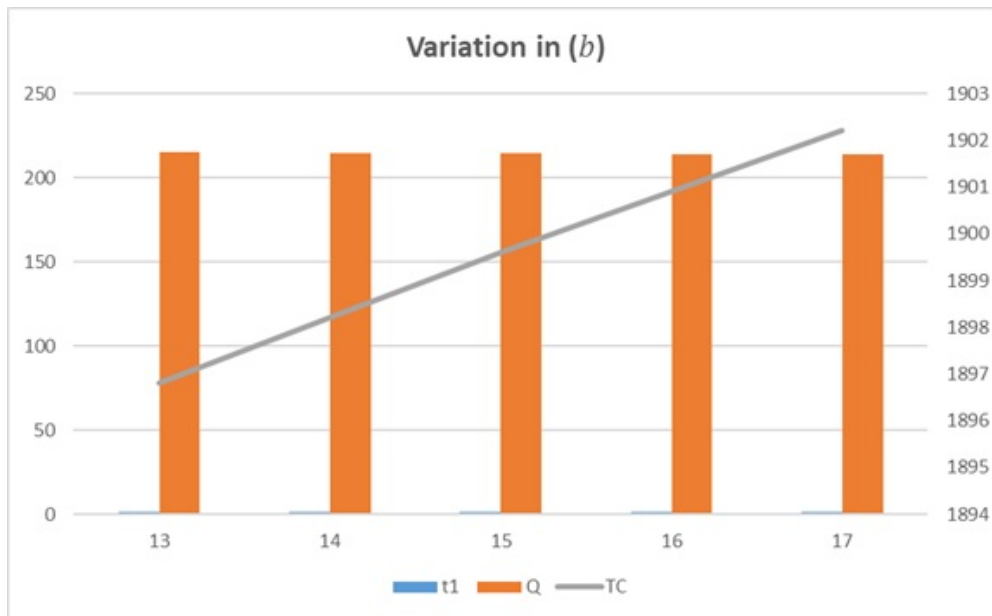


Table 5 depicts the impact of varying the parameter b on the inventory system's performance indicators t_1 , Q and TC . As the value of b increases from 13 to 17, both the optimal order quantity Q and the time before shortages occur (t_1) show a gradual decline, while the total cost TC slightly increases. This trend suggests that higher values of b , which may represent the demand sensitivity or rate of change in demand, lead to faster depletion of inventory and shorter stock availability periods. The rise in total cost occurs because more frequent replenishments are needed to meet the changing demand, increasing ordering and shortage-related expenses. Hence, the parameter b plays a crucial role in determining the trade-off between inventory availability and overall cost efficiency.

Table 6: Variation in c

c	t_1	Q	TC
2	1.5882	216.9369	1908.5
3	1.5736	215.6331	1904.0
4	1.5596	214.3753	1899.6
5	1.5461	213.1600	1895.5
6	1.5332	211.9845	1891.6

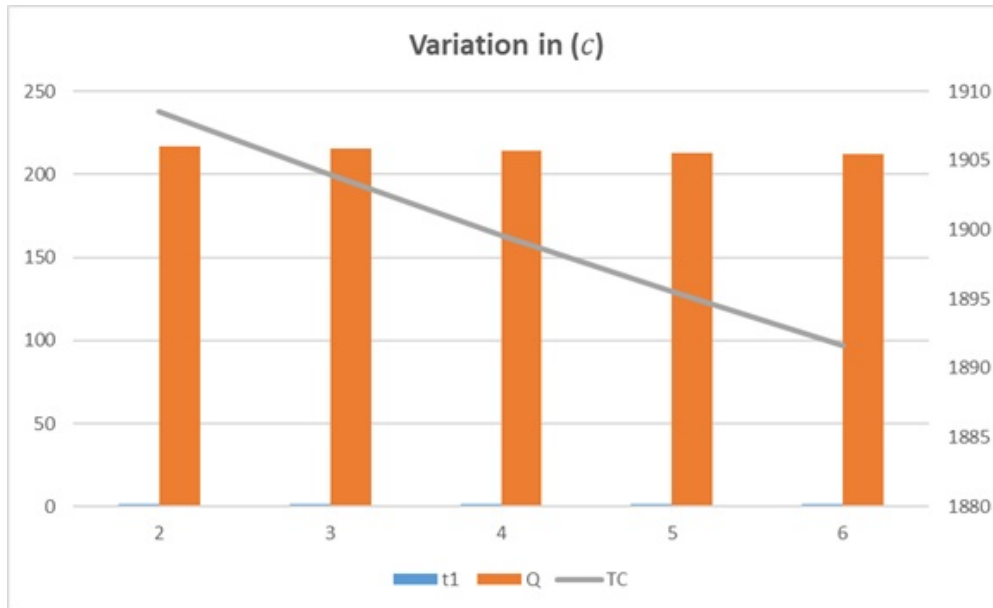


Table 6 shows the effect of varying the parameter c on the inventory system's key measures t_1 , Q and TC . As c increases from 2 to 6, all three values t_1 , Q and TC gradually decrease. This indicates that higher values of c , which may represent a cost-related or demand-influencing factor, lead to smaller optimal order quantities and shorter inventory availability periods before shortages occur. The corresponding decline in total cost (TC) suggests that the system becomes more cost-efficient as c increases, possibly due to reduced holding requirements or better inventory turnover. Overall, this trend highlights that increasing c results in a more controlled and economical inventory policy with lower total expenditure.

Table 7: Variation in d

d	t_1	Q	TC
1.8	1.5639	214.8832	1902.0
1.9	1.5617	214.6281	1900.8
2.0	1.5596	214.3753	1899.6
2.1	1.5574	214.1247	1898.4
2.2	1.5553	213.8764	1897.3

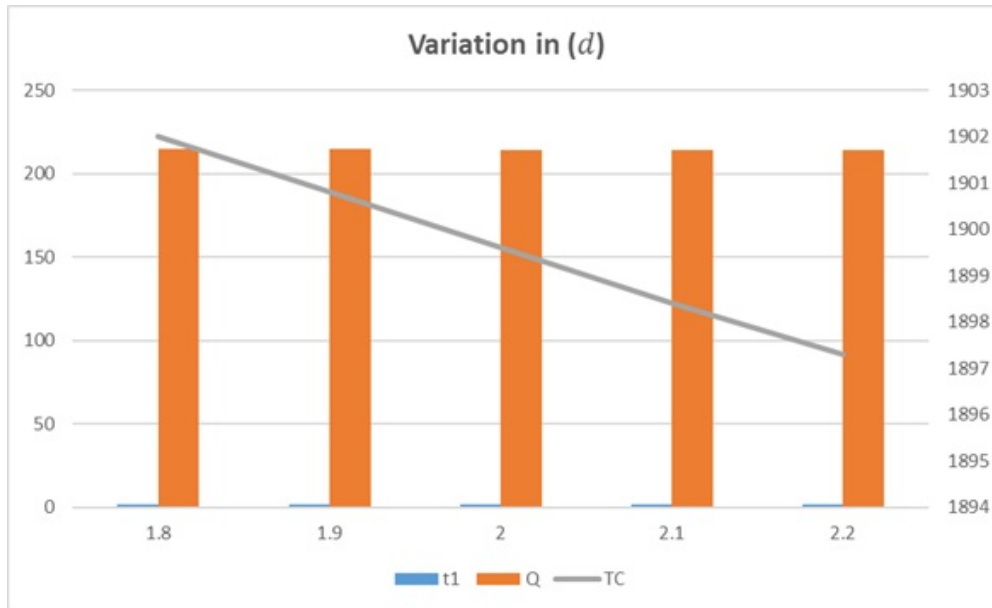


Table 7 illustrates the influence of the parameter d on the system's performance indicators t_1 , Q and TC . As the value of d increases from 1.8 to 2.2, there is a consistent decline in t_1 , Q and TC . This trend suggests that higher values of d , which may represent the demand or deterioration-related coefficient, lead to reduced order quantities and shorter periods before shortages occur. The corresponding decrease in total cost indicates that the system becomes slightly more cost-efficient as d increases. Overall, this behavior implies that an increase in d results in faster inventory depletion but lower overall inventory and cost levels, reflecting a more efficient but tighter inventory management strategy.

7. CONCLUSION

This paper presents a deterministic EOQ model that integrates a mixed demand pattern, Weibull deterioration, and time-dependent partial backlogging. The study assumes a cubic demand during the non-shortage period to reflect realistic and fluctuating customer behaviour, while a linear demand is considered during the shortage period to represent a simpler, time-sensitive purchasing trend. The deterioration of items, modelled using the Weibull distribution, provides flexibility to accommodate various product lifespans. The proposed model successfully determines the optimal replenishment policy that minimizes the total inventory cost, considering ordering, holding, deterioration, and shortage costs. Numerical examples and sensitivity analysis confirm the model's robustness and illustrate how key parameters influence the optimal order quantity and total cost. The findings offer practical insights for decision-makers managing perishable or deteriorating items in environments with changing demand and partial backlogging behaviour, thereby

contributing to more efficient and cost-effective inventory control. Future research can explore stochastic demand, variable holding costs, or multi-item systems to further enhance the model's applicability.

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