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(μ, ν)-Linear Transformations

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Abstract

Linear transformations have a vital role in modern applications of mathematics. Linear transformations encapsulate the notion of linearity. The aim of this article is to extend the notion of linear transformations to the framework of intuitionistic fuzzy linear spaces. This study also explores the isomorphism of intuitionistic fuzzy linear spaces and their structural equivalence.

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1. INTRODUCTION

Lotfi A. Zadeh [14] introduced the notion of fuzzy sets in 1965. Fuzzy set theory provides a mathematical framework to deal situations with uncertainty, vagueness and imprecision. It has now developed as a modelling language well suited for such situations. By allowing elements to possess degrees of membership within a set, the theory of fuzzy sets has advanced in a variety of ways in many disciplines.

Atanassov [1] further extended Zadeh's framework by introducing intuitionistic fuzzy sets. An intuitionistic fuzzy set is characterized by a membership function and a non-membership function, which provides a more appropriate means to model uncertainty, vagueness and hesitation. Intuitionistic fuzzy set theory has progressed into a well established and versatile field of research with broad theoretical achievements and diverse practical applications. One prominent direction of research involves the study of intuitionistic fuzzy algebraic structures.

Extending the definitions of fuzzy fields and fuzzy linear spaces proposed by Wenxiang and Tu [2], Santhosh and Ramakrishnan [11] introduced the notions of intuitionistic fuzzy fields and intuitionistic fuzzy linear spaces. This paper is to investigate the notion of linear transformations in the context of intuitionistic fuzzy linear spaces. The article is organized as follows. Section 2 consists of the notions of intuitionistic fuzzy fields, intuitionistic fuzzy linear spaces and some necessary preliminaries. The new concept (μ,ν)-linear transformations is introduced in section 3 and some fundamental properties of (μ,ν)-linear transformations are discussed. Also, some results on images of (α,β)-level sets of intuitionistic fuzzy linear spaces and of intuitionistic fuzzy convex sets are given. The isomorphism of intuitionistic fuzzy linear spaces is discussed in section 4. It is obtained that the relation of isomorphism on the class of all intuitionistic fuzzy linear spaces over an intuitionistic fuzzy field is an equivalence relation.

2. PRELIMINARIES

This section presents foundational concepts concerning intuitionistic fuzzy sets, intuitionistic fuzzy fields and intuitionistic fuzzy linear spaces.

Definition 2.1. [1] An intuitionistic fuzzy set A in a nonempty set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of nonmembership of the element $x \in X$ to A, respectively, and for every $x \in X : 0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2.2. [1] A *set of* $(\alpha,\beta) - level,$ generated by an intuitionistic fuzzy set A in X, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$, is defined as $N_{\alpha,\beta}(A) = \{x \in X : \mu_A(x) \ge \alpha, \nu_A(x) \le \beta\}$.

Definition 2.3. [4] Let f be a mapping of a set X to a set Y. If B is an intuitionistic fuzzy set in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B) = \{\langle x, \mu_B(f(x)), \nu_B(f(x))\rangle : x \in X\}$. The *image* of an intuitionistic fuzzy set A in X under f, denoted by $f(A)$, is the intuitionistic fuzzy set in Y defined by $f(A) = \left\{\left\langle y, \mu_{f(A)}(y), \nu_{f(A)}(y)\right\rangle : y \in Y\right\}$, where

$$\mu_{f(A)}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

$$\nu_{f(A)}(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise.} \end{cases}$$

Definition 2.4. [7] Let $Y \subseteq \mathbb{R}^n$, the n-dimensional Euclidean space and let A be an intuitionistic fuzzy set in Y. If for all $y_1, y_2 \in Y$ and for all $\lambda \in [0,1]$, $\mu_A(\lambda y_1 + (1-\lambda)y_2) \geq \min\{\mu_A(y_1), \mu_A(y_2)\}$ and $\nu_A(\lambda y_1 + (1-\lambda)y_2) \leq \max\{\nu_A(y_1), \nu_A(y_2)\}$, then A is called a *convex intuitionistic fuzzy set* in Y.

Definition 2.5. [11] If F is an intuitionistic fuzzy set in a field X satisfying the following conditions: for all $a, b \in X$,

(i) $\mu_F(a+b) \geq \min\{\mu_F(a), \mu_F(b)\}, \nu_F(a+b) \leq \max\{\nu_F(a), \nu_F(b)\}$

(ii) $\mu_F(-a) = \mu_F(a), \nu_F(-a) = \nu_F(a)$

(iii) $\mu_F(ab) \geq \min\{\mu_F(a), \mu_F(b)\}, \nu_F(ab) \leq \max\{\nu_F(a), \nu_F(b)\}$

(iv) $\mu_F(a^{-1}) = \mu_F(a), \nu_F(a^{-1}) = \nu_F(a)$ (if $a \neq 0$),

then (F, X) is called an *intuitionistic fuzzy field of* X.

Proposition 2.1. [11] *If* (F, X) *is an intuitionistic fuzzy field of* X*, then*

(i) $\mu_F(0) \geq \mu_F(a)$, $\nu_F(0) \leq \nu_F(a)$ *for all* $a \in X$

(ii) $\mu_F(1) \geq \mu_F(a)$, $\nu_F(1) \leq \nu_F(a)$ *for all* $a(\neq 0) \in X$

(iii) $\mu_F(0) \geq \mu_F(1)$, $\nu_F(0) \leq \nu_F(1)$.

Definition 2.6. [11] Let (F, X) be an intuitionistic fuzzy field of X and Y be a linear space over X. If V is an intuitionistic fuzzy set in Y satisfying the following conditions:

(i) $\mu_V(u+v) \geq \min\{\mu_V(u), \mu_V(v)\}, \nu_V(u+v) \leq \max\{\nu_V(u), \nu_V(v)\}$ for all $u, v \in Y$

(ii) $\mu_V(-u) = \mu_V(u), \nu_V(-u) = \nu_V(u)$ for all $u \in Y$

(iii) $\mu_V(au) \geq \min\{\mu_F(a), \mu_V(u)\}, \nu_V(au) \leq \max\{\nu_F(a), \nu_V(u)\}$ for all $a \in X$ and for all
$u \in Y$

(iv) $\mu_F(1) \geq \mu_V(0), \nu_F(1) \leq \nu_V(0)$,

then (V, Y) is said to be an *intuitionistic fuzzy linear space over* (F, X).

Proposition 2.2. [11] *If (V, Y) is an intuitionistic fuzzy linear space over (F, X), then*

(i) $\mu_F(0) \geq \mu_V(0), \nu_F(0) \leq \nu_V(0)$

(ii) $\mu_V(0) \geq \mu_V(u), \nu_V(0) \leq \nu_V(u)$ *for all* $u \in Y$

(iii) $\mu_F(0) \geq \mu_V(u), \nu_F(0) \leq \nu_V(u)$ *for all* $u \in Y$.

Theorem 2.1. [11] *Let (F, X) be an intuitionistic fuzzy field of X and Y a linear space over X. Let V be an intuitionistic fuzzy set in Y. Then (V, Y) is an intuitionistic fuzzy linear space over (F, X) if and only if*

1. $\mu_V(au + bv) \geq \min\{\mu_F(a), \mu_V(u), \mu_F(b), \mu_V(v)\}$ *and* $\nu_V(au + bv) \leq \max\{\nu_F(a), \nu_V(u), \nu_F(b), \nu_V(v)\}$ *for all* $a, b \in X$ *and for all* $u, v \in Y$.

2. $\mu_F(1) \geq \mu_V(u)$ *and* $\nu_F(1) \leq \nu_V(u)$*, for all* $u \in Y$.

3. (μ, ν)-LINEAR TRANSFORMATIONS

In this section, the notion of (μ, ν)-linear transformations is introduced and some structural properties are discussed. (μ, ν)-linear transformations are linear transformations which preserve the intuitionistic fuzzy structure of intuitionistic fuzzy linear spaces.

In [11], it is proved that if (V, Y) and (W, Z) are intuitionistic fuzzy linear spaces over an intuitionistic fuzzy field (F, X) and if $T : Y \to Z$ is a linear transformation, then $(T(V), Z)$ and $(T^{-1}(W), Y)$ are intuitionistic fuzzy linear spaces over (F, X). Also, by definition, for every $y \in Y$,

(i) $\mu_{T(V)}(Ty) = \sup\{\mu_V(y\prime) : y\prime \in Y, Ty\prime = Ty\} \geq \mu_V(y\prime)$ for all $y\prime \in Y$ with $Ty\prime = Ty$.

(ii) $\nu_{T(V)}(Ty) = \inf\{\nu_V(y\prime) : y\prime \in Y, Ty\prime = Ty\} \leq \nu_V(y\prime)$ for all $y\prime \in Y$ with $Ty\prime = Ty$.

(iii) $\mu_{T(V)}(Ty) = \mu_V(y)$ and $\nu_{T(V)}(Ty) = \nu_V(y)$ if T is injective.

(iv) If $y \in \ker T$, then $T0 = 0$. Also, $\mu_V(0) \geq \mu_V(y)$ and $\nu_V(0) \leq \nu_V(y)$. Therefore $\mu_{T(V)}(Ty) = \mu_V(0)$ and $\nu_{T(V)}(Ty) = \nu_V(0)$.

Theorem 3.1. *Every linear transformation of a linear space Y into another linear space Z together with an intuitionistic fuzzy linear space in Z induces an intuitionistic fuzzy linear space in Y.*

Proof. Assume that Y and Z are linear spaces over the field X.
Given a linear transformation $T : Y \to Z$ and an intuitionistic fuzzy a linear space (W, Z) over fuzzy field (F, X), consider the composite functions $\mu_W \circ T : Y \to [0, 1]$ and $\nu_W \circ T : Y \to [0, 1]$.
(i) For all $a, b \in X$ and for all $x, y \in Y$,

$$\begin{aligned}(\nu_W \circ T)(ax + by) &= \nu_W(T(ax + by)) = \nu_W(aTx + bTy) \\ &\leq \max\{\nu_F(a), \nu_F(b), \nu_W(Tx), \nu_W(Ty)\} \\ &= \max\{\nu_F(a), \nu_F(b), (\nu_W \circ T)(x), (\nu_W \circ T)(y)\}.\end{aligned}$$

(ii) For all $y \in Y$, $\nu_F(1) \leq \nu_W(0) \leq \nu_W(Ty) = (\nu_W \circ T)(y)$.

Analogously, $$\begin{aligned}(\mu_W \circ T)(ax + by) &\geq \max\{\mu_F(a), \mu_F(b), (\mu_W \circ T)(x), (\mu_W \circ T)(y)\}, \\ \mu_F(1) &\geq (\mu_W \circ T)(y).\end{aligned}$$

Consequently, by theorem 2.1, the intuitionistic fuzzy set in Y with membership function $\mu_W oT$ and nonmembership function $\nu_W oT$ is an intuitionistic fuzzy linear space in Y over the intuitionistic fuzzy field (F, X).

Definition 3.1. Let (V, Y) and (W, Z) be intuitionistic fuzzy linear spaces over the intuitionistic fuzzy field (F, X). If there exists a linear transformation $T : Y \to Z$ such that $\mu_W oT = \mu_V$ and $\nu_W oT = \nu_V$, then $(T, (V, Y), (W, Z), (F, X))$ is said to be a (μ, ν)-linear transformation.

Proposition 3.1. *If $(T, (V, Y), (W, Z), (F, X))$ is a (μ, ν)-linear transformation, then*

(i) $\mu_W(0) = \mu_V(0)$ *and* $\nu_W(0) = \nu_V(0)$

(ii) $\mu_V(y) = \mu_W(0)$ *and* $\nu_V(y) = \nu_W(0)$ *for all* $y \in \ker\ T$.

Proof. Since $(T, (V, Y), (W, Z), (F, X))$ is a (μ, ν)-linear transformation, we have $\mu_W oT = \mu_V$ and $\nu_W oT = \nu_V$. Therefore for all $y \in Y$, $\mu_V(y) = (\mu_W oT)(y) = \mu_W(Ty)$ and $\nu_V(y) = (\nu_W oT)(y) = \nu_W(Ty)$. Hence

(i) $\mu_W(0) = \mu_W(T0) = \mu_V(0)$ and $\nu_W(0) = \nu_V(0)$ also.

(ii) If $y \in \ker\ T$, then $\mu_V(y) = \mu_W(Ty) = \mu_W(0)$ and $\nu_V(y) = \nu_W(0)$ as well.

Proposition 3.2. *If $(T, (V, Y), (W, Z), (F, X))$ is a (μ, ν)-linear transformation, then*

(i) $T^{-1}(W) = V$

(ii) $T(V) \subseteq W$ *if* T *is injective.*

Proof. (i) For all $y \in Y$, $\nu_{T^{-1}(W)}(y) = \nu_W(Ty) = \nu_V(y)$ and likewise, $\mu_{T^{-1}(W)}(y) = \mu_V(y)$.
(ii) Assume that $T : Y \to Z$ is injective. Let $z \in Z$ be arbitrary.

If $T^{-1}(z) \neq \emptyset$, then there exists unique $y \in Y$ such that $Ty = z$. Hence

$\nu_{T(V)}(z) = \nu_{T(V)}(Ty) = \nu_V(y) = \nu_W(Ty) = \nu_W(z)$. Similarly, $\mu_{T(V)}(z) = \mu_W(z)$.

If $T^{-1}(z) = \emptyset$, then $\mu_{T(V)}(z) = 0 \leq \mu_W(z)$ and $\nu_{T(V)}(z) = 1 \geq \nu_W(z)$.

Thus $\mu_{T(V)}(z) \leq \mu_W(z)$ and $\nu_{T(V)}(z) \geq \nu_W(z)$ for all $z \in Z$, whence $T(V) \subseteq W$.

Corollary 3.1. *If* $(T, (V, Y), (W, Z), (F, X))$ *is a* (μ, ν)*-linear transformation and if* $T : Y \to Z$ *is injective, then*

$$\mu_{T(V)}(z) = \begin{cases} \mu_W(z), & \text{if } z \in Range(T) \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{T(V)}(z) = \begin{cases} \nu_W(z), & \text{if } z \in Range(T) \\ 1, & \text{otherwise.} \end{cases}$$

Proof. As in the proof of proposition 3.2(ii).

Next corollary is immediate from the definition of (α, β)-level sets and from proposition 3.2.

Corollary 3.2. *If* $(T, (V, Y), (W, Z), (F, X))$ *is a* (μ, ν)*-linear transformation, then*

(i) $N_{\alpha,\beta}(T^{-1}(W)) = N_{\alpha,\beta}(V)$

(ii) $N_{\alpha,\beta}(T(V)) \subseteq N_{\alpha,\beta}(W)$ *if* T *is injective.*

Proposition 3.3. *Let* $(T, (V, Y), (W, Z), (F, X))$ *be a* (μ, ν)*-linear transformation and let* $y_1, y_2 \in Y$*. If* $Ty_1 = Ty_2$*, then* $\mu_V(y_1) = \mu_V(y_2)$ *and* $\nu_V(y_1) = \nu_V(y_2)$*.*

Proof.
$$\begin{aligned} Ty_1 = Ty_2 &\implies \mu_W(Ty_1) = \mu_W(Ty_2) \text{ and } \nu_W(Ty_1) = \nu_W(Ty_2) \\ &\implies \mu_V(y_1) = \mu_V(y_2) \text{ and } \nu_V(y_1) = \nu_V(y_2). \end{aligned}$$

Proposition 3.4. *Let Y be a linear space in $\mathbb{R}^m$ and Z be a linear space in $\mathbb{R}^n$. If $(T,(V,Y),(W,Z),(F,\mathbb{R}))$ is a surjective (μ,ν)-linear transformation and if V is a convex intuitionistic fuzzy set in Y, then W is a convex intuitionistic fuzzy set in Z.*

Proof. Let $0 \leq \lambda \leq 1$ and let $z_1, z_2 \in Z$. Since T is surjective, $z_1 = Ty_1$ and $z_2 = Ty_2$ for some $y_1, y_2 \in Y$. Therefore

$$\begin{aligned}
\mu_W(\lambda z_1 + (1-\lambda)z_2) &= \mu_W(\lambda Ty_1 + (1-\lambda)Ty_2) = \mu_W(T(\lambda y_1 + (1-\lambda)y_2)) \\
&= \mu_V(\lambda y_1 + (1-\lambda)y_2) \\
&\geq \min\{\mu_V(y_1), \mu_V(y_2)\}, \text{since } V \text{ is convex} \\
&= \min\{\mu_W(Ty_1), \mu_W(Ty_2)\} = \min\{\mu_W(z_1), \mu_W(z_2)\}.
\end{aligned}$$

and

$$\begin{aligned}
\nu_W(\lambda z_1 + (1-\lambda)z_2) &= \nu_W(T(\lambda y_1 + (1-\lambda)y_2)) \\
&= \nu_V(\lambda y_1 + (1-\lambda)y_2) \\
&\leq \max\{\nu_V(y_1), \nu_V(y_2)\}, \text{since } V \text{ is convex} \\
&= \max\{\nu_W(Ty_1), \nu_W(Ty_2)\} = \max\{\nu_W(z_1), \nu_W(z_2)\}
\end{aligned}$$

Hence W is a convex intuitionistic fuzzy set in Z.

In the next section, the notion of isomorphism between intuitionistic fuzzy linear spaces is formulated in terms of (μ,ν)-linear transformations.

4. ISOMORPHISM OF INTUITIONISTIC FUZZY LINEAR SPACES

Definition 4.1. Two intuitionistic fuzzy linear spaces (V,Y) and (W,Z) over the intuitionistic fuzzy field (F,X) are said to be isomorphic if there exists a bijective linear transformation $T: Y \to Z$ such that $(T,(V,Y),(W,Z),(F,X))$ is a (μ,ν)-linear transformation. In this case, $(T,(V,Y),(W,Z),(F,X))$ is called an intuitionistic fuzzy linear space isomorphism. If (V,Y) and (W,Z) are isomorphic, then it is expressed in symbols by $(V,Y) \cong (W,Z)$.

Proposition 4.1. *$(T,(V,Y),(W,Z),(F,X))$ is an intuitionistic fuzzy linear space isomorphism, then $T(V) = W$.*

Proof. Let $z \in Z$ be arbitrary. Then there is a unique $y \in Y$ such that $Ty = z$. So $\nu_{T(V)}(z) = \nu_{T(V)}(Ty) = \nu_V(y) = \nu_W(Ty)) = \nu_W(z)$ and, similarly, $\mu_{T(V)}(z) = \mu_W(z)$.

Following corollary is immediate.

Corollary 4.1. $(T,(V,Y),(W,Z),(F,X))$ *is an intuitionistic fuzzy linear space isomorphism, then* $N_{\alpha,\beta}(T(V)) = N_{\alpha,\beta}(W)$.

Theorem 4.1. *The relation of isomorphism on the class of all intuitionistic fuzzy linear spaces over an intuitionistic fuzzy field is an equivalence relation.*

Proof. *Reflexivity.*

Every intuitionistic fuzzy linear space (V,Y) over an intuitionistic fuzzy field (F,X) is isomorphic to itself because the identity mapping $I : Y \to Y$ is a bijective linear transformation along with $\mu_V o\, I = \mu_V$ and $\nu_V o\, I = \nu_V$.

Symmetry.

Assume that (V_1,Y_1) and (V_2,Y_2) are intuitionistic fuzzy linear spaces over the fuzzy field (F,X) and assume that $(V_1,Y_1) \cong (V_2,Y_2)$. Then, by definition, there exists a bijective linear transformation $T : Y_1 \to Y_2$ satisfying $\mu_{V_2} \circ T = \mu_{V_1}$ and $\nu_{V_2} \circ T = \nu_{V_1}$. Its inverse $T^{-1} : Y_2 \to Y_1$ is again a bijective linear transformation. Moreover, corresponding to every $y_2 \in Y_2$, there exists a unique $y_1 \in Y_1$ such that $T^{-1}(y_2) = y_1$. This leads to $\left(\nu_{V_1} \circ T^{-1}\right)(y_2) = \nu_{V_1}\left(T^{-1}(y_2)\right) = \nu_{V_1}(y_1) = \nu_{V_2}(Ty_1) = \nu_{V_2}(y_2)$, so $\nu_{V_2} = \nu_{V_1} \circ T^{-1}$. Analogously, $\mu_{V_2} = \mu_{V_1} \circ T^{-1}$. Thus $(V_2,Y_2) \cong (V_1,Y_1)$.

Transitivity.

Let (V_1,Y_1), (V_2,Y_2) and (V_3,Y_3) be intuitionistic fuzzy linear spaces over the fuzzy field (F,X). Assume that $(V_1,Y_1) \cong (V_2,Y_2)$ and $(V_2,Y_2) \cong (V_3,Y_3)$. Then there exists a bijective linear transformation $T_1 : Y_1 \to Y_2$ such that $\mu_{V_2} \circ T_1 = \mu_{V_1}$ and $\nu_{V_2} \circ T_1 = \nu_{V_1}$, and there exists a bijective linear transformation $T_2 : Y_2 \to Y_3$ such that $\mu_{V_3} \circ T_2 = \mu_{V_2}$ and $\nu_{V_3} \circ T_2 = \nu_{V_2}$. Consequently,

$T_2 \circ T_1 : Y_1 \to Y_3$ is a bijective linear transformation with

$$\begin{aligned}\left(\nu_{V_3} \circ (T_2 \circ T_1)\right)(y_1) &= \left((\nu_{V_3} \circ T_2) \circ T_1\right)(y_1) = (\nu_{V_3} \circ T_2)(T_1 y_1) \\ &= \nu_{V_2}(T_1 y_1) = \nu_{V_1}(y_1)\end{aligned}$$

and, similarly, $\left(\mu_{V_3} \circ (T_2 \circ T_1)\right)(y_1) = \mu_{V_1}(y_1)$ for all $y_1 \in Y_1$, so that $\mu_{V_1} = \mu_{V_3} \circ (T_2 \circ T_1)$ and $\nu_{V_1} = \nu_{V_3} \circ (T_2 \circ T_1)$. Thus $(V_1,Y_1) \cong (V_3,Y_3)$.

Corollary 4.2. *The relation of isomorphism on the class of all intuitionistic fuzzy linear spaces of a linear space over an intuitionistic fuzzy field is an equivalence relation.*

5. CONCLUSION

In this study, the concepts of classical linear transformations and isomorphism of linear spaces is extended to the intuitionistic fuzzy framework. The findings indicate that such isomorphisms preserve both the linear space operations and the intuitionistic fuzziness, ensuring consistency between intuitionistic fuzzy structures and their crisp counterparts.

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