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The Efficacy of Hubble Constant in a Dark Energy Dominant Universe in Relativistic Cosmology

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Abstract

The Hubble Constant has been considered as the utmost indispensable numerical data in Astrophysics and Space Science since the application of this value in the mathematical equation which deploys the instance from this current point of time to the split second of commencement of Big Rip cosmology, as established by Astrophysicist Robert R. Caldwall, can reveal the speculation regarding the measure how hurriedly the exponential expansion of the cosmos has been executing due to the presence of dark energy's abundance which in turn helps the space researchers to through a light on the age of the cosmos and its history of genesis. Edwin Hubble initially calculated this constant from his investigatory data endorsed, regarding the stars in the year 1929 and accordingly published his noble work in a research paper in which the initiation of this numerical entity upshots a ground-breaking sensation throughout the whole world and pushed the science of Astrophysics a million miles further. This current research work deals with the introduction and investigation of a cosmological model which is bestowed upon the implementation of the well-recognized Classical Ideal Gas Law. Here, in this present paper, we set for the analysis of the various cosmological behaviors conferred by the considered model universe under the subsistence of the Classical Ideal Gas Law. During this tactic, an attempt is made in inspecting the extreme destructive fatality incurred in the universe that has been transpiring from the fruition of sturdy Phantom Energy. The model considered for this research has been grounded on the starter of bulk viscosity, denoted by the Greek letter ξ and supplementary parameters of the cosmos have been inspected such as the well-thought-out gravitational Constant G and Cosmological Constant Λ, both exhibiting the inconstant characteristics under the considered model. The exponential streaming out rate of the universe has been investigated from the analysis of these properties. One such incredible analysis encompassed based on these elementary characteristics exhibited by the universe is its cosmic inflation. It has been quite obvious that the inflation phenomenon endorses in the sturdy occurrence of bulk viscosity. Under such scenario, the governance of dark energy engrains even in the absence of the condition $\omega = -1$ where ω stands for the dimensionless parameter, satisfying the equation of state of barotropic fluid represented by the relation that connects the pressure p and energy densityρ which is $p = -\rho$. The characteristics portrayed from those of Einstein de-Sitter Cosmological Model insinuate dissimilar characteristics in the case of a cosmological inflationary epoch since during such an inflationary time epoch, the energy density ρ which is an imperative physical property suffer fluctuations w.r.t. time scale. It has been revealed as a further investigatory research result that the Gravitational Constant G and the Cosmological Constant Λ, suffer exponential increment w.r.t. time t whereas the physical characteristic, energy density suffers continual exponential deceleration w.r.t. time instances.

Keywords: Epoch, Expansion, Exponential, Hubble Constant, time.

Introduction

The Hubble constant has been emerged as one of the utmost impetus numerals in Cosmology as it sketches a realistic interpretational scenario regarding the exponential expansion of the today's discernible cosmos, fosters in predicting the initial time epoch when the cosmos had been created just after the Big Bang Cosmology, reminiscence the space researchers about the age of the cosmos and its formation history [1], [2], [3]. The Astrophysicists and space researchers have been confronting with multiple challenges in appropriately resulting the accurate value of the Hubble Constant which has been raised as one of the peak disputes in Modern Astronomy and Space Science since enumerating the exact value of Hubble Constant can transfigure the concept of the present epoch's cosmos.

It has been an innovative appraise to accurately compute the Hubble's constant, an essential numeral that measures the rate of stretching out of the cosmos and fixes the ambiguities that acts in response to answer the interrogations regarding the size, maturity, genesis, shape, numerous cosmological spectacles incurred in space. Following the laws and postulates of Fundamental Physics, space researchers have calculated the value of the Hubble constant should amounts roughly to be 68 kilometres/sec/Megaparsec [1], [3], [4]. The intense up-to-the minute experimentations inferred on the extremely located exploding stars insinuate a further précised ranging value for the Hubble Constant which has been enumerated as 69.8 km/s/Mpc while other investigations have endorsed this value to be higher than the previously computed value and approved it to be 74 km/s/Mpc [1], [2], [3].

The standardized cosmological model indicates that the scale factor of the cosmos is accelerating and in a hypothesized epoch confirming a dominance of Cosmological Constant introduced by Albert Einstein, i.e., a static universe and submissive to gravity, this has been raising a potential to augment in an exponential manner [1], [2], [3], [4]. This spreading out mechanism is uniformly identically distributed for every instance of time and has its representation by means of an unaltered or fixed entity at each point of time and that has been emerging as a constant term possessing negligible value, named as Hubble Constant, in respect of Edwin Hubble, an ex-alumnus of University of Chicago, USA. The Hubble constant has been emerged as one of the utmost impetus numerals in Cosmology as it sketches a realistic interpretational scenario regarding the exponential expansion of the today's discernible cosmos, fosters in predicting the initial time epoch when the cosmos had been created just after the Big Bang Cosmology, reminiscence the space researchers about the age of the cosmos and its formation history [1], [2], [3], [4].

The procedure that entails the numerical computation of the vastness of the stretching expanse of the cosmos in a dark energy model universe is the Hubble Constant, mathematically symbolized by H where the value of the Hubble Constant at the present epoch is represented by H_0.

The Hubble-Lemaitre Law or shortly the Hubble Law is accredited to Edwin Hubble for his research paper published in the year 1929 [5], [6] which has marked a vital upright in the arena of Astrophysics and Space science, is a correlation between the speed and distance traversed by celestial bodies like the gigantic starts, galaxies or cluster of galaxies depicting the fact that a galaxy is drifting apart from the planet Earth at a speeds that varies proportionally to its remoteness from the Earth. The consistent relationship redirects, $v = H_0 D$, where v represents recessional velocity, H_0 stands for Hubbles' Constant and D represents the accurate distance traversed by a celestial object from an observer residing on the planet Earth [5].

This current research has been carried out with the revelation of the imperative fields generated by Phantom energy, flourishing the operative upshots of bulk viscosity ξ which has been found indistinguishable to the effects or outcome based consequences when p represents the pressure term arising in the equations and that has been replenished by an effective pressure term, denoted by the symbolization p_{eff} and satisfies the relation $p_{eff} = p - 3\xi H$ where the terms arising in the relation has their usual meaning and H as usually denotes the Hubble's Constant. Correspondingly, under such privilege a good numeral of cosmological characteristics has been studied where the ascendency of Classical Ideal Gas Law forms the plinth of this present research work. It has been a well-established fact that a universe that has been experiencing an escalation in an exponential rate of streaming out in every possible direction, the viscous effects incurred in the universe are exhibiting dissipative characteristics that has been accredited to the establishment of energy i.e., the matter in this case. Additionally, during this present investigatory research work, the physical property, gravity has been coupled with viscosity and vacuum and the corresponding upshots have been implemented. An endorsement of presupposition on the stipulated restraint is that there persists a guaranteed strategy in the showground of genesis of the cosmos which is expected to progress in an interesting mechanism. This present research work has been progressed on the tactic which has been bestowed upon the cosmological models designed or articulated by a predecessor in this field of cosmological arena named Arbab in the year 2007 [7], where he formulated the models that have been plinth on the most up-to-date dark energy and Phantom Energy podiums. Here Arbab has enunciated regarding the hopefulness of such manner of rehearses and at the same instance, set on the demarcated trail to pursue a number of captivating physical characteristics relating to the fruition of the universe since the cosmic inflation phenomenon gets developed as an amalgamation of vacuum energy and bulk viscosity.

Formulating of the Problem for the Present Research:
i.e., Substantiating the Cosmological Model with the Implication of Classical Ideal Gas Law on Establishing Viscous Dark Energy Models Reinforcing the Appearances of Variable Gravitational Constant $\boldsymbol{G}$ and Cosmological Constant $\boldsymbol{\Lambda}$:

In this case, for the construction of the model, it has been taken into consideration of the action of Einstein- Hilbert with the dependability on Albert Einstein's Cosmological Constant, denoted by the Greek letter, Λ such that

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\,(R + 2\Lambda) + S_{matter} \quad \text{.................................(1)}$$

Here we have concentrated on the variation or alteration of the metric, corresponding to $g_{\mu\nu}$, where it has been considered about the authenticity of the presupposition stipulation, $f(R) = R - 2\Lambda$, that gets generated consistent with Amarzguioui *et al.* [8] has been premeditated as

$$f^{/}(R)\,R_{\mu\nu} - \frac{1}{2}\,f(R)\,g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad \text{...................................(2)}$$

Here the term $T_{\mu\nu}$ represents the energy momentum tensor of the cosmological fluid.

In the case of an ideal fluid, we shall have the following equation

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \quad \text{..(3)}$$

Equation (2) is now being contracted and Equation (3) has been utilized and in the same nature, envisaging only the 00 components, we are to obtain the equations as

$$Rf^{/}(R)\text{-}2f(R) + 8\pi GT = 0 \quad \text{...(4)}$$

and

$$f^{/}(R)\,R_{00} + \frac{1}{2}f(R) + 8\pi G T_{00} = 0 \quad \text{....................................(5)}$$

satisfying the stipulations which entail $T_{00} = \rho, T = \rho - 3p$ and $T_{ij} = -p$ for specific values of the subscripts $i, j = 1,2,3$

The Friedmann- Lemaitre- Robertson- walker Metric for a flat universe is represented by

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right)$$

Consequently, this gives rise to mathematical computations so that we shall get the followings:

$R_{00}=-3\left(\frac{\ddot{a}}{a}\right)$ $and\ R = -6\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a}\right)$, a as usually stands for the scale factor.

Implementing these conditions, Equations (4) and (5) yields the equations which are as under:

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda \quad \text{.......................................(6)}$$

And this subsequently leads to the following equation

$$3\left(\frac{\ddot{a}}{a}\right) = -4\pi G(\rho + 3p) + \Lambda \quad \text{.................................(7)}$$

The Energy Conservation Equation for the considered model is written as

$$\dot{\rho}+3\left(\frac{\dot{a}}{a}\right)(p + \rho) = 0 \quad \text{.......................................(8)}$$

The equation of state of an ideal (barotropic) fluid can be expressed as

$p = \omega\rho$

In this above Equation, p represents the pressure and ρ, the energy density, ω is a dimensionless parameter and considered as a constant entity.

For the model considered in this present research, we have introduced the Classical ideal Gas Law [9], [10], [11], [12], [13] which is as follows:
$pV = nRT$

This Ideal Gas Law may also be expressed as follows:

$$p = \rho(\gamma - 1).e \quad \text{.......................................(9)}$$

Here in this above Equation (9),

ρ is the density.
$\gamma = \frac{C_p}{C_v}$ is the adiabetic index or the ratio of specific heats.
$e = C_v T$ is the internal energy per mass i.e., the specific internal energy
C_v is the specific heat at constant volume.
C_p is the specific heat at constant pressure
T is the temperature in Kelvin Scale.

The ratio of the specific heats $\gamma = \frac{C_p}{C_v}$ is a factor in adiabetic processes and has a special significance in determining the speed of sound waves in gas. The ratio stands with a result $\gamma = 1.664$ for noble gases like He, Ne, Argon etc., all bear the same value of adiabatic index or ratio of specific heats at a temperature of 273 Kelvin, whereas this ratio rises to be 1.4 for air [14], [15].

The variation of entropy for an ideal gas refers to the fluctuations of temperature during a process or any thermodynamic reaction phenomenon. The entropy e of a monoatomic ideal gas can have its expression in terms of a renowned equation called the Sackur-Tetrode Equation [16], [17], [18], [19] expressed by

$$S \; or \; e = Nk \left[\ln \left\{ \frac{V}{N} \left(\frac{4\pi m U}{3Nh^2} \right)^{3/2} \right\} + \frac{5}{2} \right]$$

where
$N =$ Number of atoms present in the gas sample constituting the model universe
k or $kB =$ Boltzmann's Constant $= 1.3806452 \times 10^{-23}$ Joules/ Kelvin.
This constant relates the average kinetic energy of the gas particles consisting in a gas sample.
$V =$ Volume
$U =$ Internal Energy
$h =$ Plank's Constant $= 6.626 \times 10^{-34}$ joule-second. This relates the energy of a photon to its frequency. It bears the same value regardless of a classical ideal gas or any other system.

The present paper under investigation employs the Einstein's Field equations, accompanying with two physical parameters viz., Gravitational Constant G and Cosmological Constant Λ which have been flourishing the property of dependency on time scales and that further produces two numbers of Equations (6) and (7) which are independent of one another and flourishing the structure that has been indistinguishable or impossible to differentiate as flourished by the Model of a standard Cosmology. Therefore, G, Λ can now flaunt the possessions of constant terms that continue to subsist variations w.r.to time t which means that $\Lambda = \Lambda(t)$ and $G = G(t)$.

We have the Bianchi Identity as

$$\left(R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}\right)_{;\mu} = -(8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu})_{;\mu} = 0 \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Equations (2) and (3) can be implemented which gives rise the equation

$$G\dot{\rho} + 3(p+\rho)G\frac{\dot{a}}{a} + \rho\dot{G} + \frac{\dot{\Lambda}}{8\pi} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Bulk viscosity is being presented in a standardized, homogeneous perfect fluid with the replenishment of the pressure term p by introducing an effective pressure term, denoted by p_{eff} and accordingly expressed by the following relation

$$p_{\text{eff}} = p - 3\xi H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where ξ stands for the co-efficient of Bulk viscosity.

This has been conventionally modelled or validated by the equation below

$$\xi = \xi_0 \rho^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where ξ, ξ_0 are constants.

Now, we are to replace the pressure term p arising in Equation (11) with the implication of the effective pressure term p_{eff} of Equation (12) and then subsequently from Equation (8), we obtain the value of the term $p + \rho$ as

$$p + \rho = -\frac{\dot{\rho}}{3\left(\frac{\dot{a}}{a}\right)}$$

Therefore, we have

$$8\pi\dot{G}\rho + \dot{\Lambda} = 9\xi(8\pi G)H^2 \qquad \left[\text{Writing } \frac{\dot{a}}{a} = H\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Here for the Model presumed, we have considered the following ansatz

$$\Lambda = \frac{3\beta}{\rho^\alpha}, \text{ where } \alpha, \beta \text{ are constants.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

∴ Differentiating w.r.t. t and computing the first order derivative, we shall have the value as

$$\dot{\Lambda} = -3\alpha\beta\rho^{-(\alpha+1)}\,\dot{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Moreover, our presumption about the considered Model or supposition leads us to express the functional dependence of the gravitational constant G ,formulated by the equation as under:

$$8\Pi G = D\rho^{-(\alpha+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

where D is a constant.

Now applying Equation (9) and simplifying Equation (11) then finally integrating and proceeding subsequently, we have

$$G\dot{\rho} + 3[\{\rho(\gamma-1).e\} + \rho]G\frac{\dot{a}}{a} + \rho\dot{G} - \frac{3\alpha\beta\rho^{-(\alpha+1)}}{8\pi} = 0$$

$[\therefore p = \rho(\gamma-1).e$

(From Equation (9))]

$$\Rightarrow \int\frac{\dot{\rho}}{\rho} + 3[\{(\gamma-1).e\} + 1]\int\frac{\dot{a}}{a} + \int\frac{\dot{G}}{G} - \frac{3\alpha\beta}{8\pi D\rho^{-(\alpha+1)}}\frac{1}{\rho^{(\alpha+1)}}\int\frac{\dot{\rho}}{\rho} = 0$$

Putting $[G = D\rho^{-(\alpha+1)}]$

$$\Rightarrow \log\rho^{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)} = -\log D a^{3[\{(\gamma-1).e\}+1]}$$

$$\Rightarrow \rho^{A} = D^{-1}a^{-3[\{(\gamma-1).e\}+1]}$$

[Writing $A = -\alpha\left(1 + \frac{3\beta}{8\pi D}\right)$, a constant]

$\Rightarrow \rho = E\,a^{-\frac{3[\{(\gamma-1).e\}+1]}{A}}$ where $E = D^{-\frac{1}{A}}$, a constant term.

$$\Rightarrow \rho = E\,a^{-\frac{F}{A}}$$

Writing $F = 3[\{(\gamma-1).e\} + 1]$ …………………………………… (18)

Equation (17) will hold good under the condition that $E \neq 0$

i.e., $D^{-\frac{1}{A}} \neq 0 \Rightarrow A \neq 0$

$\Rightarrow -\alpha\left(1 + \frac{3\beta}{8\pi D}\right) \neq 0$; $\alpha \neq 0$ and $D \neq 0$ …… (19)

From Equation (14), we shall have

$$\frac{\dot{G}}{G} - \frac{3\alpha\beta\rho^{-(\alpha+1)}\dot{\rho}}{8\Pi G\rho} = \frac{1}{8\Pi G\rho}.9\xi(8\pi G).\frac{1}{3}\left[8\pi G\rho + \frac{3\beta}{\rho^{\alpha}}\right] \ldots\ldots(20)$$

$[\because\ \Lambda = \frac{3\beta}{\rho^{\alpha}}$

and $\dot{\Lambda} = -3\alpha\beta\rho^{-(\alpha+1)}\dot{\rho}$]

The First Order Thermodynamic Theory of Eckart stresses us to substantiate the following field equations under the surveillance of bulk viscous stresses in the universe which are

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3} \Rightarrow H = \sqrt{\frac{\rho}{3}} = \frac{1}{\sqrt{3}}\sqrt{\rho} = \frac{1}{\sqrt{3}}\rho^{\frac{1}{2}} \quad \text{.........(21)}$$

Therefore, from the first order Thermodynamic Theory of Eckart, we have the Field Equation relating the scale factor, Hubble Constant and the energy density as

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3}$$

[As illustrated in Equation (21)]
Applying Equation (21),

$$\frac{\dot{a}}{a} = \frac{1}{\sqrt{3}}\left[D^{-\frac{1}{A}}\ a^{-\frac{3[\{(\gamma-1).e\}+1]}{A}}\right]^{\frac{1}{2}}$$

[Utilizing the Equations, $\rho = E\,a^{-\frac{3[\{(\gamma-1).e\}+1]}{A}}, E = D^{-\frac{1}{A}}, A = -\alpha\left(1 + \frac{3\beta}{8\pi D}\right)$]

The expression for the scale factor $a(t)$, obtained from stepwise simplifications is as follows:

$$a = \frac{1}{\sqrt{3}} \cdot \frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}}\, t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2 \quad (22)$$

$$\Rightarrow a = \frac{1}{\sqrt{3}}\frac{1}{D\sqrt{\frac{1}{F}}} \cdot \frac{1}{t^{\frac{\alpha}{F}}} + K_2 \quad \text{.. (23)}$$

Writing $F = 3[\{(\gamma-1).e\}+1]$ and $J = 1 + \frac{3\beta}{8\Pi D}$

where D, J, F, α are all constants.

Now, from Equation (23), we have

$$\dot{a} = \frac{1}{\sqrt{3}}\frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}} \cdot \left\{-\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\} . t^{-\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}} \quad (24)$$

Also, from the first order Thermodynamic Theory of Eckart, we have the following Field Equation relating to scale factor and Hubble Constant as

$$\frac{\dot{a}}{a} = H$$

[from Equation (21)]

$$\therefore H = \frac{\frac{1}{\sqrt{3}}\frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}}\cdot\left\{-\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}.t^{-\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}}}{\frac{1}{\sqrt{3}}\cdot\frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}}t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2}$$

.(25)

$$\Rightarrow H = \frac{\left\{-\frac{\alpha.J}{F}\right\}.t^{-\left\{\frac{\alpha J}{F}+1\right\}}}{t^{\frac{-\alpha J}{F}}+K_2}$$

Writing $F = 3[\{(\gamma-1).e\}+1]$ and $J = 1+\frac{3\beta}{8\Pi D}$

where D, J, F, α are all constants.

(26)

Again, from the Eckart Theory, the Field Equation yield the equation for the energy density ρ as

$$\rho = 3H^2 = 3\left[\frac{\left\{-\frac{\alpha.J}{F}\right\}.t^{-\left\{\frac{\alpha J}{F}+1\right\}}}{t^{\frac{-\alpha J}{F}}+K_2}\right]^2$$ [Using (26)]

$$\Rightarrow \rho = 3\left[\frac{\left\{-\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}.t^{-\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}}}{t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2}\right]^2$$

$$\Rightarrow \rho = 3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2.t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right] \quad (27)$$

$$\Rightarrow \rho = 3\left[\frac{\left\{\frac{\alpha.J}{F}\right\}^2 .\ t^{\left\{\frac{\alpha.J}{F}+1\right\}^2}}{\left\{t^{\frac{\alpha J}{F}}+K_2\right\}^2}\right] .. \qquad (28)$$

Writing $F = 3[\{(\gamma-1).e\}+1]$ and $J = 1+\frac{3\beta}{8\Pi D}$

where D, J, F, α are all constants.

Now the Classical Ideal Gas Law indicates,

$$p = \rho(\gamma-1).e$$

$$= 3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2 .\ t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right].(\gamma-1).e$$

[Using Equation (27)]

$$= 3\left[\frac{\left\{\frac{\alpha^2.J^2}{F^2}\right\}.\ t^{\left\{\frac{\alpha.J}{F}+1\right\}^2}}{\left\{t^{\frac{\alpha J}{F}}+K_2\right\}^2}\right].\ (\gamma-1).e \qquad (29)$$

The expression for the ansatz from Equation (15) becomes

$$\Lambda = \frac{3\beta}{\left[3\left[\frac{\left\{\frac{\alpha.J}{F}\right\}^2 .\ t^{\left\{\frac{\alpha.J}{F}+1\right\}^2}}{\left\{t^{\frac{\alpha J}{F}}+K_2\right\}^2}\right]\right]^\alpha}$$

[Using Equation (28)]

$$= \frac{3^{(1-\alpha)}\beta.\left\{t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^{-2\alpha}}{\left[\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2 .\ t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}\right]^\alpha} \qquad (30)$$

From Equation (28) on utilizing the value of the energy densityρ, the expression for the gravitational constant is calculated to be

$$G = \frac{D}{8\pi}\,\rho^{-(\alpha+1)}$$

$$= \frac{D}{8\pi}\left[\frac{3^{-(1-\alpha^2)}\beta\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^{2\alpha(\alpha+1)}}{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^{-2\alpha(\alpha+1)}.\ t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^{-2\alpha(\alpha+1)}}}\right]$$

$$= \frac{D}{8\pi}\left[\frac{3^{-(1-\alpha^2)}\beta\{t^F+K_2\}^{2\alpha(\alpha+1)}}{\left\{\frac{\alpha.J}{F}\right\}^{-2\alpha(\alpha+1)}.\ t^{\left\{\frac{\alpha.J}{F}+1\right\}^{-2\alpha(\alpha+1)}}}\right] \quad (31)$$

Writing $F = 3[\{(\gamma-1).e\}+1]$ and $J = 1+\frac{3\beta}{8\Pi D}$
where D, J, F, α are all constants.

Computation of Co-efficient of Bulk Viscosity from Equation (13), we have

$$\xi = \xi_0\rho^n = \xi_0\left[3\left[\frac{\left\{\frac{\alpha.J}{F}\right\}^2.\ t^{\left\{\frac{\alpha.J}{F}+1\right\}^2}}{\left\{t^{\frac{\alpha.J}{F}}+K_2\right\}^2}\right]\right]^n \quad \text{[Using Equation (28)]}$$

$$= \xi_0\left[3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2.\ t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right]\right]^n \quad .(32)$$

Writing $F = 3[\{(\gamma-1).e\}+1]$
and $J = 1+\frac{3\beta}{8\Pi D}$
where D, J, F, α are all constants.

2.2: Analysis Portrayed on the Validation of the Calculated Physical Parameters for the Present Study:

- **Analysis Portrayed on Pressure:**

From the Equation (29), the pressure p of the Classical ideal Gas occupying the model universe considered satisfies the equation as $p =$

$3\left[\frac{\left\{\frac{\alpha^2.\left(1+\frac{3\beta}{8\Pi D}\right)^2}{3[\{(\gamma-1).e\}+1]^2}\right\}.\,t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right].(\gamma-1).e$ which shows that $p \propto t, D \neq 0, p \propto \gamma$ which is different for different gas molecules falling under the category of ideal gases. Another additional condition on pressure is that $[\{(\gamma-1).e\} \neq -1]$. Because in that case pressure $p \longrightarrow 0$ i.e. very negligible pressure which signifies a cosmological scenario that contributes to gravity without exerting significant pressure on other matter abundant in the cosmos since dark matter is often treated as Cold Dark Matter (CDM) component under insignificant pressure.

Whenever the pressure tends towards zero, it might lead to two imminent acknowledgeable scenarios in Cosmology and Space Science, which can be illustrated as follows:

- While investigating the physical factors, its related ambiances regarding the early cosmos or in other sense dealing with a cosmic model formulated that has been formulated encompassing a cosmological constant. In such a case, the early ambiances of the cosmos, posterior to the recombination eon which is more frequently acknowledged in the field of Cosmology as the Matter Dominated era of the cosmos where pressure was comparatively insignificant in connection to density of the cosmic fluid [20], [21], [22], [23], [24].

- Furthermore, a cosmological constant (Λ), consistently accompanied with dark energy [25], the negative repulsive force that essentially surpasses gravity, becomes comprehensible as a cosmic fluid, subsisting the characteristics of negatively aggrandized pressure which triggers the accelerated streaming out mechanism incurred in the cosmos [20], [21], [22], [23], [24], [26].
-

Throwing Light on Early universe or more precisely the Matter- Dominated Epoch of the universe:

2.3 Explicating the Recombination Stage and Its Beyond Cosmic Ambiance:

Now posterior to the recombination phenomenon, the cosmos gets transpired from a plasma state of matter to a typically neutral state of the cosmos. Under such cosmic backdrops, the density of ordinary matter along with dark matter [27] become more dominant in the cosmos that necessarily conforms to a cosmic set-up where pressure becomes negligible and essentially leads to zero.

After recombination, the universe transitions from a plasma state to a mostly neutral state, and the influence of radiation on the expansion diminishes. The density of matter (like ordinary matter and dark matter) becomes more dominant, leading to a situation where pressure is less significant.

2.4 Analyzing the $p \to \infty$ Scenario in Compliance with the Friedmann Equations:

The Friedmann Equations, eventually expresses the evolutionary phases concerning to the early epochs of the cosmos, the physical parameters of which the pressure (p) associated with the Equation of State (EoS) parameter ω, arising in the Equation $\omega = \frac{p}{\rho}$ turn out to be additionally insignificant in comparison to density (ρ) during the Matter-Dominated epoch of the universe [20], [21], [22], [23], [24], [26].

2.5 Investigating the Cosmic Models, Formulating with a Cosmological Constant:

The précised exposition of the present up-to the minute cosmos accentuates the formulation of some assortments of cosmological models that essentially clarifies the cosmos's current state where the cosmos has been under the substantial dominance of dark energy which has been predominantly accountable for the exponential expansion incurred in the cosmos. Now as the universe has been experiencing streaming apart tactic in every permissible direction, the two physical quantities, specifically the density and pressure of the cosmos endure deceleration and in due course of time tend to zero with the advancement of time instances. This is due to the magnificent intensifying volume of the cosmos [20], [21], [22], [23], [24], [26].

2.6 While Discussing with Anisotropic Models of the Cosmos:

While dealing with some anisotropic cosmic models of the cosmos which entails that as time progresses, the energy density and pressure of the cosmos might conform to zero. There also arises some probability of transpire of some supplementarily complicated characteristics exhibited by these physical parameters in such isotropic models as demanded by Shri Ram et.al. [28]

2.7 Analysis Carried Out on ansatz:

From Equation (30), the expression for the ansatz Λ gives the conditions of validity. Here,

$$\Lambda = \frac{3^{(1-\alpha)}\beta \, . \left\{ t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2 \right\}^{-2\alpha}}{\left[\left\{ \frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]} \right\}^2 . \, t^{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]} + 1 \right\}^2} \right]^{\alpha}}$$

essentially redirects the condition that $\Lambda \propto t, \alpha > 0, \beta > 0, D \neq 0$ and $\gamma > 0$ which get fluctuates accordingly w.r.t. the gas molecules constituting the model universe considered.

In Cosmology and Astrophysics, an ansatz typically stands for a mathematically précised presumption or simplification procedures which has been characteristically implemented to depict the large-scale structure of the universe. More generalized constraints usually get associated with an ansatz are the isotropy and homogeneity concerns which interprets that the universe is relatively uniform when viewed on a large scale and spatial flatness [29], [30], [31], [32].

2.8 Interpretations Portrayed on the Homogeneous and Isotropic Nature of the Universe:

Homogeneity essentially describes that the universe has been hypothesized to be identical everywhere which predicts that the properties of space remain the same from point to point in the universe [30], [33], [34], [35].

At the similar instance, the universe is supposed to be the same in all directions, redirecting that the properties of space are similar in every direction with respect to a prescribed point.

Such presuppositions have been treated as the utmost vital conditions for the Friedmann-Robertson-Lem$\hat{a}$tre- Walker metric which has been aroused as a widespread ansatz for illustrating the accelerated expansion in the cosmos. The FLRW metric assistances for different spatial geometries viz., flat, positively curved and negatively curved configurations of the observable universe. These explicit conditions depend on the cosmic models and the formulation of those cosmic models ought to be pertinent with the criterions of physical plausibility and consistencies with observations [34].

Some Specific Cosmological Models and the Associated Ansatz:

- **Models Formulated with Scalar Fields:**

Some cosmic models have been formulated so as to illustrate the ascendency of dark energy or inflation. An ansatz plays a crucial role in supposing the potential of the scalar field such as a power law or exponential potential [29], [33], [36].

- **$f(R)$ Gravity:**

In the theories relating to modified gravity, the Larangian of gravity is transfigured so that it encompasses a function containing the Ricci Scalar (R). The situation demands the supposition of an ansatz for the function $f(R)$ [36], [37], [38].

2.9 Investigations Carried Out on the Gravitational Constant G:

From the Equation (31), the expression for the Gravitational Constant G can be found as

$$G = \frac{D}{8\pi}\left[\frac{3^{-(1-\alpha^2)}\beta\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).\varepsilon\}+1]}}+K_2\right\}^{2\alpha(\alpha+1)}}{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]}\right\}^{-2\alpha(\alpha+1)}.\,t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]}+1\right\}^{-2\alpha(\alpha+1)}}}\right]$$

from which it has been analyzed that $G \propto t, D \neq 0, \alpha > 0,\ \beta > 0, G \propto \gamma$

In cosmology, the gravitational constant G plays a crucial role in understanding the evolution and structure of the universe. It's a fundamental constant that describes the strength of gravity, determining how objects with mass attract each other. While

Newton's law of universal gravitation, which uses G, is accurate for describing gravity at the scale of the solar system, it's also a key component in the equations of general relativity, which describe gravity on a cosmological scale [39], [40], [41], [42], [43].

2.10 Impact on Expansion:

Whenever, the Gravitational Constant varies with time, it redirects to a scenario of accelerated expansion incurred in the cosmos. A deceleration in the value of G,

If G(t) decreases, it could contribute to the accelerating expansion observed, as the effective gravitational force weakens over time.

2.11 Explications Done in Light of General Relativity:

Albert Einstein, in his revolutionary Theory of General Relativity, has fascinated gravity as the curvature of space-time. The utilization of the Gravitational Constant G in this framework connects the curvature of spacetime to the mass and energy content of the universe.

2.12 Measurements and Meticulousness Relating to G:

The value of G has been acknowledged with elevated precision. However, cosmologists and space researchers have been incessantly deploying in exploring the potential fluctuations or adaptations of G on cosmological scales. Research all across the sphere has been continual whether G stands truly as a constant entity throughout the cosmos or if there exist any grounds that G functions as a variable that might get fluctuated with time and space and that it erects as a sturdy novel insight into the arena of gravity and cosmology [44], [45], [46].

Gravitational Constant G has been utilized in the mathematical calculations involving the rate of expansion of the universe and fabricating the dynamics of its structure. Instances include the formation of galaxies as well as galaxy clusters. The strength of gravity also became significant in enumerating the gravitational collapse of matter in the very early epoch of the cosmos which had been expected to portray a vital role in forming the large-scale structure of the universe [44], [45], [46].

2.13 Elucidations Envisioned on Co-efficient of Bulk Viscosity:

The Co-efficient of Bulk Viscosity validates the expression from Equation (32) as follows

$$\xi = \xi_0 \left[3 \left[\frac{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{3\Pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]} \right\}^2 . t^{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{3\Pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]}+1 \right\}^2}}{\left\{ t^{\frac{\alpha\left(1+\frac{3\beta}{3\pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]}} + K_2 \right\}^2} \right] \right]^n$$

which predicts that $\xi \propto t, \xi \propto \gamma, D \neq 0$

In Cosmology and Space Science, it has been aroused as a generalized perception that the coefficient of bulk viscosity gets fluctuate with time scale. ξ, the coefficient of bulk viscosity stands as a physical parameter which characterizes the confrontation against compression or enlargement of the cosmos. As the cosmos is not in a state of endurance of proper thermodynamical equilibrium where bulk viscosity insinuates as a dissipative progression due to the deviated mechanisms transpired from the equilibrium state [47], [48], [49].

2.14 Put Forwarding the Reasons in Support of the Condition $\xi \propto t$:

- **Non-equilibrium Conditions:**

The cosmos at the very early epoch was in a state of non-thermal local equilibrium. With the progress of time, the cosmos started expanding and endured several evolutionary phases. The vivid constituents of the cosmos experience the cooling mechanisms at varied frequencies and different time instances that eventually led to deviations from the equilibrium state. The rate at which the cosmos has been subsisting a continual expansion is presumed to fluctuate the time dependence characteristic of the coefficient of bulk viscosity [50], [51]. In a simpler sense, as the cosmos had been transpiring through various evolutionary phases and experienced rapid streaming out in all directions, the time necessitated in restoring local thermal equilibrium which emerges out as a bulk viscosity in the cosmos which in turn acts as an effective pressure, sufficient to reestablish this equilibrium.

- **Energy Density of the Fluid:**

The energy density of the fluid obeying the Ideal Gas Equation with the supremacy of entropy in the system, filling up the model universe considered for this present study, plays a crucial role in getting variations with different time scales.

Thus, the coefficient of bulk viscosity whenever analyzed through the lenses of time dependence perspective, necessarily redirects the dissipative systems arousing due to the deviations from thermodynamical equilibrium state during the process of exponential expansion incurred in the cosmos. Such effects can be attributed to influence the general dynamics relating to the evolution of the cosmos, specifically during the early inflationary time eon and radiation-dominated epoch of the cosmos [47], [48], [49], [51], [52], [53].

3.0 Investigation Grounded on the Evolution of Dark Energy and Start-Up of an Epoch Under the Surveillance of Phantom Energy:

For the effective computation of the energy density ρ, expressed by

$$\rho = 3\left[\frac{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]}\right\}^{2} . \, t^{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).\varepsilon\}+1]}+1\right\}^{2}}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).\varepsilon\}+1]}}+K_2\right\}^{2}}\right]$$

we have, the constant $D \neq 0$, the constraints on the other constants are $\alpha > 0, \beta > 0$. From the expression for ρ in Equation (28), it is clear that $\rho \propto t$.

Moreover, the above pressure expression predicts a condition where $[\{(\gamma-1).e\} \neq -1]$ as in this case the energy density $\rho \longrightarrow 0$ which directs that the universe has been experiencing exponential expansion under the subsistence of dark energy.

3.1 The Implanted Condition for the Development of Phantom Energy in the Universe:

The equation of state for the progress of phantom energy satisfies the relation expressed by $p = \omega\rho c^2$ where the dimensionless parameter ω satisfies the inequality $\omega < -1$ and c stands for the speed of light waves in vacuum or air and $c \cong 3 \times 10^8$ metre/sec

Accordingly, we set for carrying out the investigation with each of these four energy conditions, as under:

4.0 Investigations Accomplished on the Pitch of the Four Different Energy Stipulations:

4.1 The Null Energy Condition:

The Null Energy Condition instructs the following inequality to be satisfied as follows:

$$p + \rho \geq 0 \quad \ldots\ldots\ldots\ldots \quad (33)$$

The values of p and ρ have been taken from Equations (29) and Equation (28) correspondingly and put in Equation (33), so we are getting

$$3\left[\frac{\left\{\frac{\alpha^2.\left(1+\frac{3\beta}{8\Pi D}\right)^2}{3[\{(\gamma-1).e\}+1]^2}\right\}.t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right].(\gamma-1).e+3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2.t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right] \geq 0$$

$$\Longrightarrow 3\left[\frac{\left\{\frac{\alpha^2.\left(1+\frac{3\beta}{8\Pi D}\right)^2}{3[\{(\gamma-1).e\}+1]^2}\right\}.t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right] \geq 0 \text{ and } \{(\gamma-1).e\} \geq 0 \qquad \ldots(34)$$

Now γ, the adiabatic index is always greater than zero. It is a positive value in Thermodynamics as it is defined as the ratio of specific heat capacities at constant pressure to constant volume, that can never be negative [54], [55], [56], [57].

However, the value of entropy e can take either positive or negative values. This variation occurs depending on whether the system is becoming more disordered i.e., positive entropy or more disordered i.e., presuming negative entropy. Essentially, a positive entropy value indicates an increase in disorder within a system while a negative value directs to a deceleration in disorder i.e., calmer and more structured situation within the system under investigation [58], [59], [60].

Based on the above condition for the model under consideration in this paper, it is obvious that the constituting gas molecules circulating inside the model universe maintaining a uniform motion and the gas molecules have been fusing in a well-ordered travelling motion, which ensures us to consider the value of entropy or the degree of randomness to be a negative numeral since the model universe under study is a homogeneous and isotropic one. Therefore, the constituent gas molecules, cannot exhibit random haphazard motion. Therefore, here we shall take negative values for entropy which yields the value of the expression $\{(\gamma-1).e\}$ to be negative, thus, the inequality $\{(\gamma-1).e\} \geq 0$ does not get satisfied i.e., $\{(\gamma-1).e\} \nless 0.$ This subsequently confirms that the inequality constraint (34) gets violated. Additional constraint imposed on the constant K_2 is that $K_2 < 0.$ Under these constrains, the Null Energy Condition (NEC) will be strictly violated. It has been a known fact that the NEC gets violated during an inflationary era. The NEC is a fundamental property of General Relativity. Thus, the condition $\{(\gamma-1).e\} < -1$, refers to the violation of the Null Energy Condition (NEC).

However, additionally the inequality (34) surely satisfies the Null Energy Condition for which the constraints imposed on the constants are $\{(\gamma-1).e\} > -1$ which is a pre − requisite for the occurence of NEC, $K_2 > 0, t \geq 0$.

4.2 Weak Energy Condition:

The weak energy condition demands the following constraint to be satisfied as under:

$$\rho \geq 0, p+\rho \geq 0 \quad \text{...(35)}$$

The first condition $\rho \geq 0$ infers that

$$3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2 . \ t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right] \geq 0$$

Here for $e < 0$ i.e., $e \in \mathcal{R}^-$and eventually for $\{(\gamma-1).e\} < -1$, the expression $[\{(\gamma-1).e\}+1]^2$ yields a term $2.(\gamma-1).e$ which leads to a negative quantity. Thus, the first condition of the inequality (35) gets violated i.e., WEC is obviously gets violated.

4.3 Embedded Mechanisms:

Practically, the violation of WEC in the cosmos transpires under the privilege of a quantifiable numeral of cosmic scenarios while formulating models under the subsistence of dark energy, wormholes and dealing with some specially formulated cosmological models. Such violations particularly get validated when the energy density of the cosmic fluid substantiates only negative values. Other possibility redirects to the cosmic ambiance when the NEC essentially a subset of WEC is not explicitly gets satisfied [61].

4.4 Illustrations Done on the Ground of Vivid Cosmic Scenarios:
Dark Energy's Ascendency and the Accelerating Cosmos:

- **Negative Pressure:**

Dark energy is the hypothetical form of energy, often characterized by negative pressure which has been extremely responsible for the accelerated expansion of the cosmos. This negatively aggrandized pressure might lead to the violation of WEC specifically while considering about satisfying the Strong Energy Condition (SEC).

- **Continual Escalation of Dark Energy:**

The quintessence models of dark energy emphasize the growth of some of the physical parameters like the energy density and pressure that have been expected to eventually violate the WEC at some particular points in space or time. A vibrant explanation that can be mentioned at this point is that the traversable wormholes, best described with the solution of Einstein's field equations, necessitated the persistence of negative energy densities which essentially violates the WEC [62], [63].

- **Cosmological Bouncing:**

Cosmic models formulated on the ground of bouncing cosmologies in which the cosmos undergoes a process of initial expansion and then contraction before experiencing expansion again, could trigger to the violations of WEC at the bounce point due to the fact the matter substances required to support the wormhole becomes negative.

4.5 In the Light of Modified Gravity:

Theories relating to Modified General Relativity such as $f(Q, L_m)$ gravity can also trigger the violation of WEC where the specific parameters and conditions play a vital role.

In essence, WEC violations are often associated with situations where the standard picture of gravity and matter doesn't hold, and exotic phenomena like dark energy, wormholes, and modifications to General Relativity are at play.

Furthermore, regarding the validity of the second inequality in (35) that can be judged from the conditions analyzed in the case of NEC as discussed in the above case. Simply, supplementarily, the second inequality $p + \rho \geq 0$ validates the same set of constrains imposed on the constants as in the above case of NEC.

4.6 Dominant Energy Condition:

The Dominant Energy Condition (DEC) instructs the following energy constraint to be satisfied which is as under:

$$\rho \geq |p| \qquad \text{...(36)}$$

Now $|p| = \left| 3 \left[\frac{\left\{ \frac{\alpha^2 . \left(1+\frac{3\beta}{8\Pi D}\right)^2}{3[\{(\gamma-1).e\}+1]^2} \right\} . t^{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]} + 1 \right\}^2}}{\left\{ t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2 \right\}^2} \right] . (\gamma - 1).e \right|$ giving rise to only positive values after simplification or application of modulus sign..

In the first case, the inequality $\rho > |p|$ undoubtedly gets disrupted, since

$$3 \left[\frac{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]} \right\}^2 . t^{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]} + 1 \right\}^2}}{\left\{ t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2 \right\}^2} \right]$$

$$\ngtr \left| 3 \left[\frac{\left\{ \frac{\alpha^2 . \left(1+\frac{3\beta}{8\Pi D}\right)^2}{3[\{(\gamma-1).e\}+1]^2} \right\} . t^{\left\{ \frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]} + 1 \right\}^2}}{\left\{ t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2 \right\}^2} \right] . (\gamma - 1).e \right|$$

Under specific cosmological scenarios, when the DEC suffers disturbances in the prevailing circumstances in the universe, cosmological theory predicts that the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology might be subject to an abrupt unexpected explanation regarding the future imminent singularity to be experienced by the universe [64], [65], [66].

Supplementarily, the cosmological revelation about the universe predicts such a circumstance in which the non-compliance of the Dominant Energy Condition in cosmological background potentially led to the ensuing plausible circumstances or set-ups in the universe which are deliberated as under:

I. A Negative Ascendency in the Values of the Energy Density:

Corresponding to some frame of references, the energy density may pertain certain negatively précised values of density parameter when the energy traverses in the course of traversal throughout a vacant or empty province of space, acquiring a velocity of dissemination or spreading out that surpasses the velocity of propagation of light waves in vacuum, as applicable [67], [68].

II. Causality Violations:

An interference or desecration in the authentication of the DEC could render to a cosmological spectacle which might be accountable in configuring some interstellar constitutions in space for instances, wormholes, superluminal travel, closed timelike curves in space etc.,

4.7 The Substantiation of Strong Energy Specification or Stipulation in the Universe:

The Strong Energy Condition (SEC) demands the simultaneous adherences of the following two inequalities:

$$\rho + p \geq 0,\ p + 3\rho \geq 0 \quad \text{...(37)}$$

From the earlier described and investigated energy conditions i.e., the Null Energy Condition and Weak Energy Condition in the universe, it has been evidenced that the Strong Energy Condition (SEC) will eventually get disobeyed where the corresponding energy restrains in the universe for specific appraises or quantities, achievable by the inequality condition $\{(\gamma - 1).e\} < -1$.

Cosmological theories have revealed that SEC gets straightaway disobeyed in our universe and at the same instance, it's been additionally violated in some inflationary growing cosmic ambientes even though this is not under the governance of an administrative influence of a scalar field.

4.8 Expecting in Building up the Plinth of the Fast-tracked Enlargement or Escalated Streaming out of the Universe under the Substantialized Violation of SEC:

The conditions under which SEC gets violated in the universe are expected to lead the accelerated streaming out of the universe at an exponential rate.

$$\therefore a(t) \propto t$$

This being the scenario when the expansion of the universe is regulated with the implications based on the dimensionless parameter, the scale factor $a(t)$ which is in general a mathematical function that has been utilized to represent the relative size of the universe at a particular time instance when compared to its size w.r.t. a reference point usually calculated relative to the present day that essentially describes the extent or degree to how much space has been inflated since the Big Bang cosmological phenomena. As the universe enlarges, the scale factor $a(t)$ intensifications with time t that signifies the extent of spreading-out of the observable cosmos.

An additional nomenclature for such measure in cosmology has been referred to as the cosmic scale factor or significantly by the Robertson–Walker scale factor [69] that has been exercised as an essential significant parameter in the equations as formulated by Alexander Friedmann. The fruition of the scale factor is a pulsating interrogative podium in cosmological domain which has been governed by the equations of General Relativity, are designed for a locally isotropic and homogeneous universe as formulated in the light of the renowned Friedmann Equations. In the extreme initial time period of progression of the universe i.e., stating from the Big

Bang cosmology then just after a few seconds, the proclamation of the cosmological inflationary epoch then proceeding with the advancement of time scales and finally subsisting under the surveillance of dark energy's dominance, this sort of cosmological background is conventional to a radiation-dominated epoch. At a subsequent time era, a vital shifting has been experienced by the universe and enters a matter-dominated time epoch. During such progression of time, this cosmological transference has been computational which reads to very closely 4 billion years ago, a consequent dark energy-dominated epoch outset [70].

The calculation of the scale factor scales with time t which is usually

calculated from the very time of evolution or genesis of cosmos and the denotation t_0 is used to refer to the current stage of developmental age of the universe which has

been calculated as 13.799 ± 0.021 Gyr

that flourishes the present value of the scale factor
$a(t)$ $a(t)$

which is generally denoted by
$a(t_0)$ $a(t_0)$

or 1.

5.0 Solutions Engrained on Inflationary Approaches:

5.1 Investigation Depicted by Taking into Account of the Scale Factor $a(t)$:

From Equation (22), the expression for the scale factor $a(t)$ is obtained as

$a\,(t) = \frac{1}{\sqrt{3}} \cdot \frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}} \, t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2$ which indicates the constraints imposed on the constants give rise to the following different conditions which are as under:

(i) For $t \geq 0, \alpha > 0, \beta > 0, K_2 \geq 0,\ D \neq 0$ and $\{(\gamma-1).e\} \neq -1$, the value of the scale factor $a(t) \longrightarrow 0$. Now when the scale factor is zero, it signifies a point in time when the universe is infinitely small and dense which is the theoretical starting point of the Big Bang Cosmology. In fact, at the very initial epoch of the genesis of the universe, the scale factor is considered to be zero representing an infinitely dense point.

(ii) For $\{(\gamma - 1).e\} = -1$, the conditions prevailed on the other constants are same as above, then in that case, the scale factor $a(t) \longrightarrow \infty$. Now based on the current space observations, the scale factor will continue to increase indefinitely, but not exactly reaches an infinite value at any finite time. In the standard cosmological model, the scale factor approaches infinity only as time proceeds to infinity redirecting that the universe is experiencing an incessant spreading out in every permissible direction.

6.0 Investigation Sketched on the Ground of Hubble's Constant:

The expression for the Hubble Constant H as obtained from Equation (25) is as under:

$$H = \frac{\left\{-\frac{\alpha\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}.t^{-\left\{\frac{\alpha\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}}}{t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}}+K_2}$$

which implies that for $t \geq 0, \alpha > 0, \beta > 0, K_2 \geq 0$, $D \neq 0$ and $\{(\gamma - 1).e\} = -1$, the value of the Hubble's Constant becomes zero i.e., $H \longrightarrow 0$ as $\{(\gamma - 1).e\} - 1 = 0$

We shall arrive at

$$H \propto t$$

The above condition entails that $H \longrightarrow \infty$ as the time gradually increases and approaches to infinity i.e., t $\longrightarrow \infty$ which is quite impracticable in realistic scenario. However, the condition regarding Hubble's Constant $H \longrightarrow \infty$ is convincingly not satisfactory or feasible in the arena of Cosmology. Since practically the value of the Hubble Constant does not actually approach to an infinite value and also does not bear a persistent constant value in correspondence to time scale. The Hubble's Constant is a mathematical measuring implement that envisages the magnitude that demonstrates the extent rendering how speedily the universe has been intensifying at a specific time illustration. Since the rate of extension of the universe alters in accordance with time factor, the value assumed by the Hubble Constant does not fundamentally endorse a constant value but varies with time.

6.1 Calculating the Age of the Universe:

The reciprocal of the Hubble's Constant can be implemented in finding the age of the universe where the age of the universe is given by the formula $t = \frac{1}{H_0}$ where H_0 is the Hubble's Constant at the present epoch t. By using this formula, the time can be calculated from the expansion of the universe which is the age of the universe.

7.0 A Universe Substantiating Under the Governance of Distinctive Time Epochs:

7.1 A Radiation-Dominated Epoch of the Universe:

The scale factor evolves in a radiation-dominated universe, for which the scale factor in the Friedmann-Lemaitre-Robertson-Walker metric has been attained on resolving the Friedmann's Equations which entails the functional dependence of $a(t)$ as:

$$a(t) \propto t^{\frac{1}{2}}$$

7.2 Matter-Dominated Epoch of the Universe:

The genesis and growth of the scale factor$a(t)$ in correspondence to a matter-dominated universe in compliance to the Friedmann-Lemaitre-Robertson-Walker metric is straightforwardly achieved as a resolution of the well-acknowledged Friedmann Equations that engross the dependence of $a(t)$ with time t satisfying the relation:

$$a(t) \propto t^{\frac{2}{3}}$$

7.3 Dark-Energy-Dominated Eon of the Universe:

In the case a dark-energy-dominated universe, the evolvement and subsequent development of the scale factor in the Friedmann-Lemaitre-Robertson-Walker metric is lucidly achievable on resolving the well-known Friedmann Equations which redirects the following functional dependence of $a(t)$ and time t as follows:

$$a(t) \propto exp(H_0 t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (38)$$

From Equation (22), we get the scale factor which is

$$a\,(t) = \frac{1}{\sqrt{3}} \cdot \frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}}\, t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2$$

Also,

$$H = \frac{\left\{-\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}.t^{-\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}}}{t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2}$$

$$\Rightarrow H = \frac{\sqrt{3}}{\left\{-\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}.t^{-\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}}}\, a\,(t)\, D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}$$

$$\Rightarrow a(t) \propto H(t)$$

where the exponents raised on the time instance t varies for different values of the adiabatic index γ exhibited by different

gas molecules satisfying the Classical Ideal Gas Laws.

which further can be implemented as $a(t) \propto exp(H_0 t)$ that indicates the pervasiveness of a Dark-Energy-Dominated Era i.e., the universe exhibits an accelerated expansion where H_0 denotes the present value of the Hubble's Constant at time t.

8.0 Investigations Endowed on the Energy-Density ρ:

The energy density ρ gives rise to the expression as under:

$$\rho = 3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2 . \, t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2\right\}^2}\right]$$

From this overhead expression for density parameter ρ, it has been evident that $\rho \propto t$ it is an absolutely clear-cut prediction that as time t increases indefinitely i.e., as $t \longrightarrow \infty$, density $\rho \longrightarrow 0$ This is the condition which suggests an ever-increasing intensification experienced by the universe due to the supremacy of the dark energy redirecting the cosmological scenario when the energy density ρ progressively creep up on to reach the value zero which is a pre-requisite that envisages Big Rip Cosmological occurrences in far-flung time ahead.

9.0 Manifestation of Pressure p on an Intensifying Universe:

We have obtained the expression for the pressure p as

$$p = 3\left[\frac{\left\{\frac{\alpha^2.\left(1+\frac{3\beta}{8\Pi D}\right)^2}{3[\{(\gamma-1).e\}+1]^2}\right\} . \, t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{t^{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2\right\}^2}\right] . \, (\gamma-1).e$$

The stipulations experienced by the constants and variables under such privilege have been discussed under:

(i) As soon as the term $\{(\gamma-1).e\}$ evaluates to -1 i.e., $3[\{(\gamma-1).e\}+1]$ eventually becomes equal to 0 when this term has been compared to the dimensionless parameter ω arising in the equation of state of the barometric fluid, we shall have the physical parameter pressure satisfying $p = 0$. In this scenario, the Friedmann's Equations have to be employed in calculating the pressure term when the shape flourished by the observable universe is flat enough. However, the Physical Cosmology engrained theoretical scenario predicts that the pressure exerted on the universe should not accurately be equal to zero, but the only plausibility arising is that it might be well-thought-out to be absolutely insignificant. In simpler sense, the pressure exactly does not become zero, however it can be considered negligible.

(ii) The equivalence relation $\{(\gamma - 1).e\} = 1$ is an assured preposition for a geometrically flat shaped cosmos and that possesses the critical density. The mathematical formula designed to calculate the critical density can be represented by $\rho_{critical} = \frac{3H^2}{8\pi G}$. The critical density is defined as the least aggregate of matter abundant in the cosmos which has been adequate to stop the spreading out of the cosmos rendering the geometrical shape of the conceivable universe to be flat. The condition $\rho_{critical} \propto H^2$ which truly infers a cosmological backdrop that helps in the ascendency of a hurried or speedy rate of exponential enlargement of the cosmos necessitating a magnificent aggrandizement of density parameter value which is enough to surmount it [71], [72].

The restriction $\{(\gamma - 1).e\} + 1 \ll 1$ predicts such a cosmological scenario when the shape of the cosmos appears as an open universe and characteristically generates an ascendency in a continually escalating enlargement of the cosmos. The density parameter of the universe will suffer deterioration more rapidly in contrast to the value of the critical density. Under this circumstance, pressure consumes negative appraises while the restrictions necessitated on other supplementary parameters stand up on satisfaction of the constraints which allow as $t \geq 0, \alpha > 0, \ \beta > 0, K_2 \geq 0$ and $D \neq 0$

In Cosmology and Astrophysics, the cosmic scenario named negative pressure substantiates a vital characteristic of dark energy in which the pressure exerted by this dark energy is essentially less than zero i.e., consumes merely only negative values, principally functioning like a repulsive force that fundamentally grounds the accelerated expansion of the cosmos. The utmost widespread enlightenment for the prevalence of negative pressure in the universe is the Cosmological Constant which is a constant energy density in the cosmos that penetrates space, consequential in a negative pressure upshot, accompanying with dark energy that performs in the opposite direction or reverse manner, instigating the universe to inflate at an amplifying rate.

9.1 Association of Negative Pressure with Dark Energy:

Negative pressure has been emerged as an effective measure of characterizing the possessions or behaviours of dark energy, as this has been the central segment that consents the dark energy to conquer the gravitational pull exerted by matter component of the universe and drives the hurrying expansion of the universe at a grander extent.

10.0 Inferences Captivated on the Universe Based on the Present Research Work:

The scale factor is a physical measure arising in the subject arena of Cosmology is an uttermost vital module in comprehending the fruition of the universe within the Friedmann-Lemaitre-Robertson-Walker Metric which is very much compassionate in modelling a locally homogeneous and isotropic universe that has been under the surveillance of accelerated expansion of the cosmos.

Significant Estimations Involving the Scale Factor in Cosmology:

I. Function of time:

The scale factor $a(t)$ designates that it gets changed with respect to time t i. e., it acts as a variable in the investigations portrayed on the escalating universe.

II. Dimensionless Quantity:

The scale factor arising in the cosmological analysis is a dimensionless numeral, suggesting that it is free from any nature of units [73], [74].

III. Interpretations Sketched on the Ground of Mathematical Appraisals:

If the numerical value of a scale factor at a prescribed instance of time turns out to be 3(say), it means that the universe at that particular time instance was thrice as small-scaled as it is measured on this current day.

IV. Evaluating Remoteness Between Interstellar Objects:

The scale factor has been utilized in Cosmology to compute the physical detachments between celestial entities in the universe at any time stages.

The enunciation and growth of the scale factor with time scale has been administered by the revolutionary Friedmann Equations that assists the interpretation of some supplementary physical parameters like the density of matter and governance of dark energy in the cosmos [73], [74].

For such cosmological phenomenon on account of which the universe experiences exponential expansion in every possible direction under such scenario, every bit of cosmological detachment assesses with a. In this present study, scale factor $a(t)$ is expressible as $a(t) = \frac{1}{\sqrt{3}} \cdot \frac{1}{D\sqrt{\frac{1}{3[\{(\gamma-1).e\}+1]}}} t^{\frac{-\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{3[\{(\gamma-1).e\}+1]}} + K_2$ as obtained from Equation (22). For a positive time scale i.e., $t > 0$ we can assume the expression attaining a value, $\{(\gamma - 1).e\} = -1$, the scale factor approaches infinity i.e., $a(t) \longrightarrow \infty$, In our current cosmological model, the supremacy of dark energy promotes the fast-tracked intensification of the universe which supplementarily boosts the scale factor on the subject of approaching infinity with time advancements. A grander attainment in the value of the scale factor resources a colossal universe. Thus, as the scale factor grows infinitely enormous, an eventual escalation persists when the universe turns out to be infinitely large and it has been predicted to expand forever.

10.1 Conclusions Furnished in Support of the Configure of Cosmological Backdrop:

In Cosmology and Astrophysics, a scale factor growing to be infinite indicates the attainment of a point geometry of uppermost enlargement, frequently stated as a cosmological Big Rip Singularity podium [73], [74] where the fabric of space-time itself gets distorted and is shredded apart, triggering all matter prevailed in the universe to become infinitely disseminated and an eventual destruction of the universe instigates which has been characteristically accompanying with cosmological set-ups assessing dark energy's surveillance satisfying a negative equation of state, causing an accelerated enlargement of the universe at a continually- intensifying rate. At Big Rip cosmological contexts, even atoms would be ripped apart from one another as a consequence of extreme expansion of the universe under the dominance of dark energy.

The Cosmological Constant Λ has got its expression as

$$\Lambda = \frac{3^{(1-\alpha)}\beta.\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^{-2\alpha}}{\left[\left\{\frac{\alpha\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2.\,t^{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}\right]^{\alpha}}$$

Constrains imposed on the physical parameters are: $t > 0, \alpha, \beta, \gamma, e, K_2 > 0,$ for the effective existence of the Cosmological Constant $\Lambda > 0$ and also from this, it has been evident that Λ scales as t, γ and e i.e., mathematically expressed as $\Lambda \propto t$, as $\Lambda \propto \gamma, \Lambda \propto e$.

The expression for the co-efficient of bulk viscosity is obtained as under for the present research which is

$$\xi = \xi_0\left[3\left[\frac{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}\right\}^2.\,t^{\left\{\frac{\alpha.\left(1+\frac{3\beta}{8\Pi D}\right)}{3[\{(\gamma-1).e\}+1]}+1\right\}^2}}{\left\{\frac{\alpha\left(1+\frac{3\beta}{8\pi D}\right)}{t^{3[\{(\gamma-1).e\}+1]}}+K_2\right\}^2}\right]\right]^n$$

interprets that $\xi \propto \rho^n$, $\xi \propto t^{2n}$

Thus, $n > 0$ or $n \in \mathcal{R}^+, K_2 \geq 0, D_1 \neq 0, \xi \propto (t^2)^n$ are the essential stipulations for the subsistence of bulk viscosity in the universe.

In Physical Cosmology, cosmological inflation is a speculation that illustrates the rapid exponential expansion of space in the very early epoch of the universe which developed just after the Big Bang Cosmology. It furnishes a mechanism for the instigation of density perturbations in the universe [75], [76].

In Cosmology, inflation and Phantom Energy which is a hypothesized form of dark energy in which the dimensionless parameter ω of the equation of state is < -1. These two phenomenon have been connected through the perception of incorporated Phantom Cosmology where a single scalar field become preferably responsible for both the rapid expansion transpired during the early inflammatory period of the universe and the present accelerated enlargement of the universe, essentially pondered by the insistence of dark energy with the significant feature that this field demonstrates phantom behavior, necessitating an ambience where the energy density gets amplified as the universe inflates, directing to a much swifter augmented acceleration rate than in comparison to a standard cosmological constant. With the evolution and developmental stages of the universe, the scalar field transitions to a phantom state starring to an accelerated expansion of the universe. The phantom energy perception elevates theoretical apprehensions concerning to causality and the stability of the universe [77], [78], [79].

Furthermore, such ambience of cosmological scenery is uncompromising in the context, portrayed throughout an inflationary time epoch, the value concerning to a Cosmological Constant intensifies at an exponential rate. On the contrary, the physical parameters of the universe like the energy density suffers a continual exponential deceleration which in due course of time approaches to zero. According to recent cosmological models, the engrained value concerning the Cosmological Constant might perceive a giant appraise or significantly grander assess during an inflationary time era in comparison to its present value.

In Cosmology, while interpreting the perception of bulk viscosity, when the viscosity has been utilized in modelling the dynamics of the universe, predominantly when the cosmic fluid satisfying or filling the model universe is not in the state of maintaining thermal equilibrium during the accelerated expansion of the universe. When a cosmic fluid inflates speedily, it might distort the thermal equilibrium, eventually steering a difference in pressure which functions to reestablish the equilibrium, this pressure difference is has been manifested in the universe as by bulk viscosity [48], [49], [80].

Thus, whenever there substantiates an incessant degrading in bulk viscous aggrandizements while surviving through an inflationary time period which necessarily has addressed the ascendency of cosmological consistencies with the space-time fabric of the universe conforming it to be locally isotropic and homogeneous in every concern which we often reasonably experience during present time instances.

In Cosmology, the Hubble Constant is an appraise of the present rate of expansion of the cosmos since this exponential expansion phenomenon has been accredited to the hypothetical repulsive force or the primary driver of space which is accountable for the accelerated expansion incurred in the cosmos i.e., the dark energy which can be investigated accurately by precisely quantifying the Hubble Constant in a mathematically analyzed manner [81], [82], [83]. In our present work, the Hubble Constant has been predominantly ascribable to the physical properties like time t, the additional constants involved in the expression for Hubble Constant are viz., are viz., α, β and γ, the ratio of specific heats and entropy e, the internal energy per unit mass of the model universe that obeys the Classical Ideal Gas Law. The characteristics flourished by this mathematically aggrandized Hubble Constant helps us in encompassing the dominance of dark energy in a précised cosmological backdrop scenario.

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The author declares no conflicts of interest for this present study.

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