International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 16 (2015) pp 36169-36174

A Survey on - How the diameter of a graph is affected by the removal and the addition of edges

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Abstract - Communication is a critical issue in the design of a parallel and distributed system. The speed of communication of an interconnection network is related to its diameter. The diameter is a measure of efficiency for studying the effects of link failures of a network with maximum time-delay or signal degradation. If some fault occurs on links or nodes of a network, then efficiency of the network will be affected. A good network must be hard to disrupt and the transmissions must remain connected even if some vertices or edges fail. Hence, to increase the efficiency of message transmission we have to decrease the diameter of a graph. Thus, the notion of diameter vulnerability has great applications in networks. A survey is given on the results concerning, how the diameter of different graph classes are affected by the removal and the addition of edges (presence of edge faults).

AMS Classification: 05C38, 05C76.

Keywords: diameter variability, diameter vulnerability, fault diameter

Introduction

An interconnection network connects the processors of a parallel and distributed system. The topological structure of a network can be modelled by a connected graph whose vertices and edges represent the sites and communication links of a network, respectively. Many graph theoretic techniques can be used to study the efficiency and reliability of a network, as discussed in [1], [2] and [3]. The diameter is a measure of efficiency for studying the effects of link failures of networks with maximum time-delay or signal degradation. The reliability of the network is characterized by the connectivity of the network. If some fault occurs on links or nodes of a network, then efficiency of the network will be affected. A good network must be hard to disrupt and should be able to communicate messages even if some faults occurs on the vertices or edges. Hence, to increase the efficiency of message transmission we have to decrease the diameter of a graph [4]. These problems deal with how these networks can communicate with a good efficiency in the presence of faults. A survey is given on the how the diameter of the different graph classes are affected by the removal and the addition of edges in the presence of fault.

Vulnerability is a measure of the ability of the system to withstand vertex or edge faults and maximum routing delay. Diameter can be used to evaluate the time delay or signal degradation in routing. In this context, the following concepts are studied.

Let G = (V,E) be a simple connected graph. The distance between u and v in G, d(u, v) is the length of a shortest path joining them. The diameter of a graph G, D(G) is the maximum distance between any two vertices in G. The diameter of a graph can be affected by the addition or the deletion of some edges. Graham and Harary [5] introduced three measurements about diameter variability. The following notations are used to describe the diameter variability [6] of a graph.

$D^{-k}(G)$: the minimum number of edges to be added to G to decrease the diameter of G by (at least) k, where $k \geq 1$.
$D^{+k}(G)$: the minimum number of edges to be deleted from G to increase the diameter of G by (at least) k, where $k \geq 1$.
$D^{0}(G)$: the maximum number of edges to be deleted from G without an increase in the diameter of G.

Illustration:

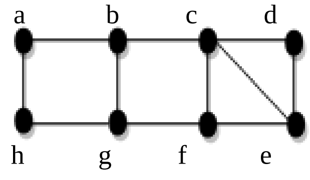

Fig 1: $D^{-1}(G) = 1$, $D^{1}(G) = 1$ and $D^{0}(G) = 3$.

The problem of determining diameter vulnerability of a graph was proposed by Chung and Garey [7]. The concept of fault diameter was introduced by M. S. Krishnamoorthy and B. Krishnamurthy [8]. The notations used are,

$F(G) = \max\{diam(G - S) / S \subseteq V(G), |S| = \kappa(G) - 1\}$
$F'(G) = \max\{diam(G - F) / F \subseteq E(G), |F| = \kappa'(G) - 1\}$.

The problem of diameter vulnerability is proved to be NP-complete by Schoone et.al [9].

Illustration:

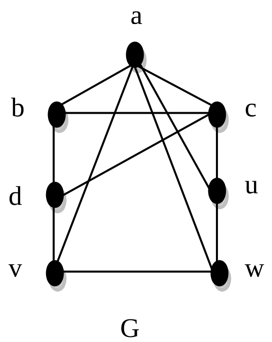

Fig 2: diam(G) = 2, $\kappa(G) = 3$ and f(G) = 4. Also, $\kappa'(G) = 3$ and $f'(G) = 4$.

1. Hypercube

A hypercube of dimension n, denoted by Q_n, is the graph whose vertex set consists of all 0-1 vectors $(v_1, v_2, ..., v_n)$, where two vertices are adjacent if they differ in precisely one coordinate.
Equivalently, $Q_1 = K_2$ and $Q_n = Q_{n-1} \square K_2$ for $n \geq 2$.

A. Changing and unchanging the diameter of a hypercube[5]

Following are the results discussing about the diameter variability of hypercube.

Theorem 1.1. The minimum number of edges to be deleted to increases the diameter of a hypercube is $D^{+1}(Q_n) = n - 1$.

Theorem 1.2. The minimum number of edges to be added to decreases the diameter of Q_n is $D^{-1}(Q_n) = 2$.

Theorem 1.3. An upper bound for $D^0(Q_n)$ is given as
$D^0(Q_n) \leq 2^n + \binom{n}{\lfloor n/2 \rfloor} - 2$.

B. Edge deletion preserving the diameter of the hypercube [10]

This paper discuss about the upper bound and lower bound of $D^0(Q_n)$.

Theorem 1.4.
$(n-2)2^{n-1} - \binom{n}{\lfloor n/2 \rfloor} + 2 \leq D^0(Q_n) \leq (n-2)2^{n-1} - \lceil (2^n - 1)/(2n - 1) \rceil + 1$.

C. Diameter variability of hypercubes [11]

In this paper J.J.Wang et. al. have discussed about the upper bound of $D^{-k}(Q_n)$.

Theorem 1.5.
$D^{-k}(Q_n) \leq 2^k + 2^{k-1}\binom{n-k}{1} + 2^{k-2}\binom{n-(k-1)}{2} + ... + 2\binom{n-2}{k-1}$.

D. Diameter variability of hypercubes [12]

Theorem 1.6. $D^{-k}(Q_n) = 2^{2k-1}$ for $1 \leq k \leq \lfloor n/2 \rfloor$.

2. Diameter variability of cycles and tori [6]

A. Changing the diameter of cycles

Deletion of any edge increases the diameter of a cycle. Hence it follows that
(1) $D^0(C_m) = 0$.

(2) $D^{+k}(C_m) = 1$, for $1 \leq k \leq m - 1 - \lfloor m/2 \rfloor$.

Now, before discussing about $D^{-k}(C_m)$, we consider the case of adding two edges to cycle (C_m). Let $D^*(C_m)$ denote the minimum diameter among the graphs obtained by adding two edges to C_m.

Lemma 2.1. Let $m \geq 5$, Then,

$$D^*(Cm) = \begin{cases} \lfloor m/4 \rfloor + 1 & \text{if } m \equiv 0, 1, 2 \bmod 4 \\ \lfloor m/4 \rfloor + 2 & \text{if } m \equiv 3 \bmod 4 \end{cases}$$

Theorem 2.2. $D^{-k}(C_m) = 2$, for $1 \leq k \leq \lfloor m/2 \rfloor - D^*(C_m)$.

B. Changing the diameter of tori

Theorem 2.3. $D^{-1}(C_m \square C_n) = 2$, for $m \geq 12$.

Theorem 2.4. $D^{-2}(C_m \square C_n) = 2$, for $m \geq 14$ and $m \neq 15$.

Theorem 2.5. $D^{+1}(C_m \square C_n) = \begin{cases} 2 \text{ when m and n are odd} \\ 3 \text{ otherwise} \end{cases}$

Theorem 2.6. $D^0(C_m \square C_n) \geq \begin{cases} mn - 2n + 1 \text{ when m is even} \\ mn - 2n \text{ when m is odd} \end{cases}$

3. Diagonal Mesh [13]

A Topology of mn diagonal mesh is a graph in which the vertex set is defined by $V(C_m) \times V(C_n)$ and edge set is defined by $\{((u_1, u_2)(v_1, v_2)) \mid (u_1, v_1) \in E(C_m)$ and $(u_2, v_2) \in E(C_n)\}$. For two cycles C_m and C_n, $C_m \otimes C_n$ is connected if and only if C_m or C_n is an odd cycle. In this paper, authors assume that at least one of m and n is odd.

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 16 (2015) pp 36169-36174
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Lemma 3.1. Let m, n be odd. Then,

$$\mathrm{Diam}(C_m \otimes C_n) = \begin{cases} m & \text{for } m = n \\ m-1 & \text{for } m+2 \le n \le 2m+1 \\ \lfloor n/2 \rfloor & \text{for } 2m+3 \ge n \end{cases}$$

Lemma 3.2. Let m be even and n be odd. Then,

$$\mathrm{Diam}(C_m \otimes C_n) = \begin{cases} n & \text{for } m/2 \le n \\ m/2 & \text{for } m/2 > n \end{cases}$$

Theorem 3.3. Let m be even and n be odd. Then,

$$D^{+1}(C_m \otimes C_n) = \begin{cases} 1 & \text{if } n = 2m+1 \\ 2 & \text{otherwise} \end{cases}$$

Theorem 3.4. Let m be even and n be odd. Then,

$$D^{+1}(C_m \otimes C_n) = \begin{cases} 1 & \text{if } n = m/2 \\ 2 & \text{otherwise} \end{cases}$$

Theorem 3.5. Let m and n be odd. Then,

$$D^{-1}(C_m \otimes C_n) \le \begin{cases} \lfloor m/2 \rfloor \lceil n/2 \rceil & \text{for } m \le n \le 2m-1 \\ 2\lfloor m/2 \rfloor \lceil n/2 \rceil & \text{for } 2m+1 \le n \end{cases}$$

Theorem 3.6. Let m be even and n be odd. Then,

$$D^{-1}(C_m \otimes C_n) = \begin{cases} (m/2)\lceil n/2 \rceil & \text{for } m/2 < n-1 \\ (m/2)n & \text{for } m/2 \ge n-1 \end{cases}$$

4. Bounded edge-connectivity and edge persistence of Cartesian product of graphs [14]

The edge-persistence $D^+(G)$ of a graph G is the minimum number of edges whose deletion from G increases the diameter of G.

Theorem 4.1. $D^+(G_1 \square G_2) \ge D^+(G_1) + D^+(G_2)$ if $\mathrm{diam}(G_1) \ge 2$ and $\mathrm{diam}(G_2) \ge 2$.

Theorem 4.2. For any $n \ge 3$ and $m \ge 2$,

$$D^+(C_n \square P_m) = \begin{cases} 1 & \text{for } n = 3 \\ 2 & \text{for } n \ge 4 \end{cases}$$

Theorem 4.3. For $n \ge 3$ and $m \ge 3$

$$D^+(C_n \square C_m) = \begin{cases} 2 & \text{if } n = 3 \text{ or } m = 3 \text{ or both n and m are odd} \\ 3 & \text{otherwise} \end{cases}$$

Theorem 4.4. $D^+(Q_n \square P_m) = n$ for $n, m \ge 2$.

Theorem 4.5. For $n \ge 1$ and $m \ge 3$

$$D^+(Q_n \square C_m) = \begin{cases} n & \text{for } m = 3 \\ n+1 & \text{for } m \ge 4 \end{cases}$$

5. Edge, vertex and mixed fault diameters [15]

Let G be a k-edge connected graph and $0 \le a < k$. The a-edge fault diameter of G is $D^E_a(G) = \max\{d(G/X) \mid X \subseteq E(G), |X| = a\}$.
Let G be a k-edge connected graph and $0 \le a < k$. The a-vertex fault diameter (or a-vertex fault diameter) of G is $D^V_a(G) = \max\{d(G/X) \mid X \subseteq V(G), |X| = a\}$.

In this paper Iztok Banic et. al. have obtained a relation between $D^E_a(G)$ and $D^V_a(G)$.

Theorem 5.1. Let G be a k-connected graph and $0 < a < k \le \kappa(G)$. Then,

$$D^E_a(G) \le D^V_a(G) + 1.$$

6. Vulnerability in graph of diameter four [16]

The line persistence of a graph G, denoted by $\rho_1(G)$, is the minimum number of edges, whose deletion increases the diameter. In this paper Geoffrey et.al. have characterized graphs of diameter four with $\rho_1(G) = 2$.

Theorem 6.1. Let G be a graph of diameter 4. Then $\rho_1(G) = 2$ if and only if for all pairs of points {u, v} of G, either there exists two line disjoint short u − v paths in G or else G contains an induced subgraph H' isomorphic to H containing u and v such that d(u, v, H') = 3.

7. The fault-diameter of Cartesian products [17]

In this paper, they have proved that, if $G_1, G_2, \ldots, G_q$ are k_1-connected, k_2-connected, . . . , k_q -connected graphs and $0 \le a1 < k1, 0 \le a2 < k2, \ldots, 0 \le a_q < k_q$ and $a = a_1 + a_2 + \ldots + a_q + (q-1)$, then the fault diameter of G, the Cartesian product of $G_1, G_2, \ldots, G_q$, with a faulty nodes satisfies the inequality $D_a(G) \le D_{a1}(G1) + D_{a2}(G2) + \ldots + D_{aq}(G_q) + 1$.

8. The edge fault-diameter of Cartesian graph bundles [18]

Let B and F be graphs. A graph G is a Cartesian graph bundle with fibre F over the base graph B if there is a graph map $p : G \to B$ such that for each vertex $v \in V(B)$, $p^{-1}(\{v\})$ is isomorphic to F , and for each edge $e = uv \in E(B)$, $p^{-1}(\{e\})$ is isomorphic to $F \square K_2$.
Let G be a k-edge connected graph and $0 < a < k$. Then we define the a-edge fault diameter of G as $\overline{D}_a(G) = \max\{d(G \setminus X) \mid X \subseteq E(G), |X| = a\}$. Following is the main result in this paper.

Theorem 8.1. Let F and B be k_F-edge connected and k_B-edge connected graphs respectively, $0 \le a < k_F$, $0 \le b < k_B$, and G a Cartesian bundle with fibre F over the base graph B. Then $\overline{D}_{a+b+1}(G) = \overline{D}_a(F) + \overline{D}_b(B) + 1$.

9. Diameter vulnerability of GC graphs [19]

(Δ;D;D';s) -problem is finding large graphs with maximum degree Δ and diameter D such that the subgraphs obtained by deleting any set of s vertices have diameter at most D'. In this work J. Gomez et. al. have studied this problem for s = 1 on some families of generalized compound graphs.

Theorem 9.1. Let w be a vertex belonging to a GC graph G of diameter D. Then, the diameter of $G - w$ is at most $D + 1$.

10. Diameter vulnerability of graphs

The problem of determining diameter vulnerability of a graph was proposed by Chung and Garey [7]. The concept of fault diameter was introduced by M.S.Krishnamoorthy and B. Krishnamurthy [8].

Let f(t, k) denote the maximum possible diameter obtained by deleting t edges from a (t + 1) -edge connected graph of diameter k.

A. Diameter vulnerability of graphs [20]

Peyrat show that $3\sqrt{2t} - 3 \le f(t, 3) \le 3\sqrt{2t} + 4$.

B. Diameter vulnerability of graphs by edge deletion [21]

H. X. Ye et al. improves the result of Peyrat and gave a bound as $4\sqrt{2t} - 6 < f(t, 3) \le \max\{59, 5\sqrt{2t} + 7\}$ for any $t \ge 4$.

This notion is also discussed in [22].

11. Diameter vulnerability of large bipartite digraph [23]

The s-diameter vulnerability, K(s; G), of a digraph G is the maximum of the diameters of the digraphs formed by removing s arbitrary vertices from G. The digraph BD(d, n) is d-regular and bipartite with partite sets $V_o = 0 \times Z_n$, and $V_1 = 1 \times Z_n$.
BD(d, n) has diameter D. $BD(d, d^{D-1} + d^{D-3})$ are the largest known bipartite digraphs with diameter D and maximum out-degree d. The main results are

Theorem 11.1. Between any pair of non-adjacent vertices of $BD(d, d^{D-1} + d^{D-3})$ there exist d disjoint paths of length at most D + 2. One of them has length equal to the distance between the vertices and at most 2 of them have length equal to D + 2.

Corollary 11.2. Let G be the bipartite digraph $BD(d, d^{D-1} + d^{D-3})$. Then the s-diameter vulnerability of G is:
$K(s; G) = l_b(s, d, 2(d^{D-1} + d^{D-3})) = D + 1$ if $d \ge 3$ and $1 \le s \le d - 3$.
$K(s; G) \le l_b(s, d, 2(d^{D-1} + d^{D-3})) + 1 = D + 2$ if $d \ge 3$ and $s = d - 2$, $d - 1$ or $d = 2$ and $s = 1$(where l_b stands for lower bound).

12. On the diameter vulnerability of Kautz digraph [24]

Let G be a graph (or digraph) with vertex set V.
The s-diameter vulnerability D(s;G) of the graph G is the maximum of the diameters of the graphs obtained by removing an arbitrary set of s vertices from G. Let V be the set of vectors with t components each in [0, d] so that any two adjacent components are distinct.The Kautz digraph K(d, t) is the digraph with vertex set V are set consisting of all arcs from the vertex $(x_1, x_2,..., x_t)$ to d other vertices $(x_2, x_3, ..., x_t, \alpha)$ where α is in [0,d] and $\alpha \ne x_t$. Number of vertices in Kautz digraph K(d, t) is $d^t + d^{t-1}$,where d is the degree and t is diameter. Following are the main results

Theorem 12.1. Let x and y be two distinct vertices in the Kautz digraph K(d, t) such that there is an arc from x to y. There exist $d - 1$ vertex - disjoint paths from x to y, not passing through arc (x, y), $d - 2$ of length at most t + 1 and one of length at most t + 2.

Theorem 12.2. Let x and y be any two vertices in K(d, t). There exist d vertex - disjoint paths from x to y, one of length at most t, $d - 2$ of length at most t + 1, and one of length at most t + 2.

Theorem 12.3. For $d \ge 2$, we have

$$D(s; K(d,t)) = \begin{cases} t & \text{for } s = 0 \\ 1 & \text{for } 1 \le s \le d-1 \text{ and } t = 1 \\ t+1 & \text{for } 1 \le s \le d-2 \text{ and } t > 1 \\ t+1 & \text{for } s = d-1 \text{ and } t = 2 \\ t+2 & \text{for } s = d-1 \text{ and } t > 3 \\ \infty & \text{for } s \ge d \end{cases}$$

13. Diameter vulnerability of iterated line digraphs [25]

If G is d-regular with d > 1, has diameter D and order n, then the iterated line graph L^kG is d-regular, has diameter D + k and order d^kn. Let G be a digraph with minimum degree $\delta > 2$ and diameter D. Let π be an integer, $0 \le \pi \le \delta - 2$ ($\pi > 1$ if G has loops). Then, $\ell_\pi = \ell_\pi(G)$, $1 \le \ell \le D$, is defined as the greatest integer such that for any two (not necessarily different) vertices x, y,

1. If $d(x, y) < \ell_\pi$, the shortest path from x to y is unique and there are at most π different paths from x to y of length d(x, y) + 1.
2. If $d(x, y) = \ell_\pi$, there is only one shortest path from x to y.

A. Diameter vulnerability of iterated line digraphs without loops

Theorem 13.1. Let G be a loop less digraph with minimum degree $\delta > 1$, diameter D = D(G) and $\ell = \ell(G)$. Then, if $k \ge D - 2\ell + 1$, the vertex-diameter vulnerability of the iterated line digraph L^kG is $K(s, L^kG) \le D(L^kG) + 2m$ for all $s = 1, 2, ..., \delta - 1$, where $m = \max\{\lceil (D+1)/2 \rceil, D - \ell\}$.

Theorem 13.2. Let G be a loopless digraph with minimum degree $\delta > 1$, diameter D = D(G) and $\ell = \ell(G)$. Then, if $k \geq D - 2\ell$, the arc-diameter vulnerability of the iterated line digraph L^kG is $\Lambda(s, L^kG) \leq D(L^kG) + 2m$ for all s = 1, 2,..., $\delta - 1$, where $m = \max\{\lceil (D+1)/2 \rceil, D - \ell\}$.

B. Diameter vulnerability of iterated line digraphs with loops

The set of vertices which are adjacent from (to) a given vertex v is denoted by $\tau^+(v)$ ($\tau^-(v)$). Let G be a digraph, then $\ell^*_1 = \ell^*_1(G)$, as the greatest integer such that for any vertex x, there exist two unique vertices $x^+ \in \tau^+(v)$ and $x^- \in \tau^-(v)$ such that for any (not necessarily different) vertex y,

1. if $d(x, y) < \ell^*_1$, then the shortest path from x to y is unique and, if there exists a path from x to y of length $d(x, y) + 1$, it is unique and its first and last arcs are, respectively, (x, x^+) and (y^-, y).
2. if $d(x, y) = \ell^*_1$, there is only one shortest path from x to y.

Theorem 13.3. Let G be a digraph with minimum degree $\delta > 1$, diameter $D = D(G)$ and $\ell^*_1 = \ell^*_1(G)$. Then, if $k \geq D - 2\ell^*_1 + 1$, the vertex-diameter vulnerability of the iterated line digraph L^kG is $K(s, L^kG) \leq D(L^kG) + 2m$ for all s = 1, 2,..., $\delta - 1$, where $m = \max\{\lceil (D+1)/2 \rceil, D - \ell^*_1\}$.

Theorem 13.4. Let G be a digraph with minimum degree $\delta > 1$, diameter $D = D(G)$ and $\ell^*_1 = \ell^*_1(G)$. Then, if $k \geq D - 2\ell^*_1$, the arc-diameter vulnerability of the iterated line digraph L^kG is $\Lambda(s, L^kG) \leq D(L^kG) + 2m$ for all s = 1, 2,..., $\delta - 1$, where $m = \max\{\lceil (D+1)/2 \rceil, D - \ell^*_1\}$.

14. Graph Products [26]

The **Cartesian product** of two graphs G and H, denoted by $G \square H$, is the graph with vertex set $V(G) \times V(H)$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent if either $u_1 = u_2$ and $v_1 - v_2 \in E(H)$ or $u_1 - u_2 \in E(G)$ and $v_1 = v_2$. For any two connected graphs G and H, $diam(G \square H) = diam(G) + diam(H)$.

The **Strong product** of two graphs G and H denoted by $G \boxtimes H$, is the graph with vertex set $V(G) \times V(H)$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent if either $u_1 = u_2$ and $v_1 - v_2 \in E(H)$ or $u_1 - u_2 \in E(G)$ and $v_1 = v_2$ or $u_1 - u_2 \in E(G)$ and $v_1 - v_2 \in E(H)$. For any connected graph G and H, $diam(G \boxtimes H) = \max\{diam(G), diam(H)\}$.

The **Lexicographic product** of two graphs G and H, denoted by $G \circ H$, is a graph with vertex set $V(G) \times V(H)$ and any two vertices (u_1, v_1), (u_2, v_2) are adjacent in $G \circ H$ if either $u_1 - u_2 \in E(G)$ or $u_1 = u_2$ and $v_1 - v_2 \in E(H)$. If G is not complete, then $diam(G \circ H) = diam(G)$ and $diam(K_n \circ G) = 2$.

It is interesting to see that even if the factors G and H of a product graph have a property $'P'$ then it is not necessary that the product $G * H$ also has that property, where $*$ denotes any of the graph products mentioned above.

Let $G \cong G_1 * G_2$, where '$*$' denotes any of the graph products mentioned above.In [27], [28], the authors have characterized graphs with

- $D^{+1}(G_1 * G_2) = 1$
- $D^{-1}(G_1 * G_2) = 1$
- Obtain a lower bound for $D^0(G_1 * G_2)$. Also, characterize the graphs for which strict equality of the bound holds.

References

[1]J. Gross, J. Yellen, Graph Theory and Applications, CRC Press, (2004).

[2]L. H. Hsu, C. K. Lin, Graph theory and interconnection networks, CRC Press, (2009).

[3]J.M. Xu, Topological Structure and Analysis of Interconnection Networks, Kluwer Academic Publishers, Dordrecht, 2001.

[4]L. Caccetta, Vulnerability of communication networks, Networks 14 (1984), pp. 141-146.

[5]N. Graham, F. Harary, Changing and unchanging the diameter of a hypercube, Discrete Appl. Math. 37/38 (1992), pp. 265 - 274.

[6]J. J. Wang, T. Y. Ho, D. Ferrero, T. Y. Sung, Diameter variability of cycles and tori, Inform. Sci. 178 (2008), pp. 2960-2967.

[7]F.R.K. Chung, M.R. Garey, Diameter bounds for altered graphs, J. Graph Theory 8 (4) (1984), pp. 511-534.

[8]M.S. Krishnamoorthy, B. Krishnamurthy, Fault diameter of interconnection networks, Comput. Math. Appl. 13 (1987), pp. 577-582.

[9]A.A. Schoone, H.L. Bodlaeder, J. Van Leeuwen, Diameter increase caused by edge deletion, J. Graph Theory 11 (3) (1987), pp. 409-427.

[10]A. Bouabdallah, C. Delormea, S. Djelloul , Edge deletion preserving the diameter of the hypercube, Discrete Appl. Math. 63 (1995), pp. 91 - 95.

[11]J.J. Wang, T.Y. Ho, T.Y. Sung, Diameter variability of hypercubes, Proceedings of the 26th Workshop on Combinatorial Mathematics and Computation Theory, Taiwan, 2009.

[12]M.Y. Ju, J. J. Wang, S. H. Chang, Diameter variability of hypercubes, Theoret. Comput. Sci. 542(2014), pp. 63-70.

[13]J.J. Wang, T.Y. Ho, T.Y. Sung, M.Y. Ju, Changing the diameter in a diagonal mesh network, J. Inform. Sci. Engineering 29 (2013), pp. 193-208.

[14]Y. Lu, J.M. Xu, X.M. Hou, Bounded edge-connectivity and edge-persistence of Cartesian product of graphs, Discrete Appl. Math. 157 (2009), pp. 3249 - 3257.

[15]I. Banic, R. Ervesc, J. Zerovnik, Edge, vertex and mixed fault diameters, Advances in Appl. Math. 43 (2009), pp. 231-238.

[16]Geoffrey E.,F. Harary, Cheng D.X., Vulnerability in graphs of diameter four, Mathl. Comput. Modelling 17 (1993), pp. 65-68.

[17]I. Banic, J. Zerovnik, The fault-diameter of Cartesian products, Advances in Appl. Math. 40 (2008), pp. 98-106.

[18]I. Banic, R. Ervesc, J. Zerovnik, The edge fault-diameter of Cartesian graph bundles, European J. Combin. 30 (2009), pp. 1054-1061.

[19]J. Gomez, I. Pelayo, C. Balbuena, Diameter vulnerability of GC graphs, Discrete Appl. Math. 130 (2003), pp. 395 - 416.

[20]C. Peyrat, Diameter vulnerability of graphs, Discrete Appl. Math. 9 (1984), pp. 245-250.

[21]H.X. Ye, C. Yang, J.M. Xu, Diameter vulnerability of graphs by edge deletion, Discrete Math. 309 (4) (2009), pp. 1001-1006.

[22]A.A. Najim, J.M. Xu, On addition and deletion of edges of graphs, J. Univ. Sci.Technol. China 36 (2006), pp. 951 - 955.

[23]C. Padro, P. Morillo, E. Llobet, Diameter-vulnerability of large bipartite digraphs, Discrete Appl. Math. 64 (1996), pp. 239-248.

[24]D.Z. Du, D.F. Hsu, Y.D. Lyuu, On the diameter vulnerability of Kautz digraphs, Discrete Math. 151 (1996), pp. 81-85.

[25]C. Padro, P. Morillo, Diameter vulnerability of iterated line digraphs, Discrete Math.149 (1996), pp. 189-204.

[26]W. Imrich, S. Klavzar, R. Hammack, Handbook of Product Graphs, CRC Press, (2011).

[27]Chithra M.R, A. Vijayakumar, The diameter variability of the Cartesian product of graphs, Discrete Math. Algorithms Appl. Vol. 6, No. 1 (2014), pp. 1450001 - 1450009.

[28]Chithra M.R, Studies on some topics in product graphs, Ph.D thesis, 2014.