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Some New Classes of Even Even and Odd Even Graceful graphs.

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Abstract
In this paper we find the following new classes of Even-even and odd-even graceful graphs:

i) A Bistar $B_{r,r}$ is **even even graceful**.
ii) Every comb graph is **even even** graceful.
iii) The coconut tree **CT (2m, 3**) is **even even** graceful**.**
iv) $S_m + C_3$ is **odd even** graceful.
v) $C_3 \odot 2K_1 + S_m$ is **odd even** graceful.
vi) $C_n + K_{1,r}$, $r = 1, 2, ---, n-1$ is **odd even** graceful.

Introduction

Most graph labelling methods trace their origin to one introduced by Rosa [1] in 1967. Rosa [1] called a function f a β-valuation of a graph G with q edges if f is an injection from the vertices of G to the set *{0, 1... q}* such that, when each edge *xy* is assigned the label $|f(x) - f(y)|$,the resulting edge labels are distinct. Golomb subsequently called such labelling graceful and this is now the popular term. Here we have considered the type of graceful labelling as Even-even and Odd-even graceful.

Definition1. 1: A graph is even-even graceful if there exists an injective map $f: E(G) \to \{2, 4 \ldots 2q\}$ so that the induced map $f: V(G) \to \{0, 2, \ldots, (2k-2)\}$ defined by $f^*(x) \equiv \Sigma f(x, y) \pmod{2k}$ where $k = \max\{p, q\}$ makes all distinct and even.

Definition 1. 2: The odd-even graceful labeling of a graph G with q edges is an injection $f: V(G) \to \{1, 3, 5, \ldots, 2q + 1\}$ such that, when each edge *uv* is assigned the label $|f(u)-f(v)|$, the resulting edge labels are $\{2, 4, 6, \ldots, 2q\}$. A graph which admits an odd-even graceful labelling is called an odd-even graceful graph.

Definition1. 3: A Bistar is a tree with two internal nodes and ***n*** leaves.

Notation:

$\boldsymbol{B_{r,r}}$: A *bistar* containing **2*n*** vertices whose two internal vertices are separately joined by *r* vertices each.

$\boldsymbol{P_n \odot K_1}$: A *comb* which is obtained by joining ***n*** pendant edges to n vertices of a path $\boldsymbol{P_n}$.

CT (2m, 3): A particular class of coconut tree obtained by joining 2m pendant vertices at one end vertex of the path $\boldsymbol{P_3}$.

$S_m + C_3$ **:** The star S_m whose central vertex is joined to one vertex of C_3 .

$C_n + K_{1,r}$: $\boldsymbol{C_n}$ be a cycle of length *n* whose one end vertex coincides with the central vertex of $K_{1,r}$ and the *r* vertices of $\boldsymbol{K_{1,r}}$ coincide with *n-1* vertices of $\boldsymbol{C_n}$ lying on the opposite edge of the central vertex.

Results

Theorem2. 1: A bistar $B_{r,r}$ containing 2n vertices is **even even** graceful where $2n = 2r + 2$.

Proof : Let $B_{r,r}$ be a bistar containing 2n vertices where $2n = 2r + 2$. Let $\{e_1, e_2, e_3, ---, e_n\}$ be the edge set of $B_{r,r}$. Let q = total number of edges of $B_{r,r}$ = 2n-1 = 2r +1. The internal vertices of $B_{r,r}$ have vertex labelling as 0, 2q. Now the vertex labelling of the r vertices joined to the internal vertex with labelled 0 such that $f(u_i) + f(u_{r-i+1}) = 4r + 2 = 2q$, $i = 1, 2, ---, --, \frac{r}{2}$. Similarly the other end of the internal vertex labelled with 2q join to the r vertices such that $f(v_j) + f(v_{r-j+1}) = 4r + 2 = 2q$, $j = 1, 2, ------, \frac{r}{2}$. Now the vertex labelling of the two stars joined to two internal vertices of the Bistar is given by

$$f(u_i) = \begin{cases} 4i - 2 \ , & i = 1, 2, ------, \frac{r}{2} \\ 4i \ , i = \frac{r}{2} + m & , m = 1, 2, ---, \frac{r}{2} \end{cases}$$

$$f(v_j) = \begin{cases} 4j \ , & j = 1, 2, ------, \frac{r}{2} \\ 4j - 2 \ , j = \frac{r}{2} + k & , k = 1, 2, ---, \frac{r}{2} \end{cases}$$

Now considering the edges join to the internal vertex with 0 labelled, their edge labelling is given as *2, 6, 10,-------------, 2r-2, 2r+4, 2r+8,-------------, 4r = 2q-2*. Again the labelling of the edges joined to the other internal vertex labelled with 2q are given as *2q-4, 2q-8, 2q-12, --- -, 2q-2r , --------, 8, 4*. So the edge labelling of the **Bistar** consists of the labelled set { *2, 4, 6, 8, 10,--------------, 2q-4, 2q-2, 2q }*. Hence the **Bistar** defined above is **even even graceful.**

Illustration 1:

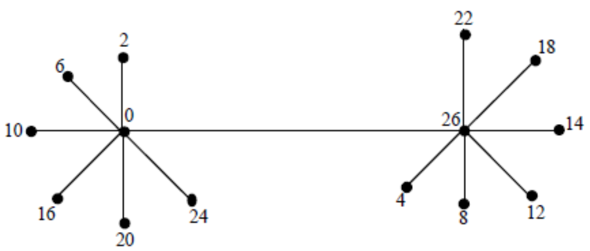

Figure 1: $B_{6,6}$ is **even even graceful**.

Theorem 2. 2: Every comb graph is even even graceful.

Proof: Let $P_n \odot K_1$ be a comb graph with *2n* vertices and *2n-1* edges as shown in the following figure.

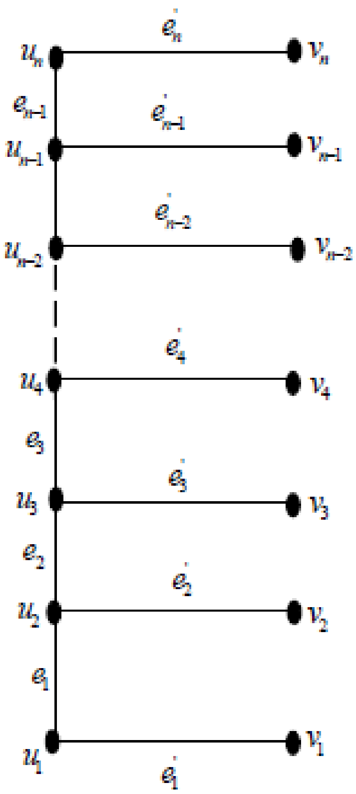

Figure 2: A Comb containing 2n vertices.

The vertex labelling of the comb $P_n \odot K_1$ is defined as follows

$$f(u_i)=\begin{cases}2i-2\ , & i \text{ is odd}\\ 4n-2i\ , & i \text{ is even}\end{cases}$$

$$f(v_j)=\begin{cases}2j-2\ , & j \text{ is even}\\ 4n-2j\ , & j \text{ is odd}\end{cases}$$

Now the edge labelling for the comb $P_n \odot K_1$ is given by

$$e_i = 4n-4i\ ,\ i=1, 2, ------, n-1$$

$$e_i^{'} = 4n-4i+2\ ,\ i=1, 2, -----, n$$

$$i=1\ ,\ e_1 = 4n-4 = 2(2n-1)-2 = 2q-2$$

$$i=2\ ,\ e_2 = 4n-8 = 2(2n-1)-6 = 2q-6$$

$$i=3\ ,\ e_3 = 4n-12 = 2(2n-1)-10 = 2q-10$$

$$i=n-1,\ e_{n-1} = 4n-4(n-1) = 4$$

$$j=1\ ,\quad e_1^{'} = 4n-2 = 2(2n-1) = 2q$$

$$j=2\ ,\quad e_2^{'} = 4n-6 = 2(2n-1)-4 = 2q-4$$

$$j=3\ ,\quad e_3^{'} = 4n-10 = 2(2n-1)-8 = 2q-8$$

$$j=n\ ,\qquad e_n^{'} = 4n-4n+2 = 2$$

So the edge labelling of the comb consists of the *set {2, 4, 6, 8, ----------, 2q-10, 2q-8, 2q-6, 2q-4, 2q-2, 2q}* with vertices labelled with *0, 2, 4, 6, ----------, 2q*. Hence the Comb $P_n \odot K_1$ is even even graceful.

Illustration *2:*

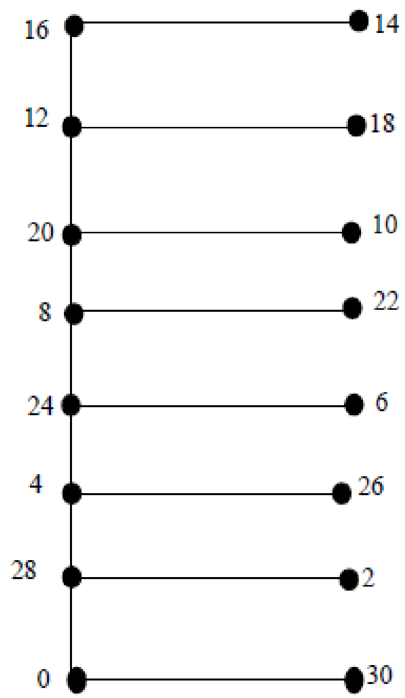

Figure 3: $P_8 \odot K_1$ is even even graceful.

Theorem 2. 3: The coconut tree **CT (2m, 3)** is even even graceful**.**

Proof: Let **CT (2m, 3**) be a particular class of coconut tree obtained by joining 2m pendant vertices at one end vertex of the path P_3.

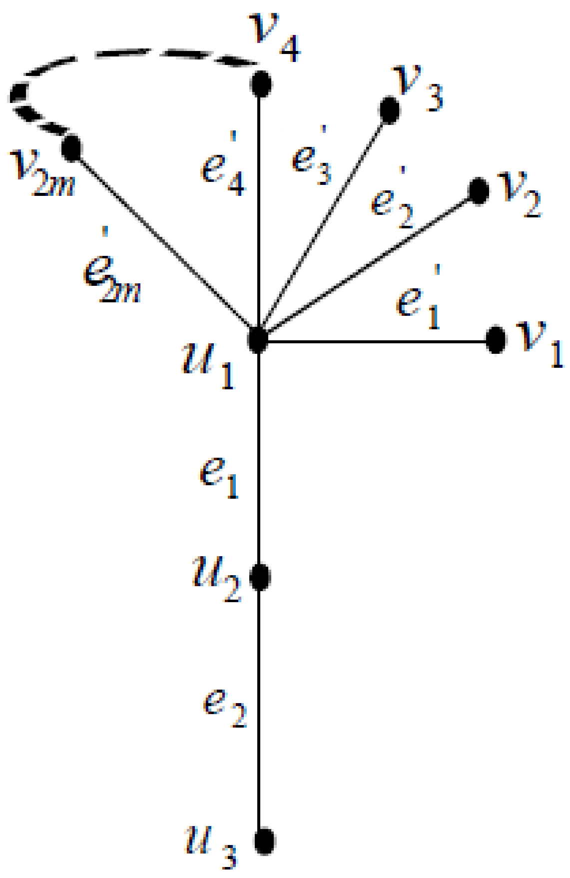

Figure 4: A generalized CT (2m, 3).

The vertex labelling of **CT (2m, 3**) is given by

$$f(u_i) = \begin{cases} 0 \ , & i = 1 \\ 4m+4 \ , & i = 2 \\ 2m+2 \ , & i = 3 \end{cases}$$

$$f(v_j) = \begin{cases} 4m-2j+4 \ , & j = 1, 2, ---, m \\ 4m-2j+2 \ , & j = m+k \ , \ k = 1, 2, ---, m \end{cases}$$

The edge labelling of **CT (2m, 3**) is given by

$$e_1 = 4m+4 = 2(2m+2) = 2q$$

$$e_2 = 2m+2$$

$$e'_j = \begin{cases} 4m-2j+4 \ , & j = 1, 2, -----------, m \\ 4m-2j+2 \ , & j = m+k \ , \ k = 1, 2, ---, m \end{cases}$$

$$j = 1 \ , \ e'_1 = 4m+2 = 2q-2$$

$$j = 2 \ , \ e'_2 = 4m = 2q-4$$

For

$$j = m \ , \ e'_m = 4m+4-2m = 2m+4$$

$$j = m+1 \ , \ e'_{m+1} = 4m-2(m+1)+2 = 2m$$

$$j = m+2 \ , \ e'_{m+2} = 4m-2(m+2)+2 = 2m-2$$

$$j = 2m \ , \ e'_{2m} = 4m-2(m+m)+2 = 2$$

So the edge labelling of **CT (2m, 3**) consists of distinct even integers ranging from *2, 4, 6,------------, 2q-4, 2q-2, 2q*. Hence **CT (2m, 3**) is even even graceful.

Illustration 3:

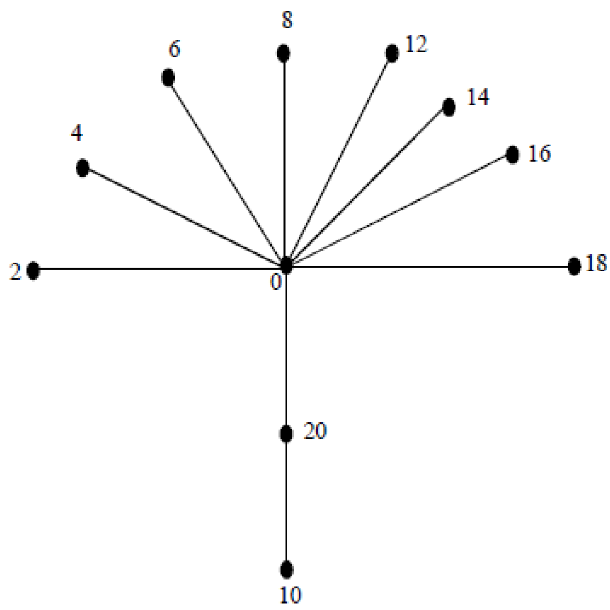

Figure 5: CT (8, 3) is even even graceful.

Theorem-2. 4: $S_m + C_3$ is odd even graceful.

Proof: Let S_m be the star containing m vertices $u_1, u_2, u_3, -----u_m$ whose central vertex is joined to one end vertex of C_3. The following figure represents $S_m + C_3$

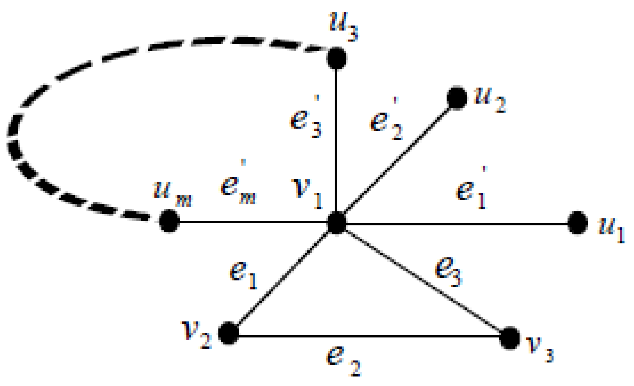

Figure 6: $S_m + C_3$

The vertex labelling of $S_m + C_3$ are given by
$f(u_i) = 2i + 7$, $i = 1, 2, ----, m$,

$$f(v_j) = \begin{cases} 1, & j = 1 \\ 5, & j = 2 \\ 7, & j = 3 \end{cases}$$

Now the edge labelling of $S_m + C_3$ **is** given by

$e_1 = 6$ $e_i' = 2i + 6$, $k = 1, 2, ----, m$

$e_2 = 2$ $e_1' = 8$,

$e_3 = 4$ $e_2' = 10$

$e_3' = 12$

$----$

$----$

$e_m' = 2m + 6$

Now the edges of $S_m + C_3$ are labelled with even integers ranging from *2, 4, 6, 8, 10,* ---------------up to *2q* while the vertices are labelled with odd integers *from 1, 3, 5, 7,*-------------up to*2q+1*. Hence $S_m + C_3$ is **odd even** graceful**.**

Illustration 4

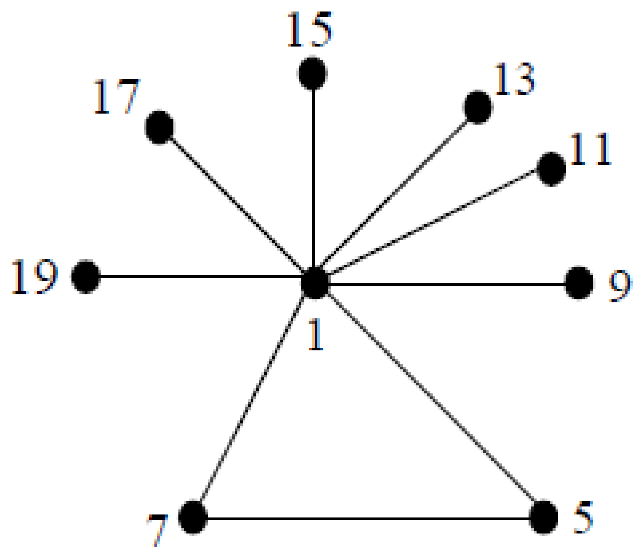

Figure 7: $S_6 + C_3$ is **odd even graceful.**

Theorem 2. 5: $C_3 \odot 2K_1 + S_m$ is **odd even** graceful.

Proof: Let $u_i, i = 1, 2, .3$ be the internal vertices and v_j , $j = 1, 2, 3, ----$ be the external vertices of $C_3 \odot 2K_1 + S_m$.

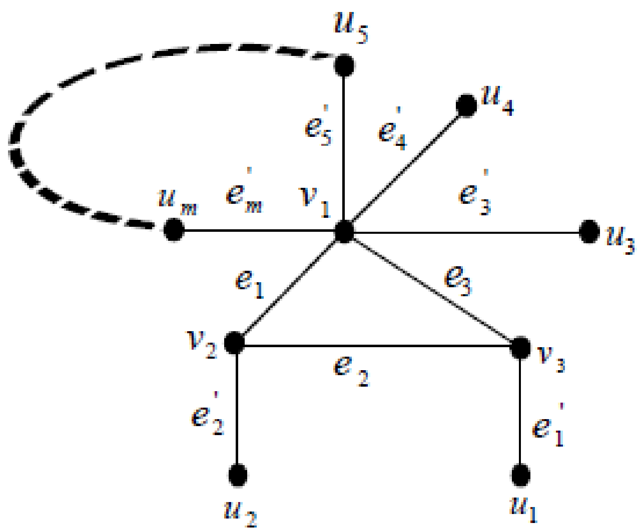

Figure 8: $C_3 \odot 2K_1 + S_m$.

The vertex labelling of the above graph is given as

$$f(u_i)=\begin{cases}1, & i=1\\ 3, & i=2\\ 11, & i=3\end{cases}$$

$$f(v_j)=\begin{cases}7\ , & j=1\\ 9\ , & j=2\\ 2q-2j+1, & j=3,4,5,-----\end{cases}$$

The edge labelling of the above graph is given by

$e_1=2\quad e_j^{'}=2q-2j\ ,\ j=3,4,5,----\ ,\ r$

$e_2=8\quad e_1^{'}=4\quad ,\ e_3^{'}=12\ ,\ -----,-------$

$e_3=10\ e_2^{'}=6\quad ,\ e_4^{'}=14\ ,\ e_{r-1}^{'}=2q-2\ ,\ e_r^{'}=2q$

Now since all the edges are labelled with even integers ranges from *2, 4, 6, ----------, 2q-2, 2q* whereas the vertices are labelled with odd integers ranging from *1, 3, 5,------, 2q + 1*. Hence the graph defined above is odd even graceful.

Illustration 5

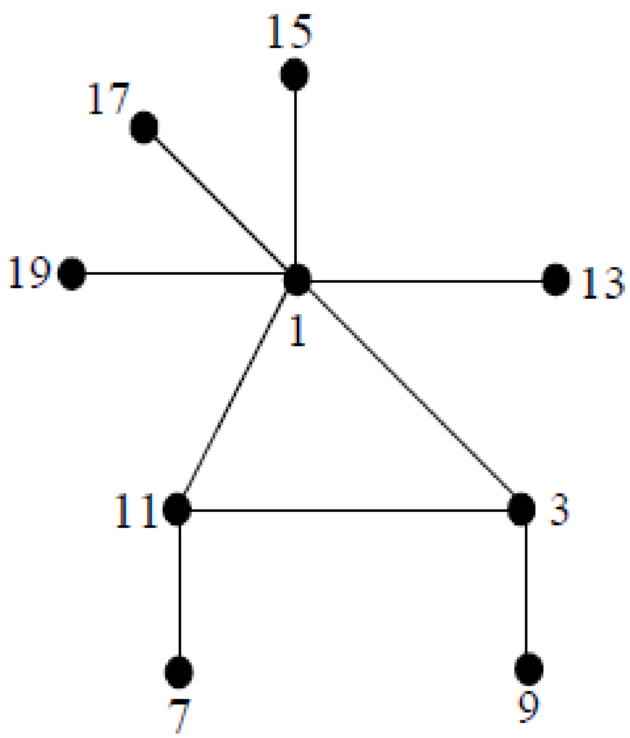

Figure 9: $C_3 \odot 2K_1 + S_4$ **is odd even graceful.**

Theorem 2. 6: $C_n + K_{1,\,r}\ ,\ r=1,2,---,\ n-1$ is odd even graceful.

Proof: Let C_n be a cycle of length n whose one end vertex coincides with the central vertex of $K_{1,r}$ and the *r* vertices of $K_{1,r}$ coincide with *n-1* vertices of C_n lying on the opposite edge of the central vertex.

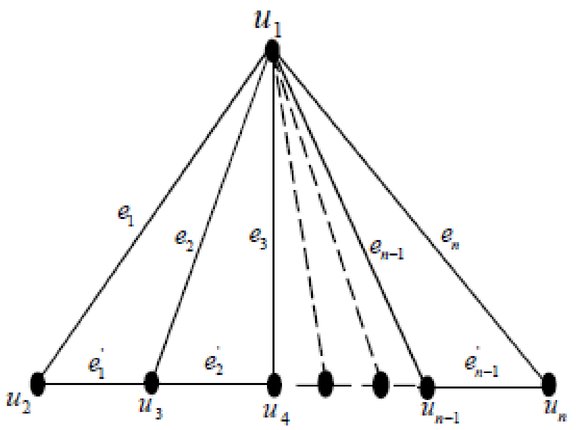

Figure 10: $C_n + K_{1,\,r}$

Now the vertex labelling of then-1 vertices of the cycle C_n is given by

$$f(u_i)=\begin{cases}2i-1\ , & i\ \ is\ \ even\\ 2q-2i+2 & ,\ i\ \ is\ \ odd\end{cases}$$

Again the edge labelling of $C_n + K_{1,\,r}$ is given by

$$e_i=\begin{cases}2q-2i+2\ , & 1,\ 3,5,----,\ 2r-1\\ 2i-2 & ,\ 2,\ 4,6,----,\ 2r-2\end{cases}$$

$e_j^{'}=2q-4i+2\ ,\ i=1,\ 2,------,\ r-1$

$e_1=2q$

$e_2=2$ -----------,------------, $e_{2r-1}=2q-4r+4$

$e_3=2q-4$

$e_1^{'}=2q-2,\ e_2^{'}=2q-6,$

$e_3^{'}=2q-10,------,e_{r-1}^{'}=2q-4r+6$.

Since the vertex labelling of $C_n + K_{1,\,r}$ are odd integers from 1 to 2q + 1 whereas the edge labelling are even integers from 2, 4, 6, ---------up to 2q. Hence $C_n + K_{1,\,r}$ is odd even graceful.

Illustration 6

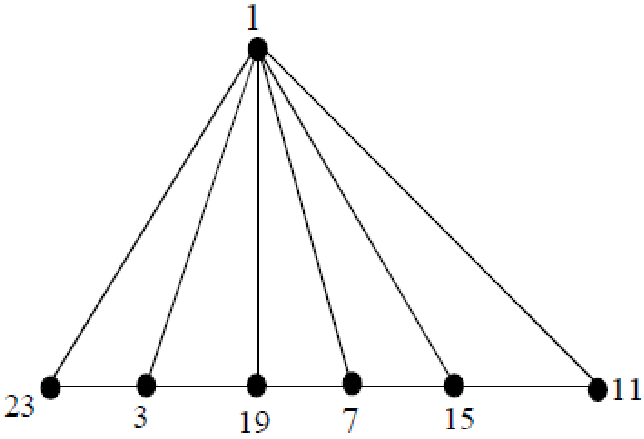

Figure 11: $C_7 + K_{1,6}$ **is odd even graceful.**

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