International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 21 (2015) pp 42388-42395

Comparison of Autoregressive (AR), Vector Autoregressive (VAR), Space Time Autoregressive (STAR), and Generalized Space Time Autoregressive (GSTAR) in Forecasting (Case: Simulation study with Autoregressive pattern)

Ferdian Bangkit Wijaya

PG Scholar Department of Statistics Bogor Agricultural University West Java District ferdian.bangkit.wijaya@gmail.com

I Made Sumertajaya

Lecturer Department of Statistics Bogor Agricultural University West Java District imsjaya@yahoo.com

Erfiani

Lecturer Department of Statistics Bogor Agricultural University West Java District erfiani_ipb@yahoo.com

Abstract
Modeling is a process of simplifying an event. One of the purposes of modeling is forecasting. A good model is capable of explaining data entirely in a simple way. A good forecasting is born from a precise modeling. This article will discuss the evaluation of time series data modeling without considering location factor, i.e. univariate time series data modeling in each location using Autoregressive (AR), multivariate time series data modeling in all locations using Vector Autoregressive (VAR), in addition to modeling which takes into account location factor, i.e. space-time modeling using Space Time Autoregressive (STAR) and Generalized Space Time Autoregressive (GSTAR) models. The scope of problem in this article is only determined autoregressive time series pattern in each location. In addition, the study only one variable and used Queen Contiguity spatial weight matrix. Data used was derived from simulation by generating first order stationary autoregressive pattern data, AR(1). Generated locations were 3, each of which with 100 time series, 10 percent data was used as evaluation tool between methods, i.e. by determining Root Mean Square Error (RMSE) for forecasting. The method was repeated 100 times to determine the consistency of RMSE. The results show that STAR method is the best among AR, VAR, and GSTAR methods for cases of same time order, indicated by the smallest RMSE frequency of STAR had the most, i.e. 37% in location 1, 32% in locations 2 and 3.

Keywords : Autoregressive (AR), Generalized Space Time Autoregressive (GSTAR), Root Mean Square Error (RMSE), Space Time Autoregressive (STAR), Vector Autoregressive (VAR)

Introduction

Modeling is a simplification of a system built with an intended goal in mind. The model should be able to answer questions in place of the actual system. One of the purposes of modeling is forecasting. A good model is capable of explaining data entirely in a simple way. A good forecasting is born from a precise modeling, or in other word the accuracy of forecasting depends on modeling process. Modeling process can be seen from its independency against space and time. Time series data modeling is a time-oriented or chronological sequance of observations on a location interest. Generally, time series data modeling consists of two methods, i.e. univariate and multivariate. Univariate time series refers to a time series that consists of observations from single variables which recorded sequentially over equal time increments. This modeling such as Box-Jenkins ARIMA. While, the time series data in many empirical studies consists of observations from several variables. This model is called as multivariate time series modeling. This modeling such as Vector Autoregressive Moving Average (VARMA)in [6].

Observation on an event is often carried out not only based on time series data in a location, but also data in several locations. Such data is known as space-time (panel) data. Panel data is basically a mixture of cross section data and time series data. Space Time Autoregressive (STAR) modeling method is originally introduced by Pfeifer and Deutch in [2]. In their scientific journal entitled "A three Stage Iterative Procedure for Space Time Modeling", this procedure consists of three steps, i.e. the identification of space time model, the estimation of the parameters, and the diagnose of model checking. This modeling method, however, has shortcoming, i.e. it requires same parameter values for all areas/locations. Ruchjana in [3] introduced the extension of STAR model which requires different model parameters for each area/location in an article entitled "*Pemodelan Kurva Produksi Minyak Bumi Menggunakan Model Generalisasi STAR*" which was followed by Suhartono in [5] who studied on the optimization of location/area weight in GSTAR modeling method.

A forecast is a prediction of some future event or events. Forecasting is an important problem that spans many fields including business, industry, economics, finance, etc. Quantitative forecasting techniques make formal use of historical data and a forecasting model. The model formally summarizes patterns in the data and expresses a statistical relationship between previous and current values of the variable in [1]. This research will discuss the evaluation of univariate through Autoregressive (AR) approach and multivariate through Vector Autoregressive (VAR) approached, and taken into account the independency of space-time with Space Time Autoregressive Modeling (STAR and GSTAR). The approach evaluation can be seen from the quality of the model to forecast, indicated by Root Mean Square Error (RMSE). This research only used single variable in three locations. Modeling is made without considering location factor as partial in each location, i.e AR modeling. Other ways, we assumed the locations as relationship variables and then made the model as simultan in all location, i.e VAR modeling. Last, we account locations effect with single variable. i.e STAR and GSTAR.

Preliminary Theory

1. Autoregressive (AR)

Autoregressive (AR) model is a special case of scalar time series Autoregressive Moving Average (ARMA). If the observation of time series at time-t depends only on the first time lag of observation, then it is called the first order autoregressive or an autoregressive first order, denoted by AR (1). Furthermore, for the constant mean μ, shows the AR (1) as :

$$z_t = \emptyset_1 z_{t-1} + \varepsilon_t \qquad (1)$$

Or using the backshift operator, shows the AR (1) as :

$$(1 - \emptyset_1 B) z_t = \varepsilon_t \qquad (2)$$

where :z_t is observation at time-t, z_{t-1} is observation at time-(t-1), $\emptyset_1$is autoregressiveparameter order-1, and ε_t is white noise, where $\varepsilon_t \sim N(0,1)$
Identification AR (1) model can be seen from Partial Autocorrelation Function (PACF) plot and Autocorrelation Function (ACF) plot. AR (1) model had pattern in PACF is cuts off at first lag while the pattern in ACF is tails off. Stationary in AR (1) model can be seen from ACF plot too, if ACF down very slowly then the data indicated not stationer, so differencing process must be made for modeling the data. Formal test to indicated stationary is unit root Augmented Dickey Fuller test where alternative hypotesis is the data is stationer. The stationary also can be seen from its parameter value. The paremeter absolute value must be lower than 1, $|\emptyset_1| < 1$. The election of the best model used the smallest value of Bayesian Information Criterion (BIC). The estimation of parameter AR(1) used Ordinary Least Square in [6].

2. Vector Autoregressive (VAR)

Vector Autoregressive (VAR) model is a approachment quantitative forecasting which usually applied in multivariate time series modeling. This model explains the relationship between observations at the variable itself at a previous time and also its association with other variables at a previous time. If the observation in one location at time't' depends on the first time lag of observation and observations in other location, then it is called the first order vector autoregressive or a vector autoregressive order one, denoted by VAR (1). Furthermore, the VAR (1) as:

$$\mathbf{z_t} = \mathbf{\Phi_1 z_{t-1}} + \boldsymbol{\varepsilon_t} \qquad \textbf{(3)}$$

Or using the backshift operator, shows the VAR (1) as :

$$(\mathrm{I} - \mathbf{\Phi_1}\mathrm{B})\mathbf{z_t} = \boldsymbol{\varepsilon_t} \qquad \textbf{(4)}$$

Where : $\mathbf{z_t}$ is vector of observation at time-t, location-n with size$(n \times 1)$, $\mathbf{z_{t-1}}$is vector of observation at time-(t-1), $\mathbf{\Phi_1}$is matrix of VAR parameter order-1 with size$(n \times n)$, $\boldsymbol{\varepsilon_t}$ is white noise vector, where $\varepsilon_t \sim MN(0,1)$ with size $(n \times 1)$
For example, if a number of location is 2 then VAR (1) show as :

$$z_t = \Phi_1 z_{t-1} + \varepsilon_t$$

where:
$z_t = \begin{pmatrix} z1_t \\ z2_t \end{pmatrix}$, $z_{t-1} = \begin{pmatrix} z1_{t-1} \\ z2_{t-1} \end{pmatrix}$, $\varepsilon_t = \begin{pmatrix} \varepsilon 1_t \\ \varepsilon 2_t \end{pmatrix}$, and $\Phi_1 = \begin{pmatrix} \Phi_{11}{}^1 & \Phi_{12}{}^1 \\ \Phi_{21}{}^1 & \Phi_{22}{}^1 \end{pmatrix}$

Identification VAR (1) model can be seen from Multivariate Partial Autocorrelation Function (MPACF) plot. VAR (1) model had pattern in MPACF is cuts off at first lag. Stationary in VAR (1) model can be seen from the eigen value of parameter matrix as :

$$\text{eigval } (\Phi_1) = \begin{pmatrix} \Phi_{11}{}^1 & \Phi_{12}{}^1 \\ \Phi_{21}{}^1 & \Phi_{22}{}^1 \end{pmatrix} < 1 \qquad (5)$$

If eigen value higher than 1, so the data indicated not stationer, differencing process must be made for modeling the data.The election of the best model used the smallest value of Akaike Information Corrected Criterion (AICC). The estimation of parameter AR(1) used Ordinary Least Square in [6].

3. Space Time Autoregressive (STAR)

Space Time Autoregressive (STAR) is one of space time modeling. That's model includes combining elements of time and location dependencies. STAR (1;1) model is combine autoregressive first order model and spatial first order. Shows the STAR (1;1) as :

$$z_t = \emptyset_{10} z_{t-1} + \emptyset_{11} W^1 z_{t-1} + \varepsilon_t \qquad (6)$$

where: $\mathbf{z_t}$ is vector of observation at time-t, location-n with size$(n \times 1)$,$\emptyset_{10}$is autoregressive parameter, time order-1 and space order-0, $\emptyset_{11}$is autoregressive parameter, time order-1 and space order-1, $\mathbf{W^1}$ is spatial weighted matrix with size (n x n).
For example, if a number of location is 2 then STAR (1;1) show as :

$$z_t = \emptyset_{10} z_{t-1} + \emptyset_{11} W^1 z_{t-1} + \varepsilon_t$$

Where: $z_t = \begin{pmatrix} z1_t \\ z2_t \end{pmatrix}$, $z_{t-1} = \begin{pmatrix} z1_{t-1} \\ z2_{t-1} \end{pmatrix}$, $\varepsilon_t = \begin{pmatrix} \varepsilon 1_t \\ \varepsilon 2_t \end{pmatrix}$, and $W = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}$

Stationary STAR (1;1) model can be seen from the stationary in each location. Ruchjana in [4] had approached the stationary STAR (1;1) through VAR (1), and can be concluded that STAR (1;1) model said stationer if $|\emptyset_{10}|+|\emptyset_{11}| < 1$. The estimation of parameter STAR(1;1) used Ordinary Least Square with the value of parameter is same in each location.

4. *Generalized Space Time Autoregressive (GSTAR)*

Generalized Space Time Autoregressive (GSTAR) is an extension of STAR model with the assumption that the parameters of space and time changes in each location. Same as STAR model, GSTAR model is combine autoregressive first order model and spatial first order. Shows the GSTAR (1;1) as :

$$z_t = \emptyset_{10} z_{t-1} + \emptyset_{11} W^1 z_{t-1} + \varepsilon_t \quad (7)$$

Where : $\mathbf{z_t}$ is vector of observation at time-t, location-n with size$(\mathbf{n} \times \mathbf{1})$, $\emptyset_{10}$is diagonal matrix of autoregressive parameter, time order-1 and space order-0 with size (n x n), $\emptyset_{11}$is autoregressive parameter diagonal matrix, time order-1 and space order-1 with size (n x n), $\mathbf{W^1}$ is spatial weighted matrix with size (n x n).
For example, if a number of location is 2 then GSTAR (1;1) show as :

$$z_t = \emptyset_{10} z_{t-1} + \emptyset_{11} W^1 z_{t-1} + \varepsilon_t$$

where:
$\mathbf{z_t} = \begin{pmatrix} z1_t \\ z2_t \end{pmatrix}$, $\mathbf{z_{t-1}} = \begin{pmatrix} z1_{t-1} \\ z2_{t-1} \end{pmatrix}$, $\boldsymbol{\varepsilon_t} = \begin{pmatrix} \varepsilon 1_t \\ \varepsilon 2_t \end{pmatrix}$,
$\boldsymbol{\emptyset_{10}} = \begin{pmatrix} \emptyset_{10}{}^1 & 0 \\ 0 & \emptyset_{10}{}^2 \end{pmatrix}$, $\boldsymbol{\emptyset_{11}} = \begin{pmatrix} \emptyset_{11}{}^1 & 0 \\ 0 & \emptyset_{11}{}^2 \end{pmatrix}$, and $\mathbf{W} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}$

Stationary GSTAR (1;1) model can be seen from the stationary in each location. The estimation of parameter STAR(1;1) used Ordinary Least Square with the value of parameter is same in each location in [4].

5. *Queen Contiguity Weighted Matrix*

Queen Contiguity is a matrix that describes the relationship between locations, 1 is given if location-i directly close with location-j, while 0 for not directly close, this matrix is normalized so that the value of this matrix element is $W_{ij} = \frac{c_{ij}}{c_{i.}}$, where sum of line is 1. A simple example of this matrix is numbered two locations directly close to each other follow as:
$W = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

6. *Bayesian Information Criterion (BIC), and Akaike Information Corrected Criterion (AICC)*

Each candidate models in AR(p), VAR(p), STAR(p;1), and GSTAR(p;1) had been chosen the smallest Bayesian Information Criterion (BIC) for AR(p) model and the smallest Akaike Information Corrected Criterion (AICC) for VAR(p) model, the process as overfitting so look for time order whose the smallest value of BIC dan AICC, the smaller the value, the accuracy of the model in the better predict.

Formula for BIC and AICC as :

$$BIC = \log(|\Sigma|) + \frac{r \log(T)}{T} \quad (8)$$

$$AICC = \log(|\Sigma|) + \frac{2r}{\left(T - r/_k\right)} \quad (9)$$

where : is matrix of maximum likelihood estimator for Σ , r is number of estimate parameter, T is number of observation, k is number of dependent variable in [6].

7. *Comparison of model performance*

The goodness performance model can bee seen from Root Mean Square Error (RMSE) value. AR, VAR, STAR, and GSTAR had a forecasting RMSE value, i.e how well the model can predict the values in the future. The smallest RMSE values show the most excellent model in forecasting. Formula for Root Mean Square Error as :

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(Z_t - \hat{Z}_t)^2} \quad (10)$$

Where,T is number of observation, Z_t is actual observation at time-t, $\hat{Z}_t$ is predict observation at time-t in [6].

Simulation

Data in this study was from the result of simulation which was designed in such a way to have same Autoregressive order in each location, i.e. time order value of 1 and spatial order value of 1. A total of three locations were used. SAS 9.3 software was used for analysis. The process of data generation is as follow:

1. Make the subjective design of location position as follow.

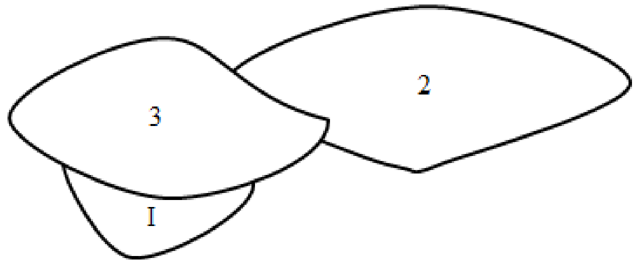

Fig 1. The design of location position

2. Generate one of Z_{t-1} value as initial value where Z_{t-1} ~ N (0.1).
3. Determine Autoregressive parameter, AR(1) which is stationary in each location, i.e.
   Location 1: $\emptyset_1 = 0.6$
   Location 2: $\emptyset_1 = 0.7$
   Location 3: $\emptyset_1 = 0.8$
4. Generate a white noise value where ε_t ~ N (0.1).
5. Compute Z_t value in each location by following equation below:
   $Z_t = \emptyset_1 Z_{t-1} + \varepsilon_t$
6. Convert Z_{t-1} value to Z_t value as the result of step 4.
7. Repeat step 3-5 100 times (T=100).
8. Determine the matrix of cholesky correlation simulation to make each location correlates each other as follow.
   $$K = \begin{pmatrix} 1 & 0.25 & 0.85 \\ 0.25 & 1 & 0.70 \\ 0.85 & 0.70 & 1 \end{pmatrix}$$
9. Carry out cholesky factorization in such so Z_t value in location 1, 2, and 3 has spatial correlation of K.
10. Data is ready to use in AR, VAR, STAR, and GSTAR modeling methods.

Methods

Steps used in this analysis were:

1. Carry out the process of data generation as explained in sub chapter Data.
2. Divided the data, 90% for modeling, and 10% to find RMSE of forecasting.
3. Carry out Autoregressive (AR) and Vector Autoregressive(VAR) modeling methods followed by determining the value of Root Mean Square Error (RMSE) and BIC and AICC to define the benefit of the model to determine data with same autoregressive order.
4. Determine Queen Contiguityspatial weight matrix according to position design in Figure 1 as :
   $$W = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$
5. Carry out Space Time Autoregressive (STAR) and Generalized Space Time Autoregressive (GSTAR) modeling methods with same time and spatial orders (1;1) followed by determining the value of Root Mean Square Error (RMSE) to define the benefit of the model.
6. Repeat step 1-4 for 100 times and compare RMSE of forecasting for each model.

Results and Discussion

Discussion for each model is result of example from one repetition analysis. Analysis starts from checking stationary until forecasting. In section comparison model performance, the results of Root Mean Squared Error is by 100 repetitions according to procedure in Materials and Methods section.

1. Autoregressive (AR) Model

This modeling is made separately at each location, which is the parameter erstimation suspected partially at three locations and then look for the accuracy of forecasting. The first thing to do is to split the 100 data into two parts, for modeling construct and determined accuracy of forecasting other i.e 90 to modeling and 10 to forecasting. After the data is generated, viewed back the stationary. The stationary is determined by formal test, it is Augmented Dickey Fuller test which the null hypothesis is the data not stationer. Stationary test results in each location can be seen at Table 1.

TABLE 1. Stationary of AR(1)

Locations	Rho	p-value	Tau	p-value
1	-39.0816	<.0001	-4.42	<.0001
2	-24.4946	0.0002	-3.47	0.0007
3	-34.0587	<.0001	-4.08	<.0001

Table 1 explained that data in locations 1, 2, and 3 had met the assumption of stationarity, this can be seen from the p-value, either in statistical Rho or Tau which has a value (<0.0001) is always less than α (0.05), so that the data is stationer on average. After checked stationary, then carried autoregressive (AR) model parameter estimation used Ordinary Least Squares (OLS) method. This estimation as follows :

$$z_t = \emptyset_1 z_{t-1} + \varepsilon_t$$

Based on OLS result that parameter of AR(1) in each location is significant. It's can bee seen from p-value in location 1 (<0.0001), location 2 (0.0009), location 3 (0.0002) are less thanα (0.05). Thus, the autoregressive model in each location that are formed :

Location 1: $z1_t = 0.64637\, z1_{t-1} + \varepsilon_t$
Location 2: $z2_t = 0.71949\, z2_{t-1} + \varepsilon_t$
Location 3: $z3_t = 0.65416\, z3_{t-1} + \varepsilon_t$

These models already meet the criteria of residual assumption that independent and follow normal distribution. Tentative model selection determined from the Bayesian Information Criterion (BIC) of any combination of the order time, the smallest BIC value show that the best AR model. Based on analysis showed that the smallest BIC value in each location is AR(1), exactly-0.10579 in location 1,-0.10162 in location 2, and-0.11415in location 3. This is consistent with the data which generated at first.

Forecasting process carried out from 10 percent divided data generation. Forecasting is done in accordance with the model obtained. After the result of predictions then look for the value of RMSE with 10 original data generation according to the formula (10). The following is an example of forecasting time 91-th in location 1. If known from data generated 90-th in each location follow as :

z1(90) = 1.10354, z2(90) = 0.8278, and z3(90) = 0.89882 then forecast for time 91-th in location 1 as $z1_{(91)} = 0.64637\, z1_{(90)}$,

i.e $z1_{(91)} = 0.64637 \times 1.10354 = 0.7133$.The same method is used to get the forecast value of time 92-th to 100-

th, so that the value of forecasting in each location can be seen at Table 2.

TABLE 2. Forecasting value (AR) in each location

Observation	Locations		
	1	2	3
91	0.71330	0.59560	0.58800
92	0.46110	0.42850	0.38460
93	0.29800	0.30830	0.25160
94	0.19260	0.22180	0.16460
95	0.12450	0.15960	0.10770
96	0.08050	0.11480	0.07040
97	0.05200	0.08260	0.04610
98	0.03360	0.05940	0.03010
99	0.02170	0.04280	0.01970
100	0.01400	0.03080	0.01290

After forecasting values obtained, the next step is to find the accuracy of forecasting with Root Mean Squares Error value. it is basically comparing the values obtained from the forecasting model with the original values of the generation process. This value is used to look at the performance of forecasting models. If known the forecasting values at Table 2 and the original values from 91-th to 100-th at Table 3 below, so the determination of RMSE in location 1 as :

$$\mathrm{RMSE} = \sqrt{\frac{(1.4118 - 0.7133)^2 + (0.5692 - 0.4611)^2 + \ldots + (0.28352 - 0.014)^2}{10}}$$

$$= 0.71827$$

TABLE 3. Original value from 10% data generated

Observation	Locations		
	1	2	3
91	1.4118	0.41507	0.68711
92	0.5692	0.36166	0.20667
93	-0.12474	-0.378	-0.57931
94	0.4194	0.64387	0.50875
95	-0.61449	-2.75118	-2.24731
96	0.07767	-4.02016	-2.23841
97	1.28625	-3.38057	-1.06088
98	1.14235	-3.1186	-1.15384
99	-1.0069	-0.83444	-1.70647
100	0.28352	0.89855	0.43823

The same method is used to get the RMSE value of location 2 and 3, so that the value are 2.23323 in location 2 and 1.32344in location 3.

2. Vector Autoregressive (VAR) Model

This modeling is constructed simultaneously at all locations, it means the parameter estimation is estimate as simultan in 3 locations and then look for the precision of forecasting. Stationary of VAR model can be seen from eigen value of parameter matrix, as follow formula 5. VAR model is said to be stationary if eigen value of parameter matrix is less than one. The results of the eigen value can be seen at Table 4.

TABLE 4. Stationary of VAR (1)

Eigen	Real	Imaginary	Modulus	Radian	Degree
1	0.7314	0.0485	0.7330	0.0662	3.7913
2	0.7314	-0.0485	0.7330	-0.0662	-3.7913
3	0.6436	0.0000	0.6436	0.0000	0.0000

Table 4 explained that VAR (1) model has stable process or stationer. it's can be seen from 3 modulus eigen value of parameter matrix. All of its less than one. Therefore, it did not need differencing process for further analysis. Paremeter estimation for VAR model also used Ordinary Least Squares (OLS) method. The parameter model estimated as follow :

$$z_t = \Phi_1 z_{t-1} + \varepsilon_t$$

More detail formula as follow :

Location 1 : $z1_t = \emptyset_{11} z1_{t-1} + \emptyset_{12} z2_{t-1} + \emptyset_{13} z3_{t-1} + \varepsilon_t$

Location 2 : $z2_t = \emptyset_{21} z1_{t-1} + \emptyset_{22} z2_{t-1} + \emptyset_{23} z3_{t-1} + \varepsilon_t$

Location 3 : $z3_t = \emptyset_{31} z1_{t-1} + \emptyset_{32} z2_{t-1} + \emptyset_{33} z3_{t-1} + \varepsilon_t$

Based on OLS result that parameter which significant of VAR (1) is only $z2_{t-1}$ in location 2. It's can bee seen from p-value (0.0042) is less than α (0.05), but in this research is focused in forecasting. Therefore all parameter include in VAR model. Thus, the vector autoregressive model in each location that are formed :

Location 1: $z1_t = 0.62732\ z1_{t-1} - 0.0171\ z2_{t-1} + 0.02665\ z3_{t-1} + \varepsilon_t$

Location 2: $z2_t = -0.1344\, z1_{t-1} + 0.67176\, z2_{t-1} + 0.11194\, z3_{t-1} + \varepsilon_t$

Location 3: $z3_t = -0.125\, z1_{t-1} - 0.0673\, z2_{t-1} + 0.80723\, z3_{t-1} + \varepsilon_t$

These models already meet the criteria of residual assumption that independent and follow normal distribution. Tentative model selection determined from the Akaike Information Corrected Criterion (AICC) of any combination of the order time, the smallest AICC value show that the best VAR model. Based on analysis showed that the smallest AICC value in all location is VAR(1), exactly-3.76197 in location 1,2,3 as simultan. This is consistent with thedata which generated at first in each location.

Forecasting process in VAR model same as AR model. If known from data generated 90-th in each location follow as : z1(90) = 1.10354, z2(90) = 0.8278, and z3(90) = 0.89882 then forecast for time 91-th in location 1 as $z1_{(91)} = 0.62732\ z1_{(90)} - 0.0171\ z2_{(90)} + 0.02665\ z3_{(90)}$, ie $z1_{91} = (0.62732 \times 1.10354) - (0.0171 \times 0.8278) + (0.02665 \times 0.89882) = 0.70203$. The same method is used to get the forecast value of time 92-th to 100-th, so that the value of forecasting in each location can be seen at Table 5.

TABLE 5. Forecasting value (VAR) in each location

Observation	Locations		
	1	2	3
91	0.70203	0.50840	0.53191
92	0.44586	0.30672	0.30741
93	0.28263	0.18053	0.17178
94	0.17878	0.10252	0.09119
95	0.11283	0.05505	0.04436
96	0.07102	0.02678	0.01801
97	0.04457	0.01046	0.00386
98	0.02788	0.00147	-0.00316
99	0.01738	-0.00311	-0.00614
100	0.01079	-0.00511	-0.00692

After forecasting values obtained, the next step is to find the accuracy of forecasting with Root Mean Squares Error value. If known the forecasting values at Table 5 and the original values from 91-th to 100-th at Table 3, so the determination of RMSE in location 1 as :

$$\mathrm{RMSE} = \sqrt{\frac{(1.4118 - 0.70203)^2 + (0.5692 - 0.44586)^2 + \ldots + (0.28352 - 0.01079)^2}{10}}$$

$= 0.71967$

The same method is used to get the RMSE value of location 2 and 3, so that the value are 2.18200 in location 2 and 1.29085 in location 3.

3. Space Time Autoregressive (STAR) Model

Space time modeling is constructed simultaneously at all locations, it means the parameter estimation is estimate as simultan in 3 locations and then look for the precision of forecasting. Unlike VAR model, this modeling incorporating of the proximity locations effect in its calculations. Queen Contiguity, spatial weighted matrix is defined as a description of relationship this locations. This matrix is constructed follow as design of position at Figure 1. So that spatial weighted matrix as :

$$W = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

It means that location 1 and 3 are directly close as well as location 2 and 3. Location 3 is directly close with 1 and 2, so gived same weight, i.e 0.5. The location which not directly close is gived by 0 value. The simulation design adapts to this spatial weighted. Its can be seen from determination of Cholesky decomposition where the nearest neighbour is gived higher correlation value than far neighbour.

Paremeter estimation for STAR(1;1) model also used Ordinary Least Squares (OLS) method. The parameter model estimated as follow :

$$z_t = \emptyset_{10} z_{t-1} + \emptyset_{11} W^1 z_{t-1} + \varepsilon_t$$

More detail formula as follow :

Location 1: $z1_t = \emptyset_{10} z1_{t-1} + \emptyset_{11} z3_{t-1} + \varepsilon_t$

Location 2: $z2_t = \emptyset_{10} z2_{t-1} + \emptyset_{11} z3_{t-1} + \varepsilon_t$

Location 3: $z3_t = \emptyset_{10} z3_{t-1} + \frac{1}{2}\emptyset_{11} z1_{t-1} + \frac{1}{2}\emptyset_{11} z2_{t-1} + \varepsilon_t$

Based on OLS result that parameter which significant of STAR (1;1) is only $\emptyset_{10}$ in all locations. It's can bee seen from p-value (<0.0001) is less than α (0.05), but in this research is focused in forecasting. Therefore all parameter include in STAR(1;1) model. Thus, the space time autoregressive model in each location that are formed :

Location 1: $z1_t = 0.713676\, z1_{t-1} - 0.05151\, z3_{t-1} + \varepsilon_t$

Location 2: $z2_t = 0.713676\, z2_{t-1} - 0.05151\, z3_{t-1} + \varepsilon_t$

Location 3: $z3_t = 0.713676\, z3_{t-1} - \frac{1}{2}(0.05151) z1_{t-1} - \frac{1}{2}(0.05151) z2_{t-1} + \varepsilon_t$

These models already meet the criteria of residual assumption that independent and follow normal distribution, so that forecasting can be made. Forecasting process in space time modeling same as time series modeling. If known from data generated 90-th in each location follow as : z1(90) = 1.10354, z2(90) = 0.8278, and z3(90) = 0.89882 then forecast for time 91-th in location 1 as $z1_{(91)} = 0.713676\, z1_{(90)} - 0.05151\, z3_{(90)}$ ie $z1_{91} = (0.713676 \times 1.10354) - (0.05151 \times 0.89882) = 0.74126$. The same method is used to get the forecast value of time 92-th to 100-th, so that the value of forecasting in each location can be seen at Table 6.

TABLE 6. Forecasting value (STAR) in each location

Observation	Locations		
	1	2	3
91	0.741267	0.544482	0.591721
92	0.498543	0.358103	0.389181
93	0.335751	0.235522	0.255685
94	0.226446	0.154915	0.167762
95	0.152967	0.101917	0.109905
96	0.103508	0.067074	0.071872
97	0.070169	0.044167	0.046900
98	0.047662	0.029105	0.030526
99	0.032443	0.019199	0.019809
100	0.022133	0.012682	0.012807

After forecasting values obtained, the next step is to find the accuracy of forecasting with Root Mean Squares Error value. If known the forecasting values at Table 6 and the original values from 91-th to 100-th at Table 3, so the determination of RMSE in location 1 as :

$$\mathrm{RMSE} = \sqrt{\frac{(1.4118 - 0.74126)^2 + (0.5692 - 0.49854)^2 + \ldots + (0.28352 - 0.02213)^2}{10}}$$

$= 0.71549$

The same method is used to get the RMSE value of location 2 and 3, so that the value are 2.20518 in location 2 and 1.32441 in location 3.

4. Generalized Space Time Autoregressive (GSTAR) Model

GSTAR modeling is basically same as STAR modeling, but the difference is parameters in GSTAR is gived different weight at each location, it means that parameter in formula (7) is matrix form, not a constant form. The spatial weight queen contiguity is determined same as STAR model.

Paremeter estimation for GSTAR(1;1) model also used Ordinary Least Squares (OLS) method. The parameter model estimated as follow :

$$z_t = \emptyset_{10} z_{t-1} + \emptyset_{11} W^1 z_{t-1} + \varepsilon_t$$

where,

$$\boldsymbol{\emptyset_{10}} = \begin{pmatrix} {\emptyset_{10}}^1 & 0 & 0 \\ 0 & {\emptyset_{10}}^2 & 0 \\ 0 & 0 & {\emptyset_{10}}^3 \end{pmatrix}, \boldsymbol{\emptyset_{11}} = \begin{pmatrix} {\emptyset_{11}}^1 & 0 & 0 \\ 0 & {\emptyset_{11}}^2 & 0 \\ 0 & 0 & {\emptyset_{11}}^3 \end{pmatrix},$$

$$W = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

More detail formula as follow :

Location 1: $z1_t = {\emptyset_{10}}^1 z1_{t-1} + {\emptyset_{11}}^1 z3_{t-1} + \varepsilon_t$

Location 2: $z2_t = {\emptyset_{10}}^2 z2_{t-1} + {\emptyset_{11}}^2 z3_{t-1} + \varepsilon_t$

Location 3: $z3_t = {\emptyset_{10}}^3 z3_{t-1} + \frac{1}{2}{\emptyset_{11}}^3 z1_{t-1} + \frac{1}{2}{\emptyset_{11}}^3 z2_{t-1} + \varepsilon_t$

Based on OLS result that parameter which significant of GSTAR (1;1) is ${\emptyset_{10}}^1$, ${\emptyset_{10}}^2$, ${\emptyset_{10}}^3$ in all locations. It's can bee seen from p-value (<0.0001) is less than α (0.05), but in this research is focused in forecasting. Therefore all parameter include in GSTAR(1;1) model. Thus, the generalized space time autoregressive model in each location that are formed :

Location 1: $z1_t = 0.650678\, z1_{t-1} - 0.00531\, z3_{t-1} + \varepsilon_t$

Location 2: $z2_t = 0.752957\, z2_{t-1} - 0.05913\, z3_{t-1} + \varepsilon_t$

Location 3: $z3_t = 0.697621\, z3_{t-1} - \frac{1}{2}(0.05559) z1_{t-1} - \frac{1}{2}(0.05559) z2_{t-1} + \varepsilon_t$

These models already meet the criteria of residual assumption that independent and follow normal distribution, so that forecasting can be made. Forecasting process in GSTAR modeling same as STAR modeling. If known from data generated 90-th in each location follow as : z1(90) = 1.10354, z2(90) = 0.8278, and z3(90) = 0.89882 then forecast for time 91-th in location 1 as $z1_{(91)} = 0.650678\, z1_{(90)} - 0.00531\, z3_{(90)}$, ie $z1_t = (0.650678 \times 1.10354) - (0.00531 \times 0.89882) = 0.713272$. The same method is used to get the forecast value of time 92-th to 100-th, so that the value of forecasting in each location can be seen at Table 7.

TABLE 7 Forecasting value (GSTAR) in each location

Observasi (t)	Peramalan		
	z1(t)	z2(t)	z3(t)
91	0.7132723	0.570153	0.573354
92	0.4610649	0.395398	0.364311
93	0.2980696	0.276176	0.230346
94	0.1927237	0.194328	0.144733
95	0.1246323	0.137762	0.090211
96	0.0806163	0.098395	0.05564
97	0.0521597	0.070797	0.03384
98	0.0337594	0.051306	0.02019
99	0.0218593	0.037438	0.011721
100	0.0141611	0.027496	0.006528

If known the forecasting values at Table 7 and the original values from 91-th to 100-th at Table 3, so the determination of RMSE in location 1 as :

$$\text{RMSE} = \sqrt{\frac{(1.4118 - 0.713272)^2 + (0.5692 - 0.461065)^2 + \ldots + (0.28352 - 0.0141611)^2}{10}}$$

$$= 0.71826$$

The same method is used to get the RMSE value of location 2 and 3, so that the value are 2.22357 in location 2 and 1.31407 in location 3.

5. Comparison of Model Performance

The calculation of root mean squares error (RMSE), especially RMSE of forecasting is used to compare the goodness of forecasting accuracy among models of AR, VAR, STAR, and GSTAR. The modeling described earlier is for a set of sample data in one repetition. This modeling is repeated 100 times to see consistency of RMSE. This repetition used macros SAS system to facilitate the analysis. The best choice of methods can be seen from consistency of the smallest RMSE from 100 times repetition.

Figure 2 explains the distribution of each space and time model data. Based on the boxplot can be seen on the first location, there are 3 sets of data for each model that cause the increasing of RMSE, that makes all of the four models shift to the right. Location 2 for each of the model has 1 set of data that resulted in the increasing value of RMSE that cause a shift to the right, as the location 3 has no set of data that caused neither the increasing nor decreasing of the RMSE value, that caused four of the models are normally distributed. Overall, all the distribution of RMSE values each of the location have identical characteristics--AR, VAR, STAR, and GSTAR.

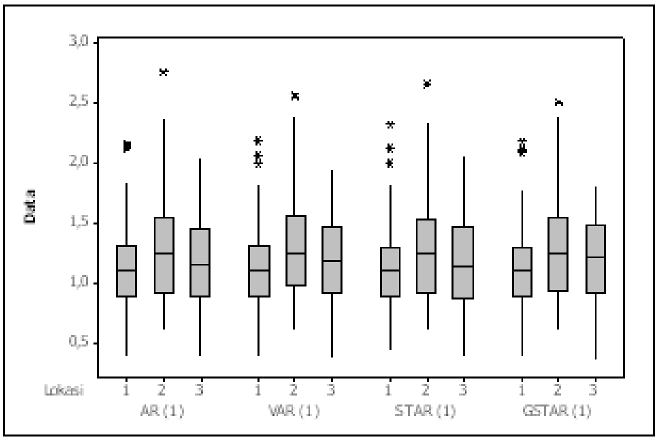

Fig 2. Boxplot RMSE same order in each location

Table 8 describe statistics of RMSE in 100 times repetition for each location. The determination of best model is not considered from mean because the values for all models do not differ much. This had been proven by F-test that null hypothesis is received. If looked from the values of varians, all methods is same. Its had been proven by Bartlett test that null hypothesis is received. Based on the same of varians, the consistency of RMSE values is determined by the smallest frequency had the most. For example in location 3, it means that from 100 repetition, 32 is a number of the smallest RMSE in STAR method, 68 is divided in other methods, i.e 18 in AR, 28 in VAR, 22 in GSTAR. The comparison between VAR and STAR is close enough in all location, but the simple model had been chosen as the best model. Furthermore STAR (1;1) is the best method for forecasting with same autoregressive order.

TABLE 8 RMSE of forecasting in 100 times

Location	Statistics	Methods			
		AR	VAR(1)	STAR(1)	GSTAR(1)
1	Mean	1.11279	1.12107	1.12022	1.11560
	Varians	0.11118	0.11134	0.11441	0.11079
	Frequency	22	37*	26	15
2	Mean	1.27614	1.28674	1.28260	1.28301
	Varians	0.18789	0.18468	0.19000	0.18654
	Frequency	24	28	32*	16
3	Mean	1.16722	1.18682	1.16490	1.19006
	Varians	0.11977	0.12924	0.12026	0.11639
	Frequency	18	28	32*	22

Conclusion and Remarks

Space and time modeling can be constructed using the Autoregressive (AR), Vector Autoregressive (VAR), Space Time Autoregressive (STAR), and Generalized Space Time Autoregressive (GSTAR). The forecasting become one of the goals of the modeling. The precision of forecasting can be measured by Root Mean Square Error (RMSE). The smallest RMSE value is indicated a good model. The precision of forecasting can be seen from consistency the repeatation of RMSE value, i.e the most frequency of the smallest RMSE value. Each modeling method had a different value of forecasting accuracy. Modeling with same order in each location is resulted STAR (1;1) model is the best for forecasting.

Suggestion

Differences in autoregressive pattern is required for further studies, in addition to the necessary development pattern, such as moving average, integrated, or seasonal.

References

[1] Montgomery, Jennings, Kulahci. Introduction toTime Series Analysis and Forecasting. New York:Wiley. 2008

[2] Pfeifer PE, Deutsch SJ. A Three Stage Iterative Procedure for Space-Time Modelling. Technometrics. 22(1) : 35-47.1980

[3] Ruchjana BN. *Pemodelan Kurva Produksi Minyak Bumi Menggunakan Model Generalisasi S-TAR*. 2002

[4] Ruchjana BN. The Stationarity of the S-TAR (1;1) Model. 2003

[5] Suhartono, Wutsqa DU.*Perbandingan Model VAR dan STAR pada Peramalan Produksi Teh di Jawa Barat*. Surabaya (ID): Institut Teknologi Sepuluh Nopember, ITS.2008.

[6] Wei W. Time series analysis: univariate and multivarariate methods. New York (US): Addison-Wesley Publishing Co.2006