

# Mathematical Exposition Coupled with Empirical Testing of Heuristically Designed Portfolio Optimization Model

Dr. Soma Panja

*Assistant Professor, Department of Management Studies, National Institute of Technology Silchar, Silchar, Assam*

## Abstract

Portfolio optimization coupled with its various dimensions and categorization has its immense contribution in shaping the context and dimensions of theoretical and practical maneuvers of financial engineering field. Decision science as an application field has its own area and sphere of influence in shaping the decision making process, its understanding and its manipulations wherever it was felt applicable by the decision makers. Drawing upon the experiences on these two fields the present intervention aimed at an effort of hybrid modeling, marrying in the concepts of portfolio optimization in to the realm of decision sciences with very basic fundamental yet very powerful mathematical exposition. The empirical results demonstrate that the mathematical exposition coupled with heuristic design of the portfolio optimization model was capable enough to generate results by which an investor can reach near to the optimum portfolio heuristically, provided they are aware of their coefficient of optimism. Rather we can be ambitious to infer that the investor should know their propensity to take risk.

**Key words:** Portfolio Optimization, optimum portfolio, heuristic portfolio, optimistic portfolio, pessimistic portfolio, decision theory, coefficient of optimism, portfolio weight.

## Introduction

Financial engineering as an application field has commanded its wide acceptability amongst the theorist and scientific community for its academic rigor and objectifying capabilities of the erstwhile not so manageable and explainable phenomenon. Although the mathematical dimensions of financial engineering seem to be appealing much to the intellectual faculties of the scientific community at large, its limited appeal to the mass is somewhat compelling. The dynamism of this work does not aim at demeaning the status of financial engineering as a highly sophisticated technique encompassing mathematical and statistical modeling techniques rather it adds to its flavor by trying to make its application in the most widely used field of portfolio optimization. The design of financial portfolio consists of considering and including two or more than two assets. According to the pioneers of portfolio theory, Markowitz (1959), investment in a portfolio always gives more financial security than investment in one asset since diversification of risk in its mathematical underplay can have its effect in portfolio of assets only. For a given set of assets it is mathematically as well as practically possible to design large number of portfolio. However, as the combination of assets changes, the risk-return characteristics of the portfolios exhibit varying dimensions. Based on the seminal work of

Markowitz researchers have found the motivation to contribute to the field of portfolio optimization. The outcome of the umpteen number of research has yielded many ways and means of selecting an optimum portfolio. But, due to the innumerable parameters involved in selection of a portfolio coupled with statistical and mathematical dynamicity decision making becomes a complicated task. Upon thoughtful observation it is understood that the main concern for selecting an optimum portfolio is selecting an appropriate portfolio weight (the function of wealth invested in individual asset). Precise estimation of weights reduces unnecessary transaction costs and helps to construct the optimum portfolio. Markowitz (1952) first developed the theory of portfolio optimization. He proposed the Mean-Variance model to select an optimum portfolio. After his pioneering work, many researches in portfolio optimization kept on considering the Mean-Variance model. However, some researchers are developing new methods of portfolio optimization. In many ways it is possible to design an optimum portfolio, heuristically also, we can reach to an optimum solution. Arriving at a heuristic solution may be possible at a given point of time and at a given set of conditions but the same may not be a uniformly optimal one.

Development of a Mean – Variance framework of modern portfolio optimization more than 55 years ago by Markowitz, based on some assumptions, was designed in such a way so as to keep simplicity as the key part of its appeal. However, the consideration of more sophisticated risk measures like VaR or CVaR or putting constraints including limits on the maximum number of different assets in a portfolio and minimum holding size, have made it all but impossible to optimize portfolios with classical techniques. Since the crucial work of Markowitz and Sharpe, Mean – Variance have been the central focuses of portfolio analysis, most authors find it appealing to consider the Mean – Variance technique to select an optimum portfolio. The intervention of the present work do take its basic motivation from the classical mean-variance model of Markowitz, but, the mathematical exposition entails upon its elaboration by incorporating decision theory in portfolio optimization model and hence in its management. The present work aims at financial re-engineering of the optimization model and develops upon its heuristically designed portfolio to exhibit characteristics of the developed portfolio in comparison to the portfolio constructed using classical model.

## Theoretical Underpinnings

The designing of the mathematical formulation and its empirical testing needs to be backed by the fundamental works related to the theory of portfolio optimization. Keeping

in perspective the relevance of theoretical underpinnings, it is observed that after the pioneering work of Markowitz in portfolio optimization, W. Sharpe et. all (1963) developed CAPM model based on the same classical approach. Later on Sharpe developed Single Index Model conveying the idea that stock prices move with the common co-movement with a single index. Ross in 1976 proposed the model called Arbitrage Pricing Theory which states that the return on any stock is linearly related to asset of systematic factor or risk factor. The difficulty of precisely estimating the expected portfolio return was described by Merton (1980). Merton was able to reduce the portfolio problem to a stochastic optimum control problem, solvable by using stochastic control methodology. These revolutionary works provided the much needed direction to the researcher in the field of portfolio optimization. On further elaboration on the review the revelation pointed to the fact that based on the development of all these classical works, certain new areas of work is evolving in the field of portfolio optimization.

Sahalia & Brandt (2001) deliberated the asset allocation problem with partly predictable conditional moments of return. They studied the effect of predictive variables on the optimal portfolio weights and its direct determination of dependency. The predictive variables were combined into a single index which helped the investors to determine the correct combinations of important economic variables. A model portfolio for selection which is very easy to implement like the classical Markowitz model is proposed by Brandt and Clara (2006). They increased the number of asset in the portfolio and statistically optimized the managed portfolio in that increased asset space. The static choice of conditional portfolio and timing of portfolio which are managed portfolios represent a dynamic strategy that closely approximates the optimal dynamic strategy for horizons up to five years. As the classical models did not say much about the optimal portfolio weights, it becomes difficult to measure the optimum portfolio weight. If weights cannot be measured perfectly then it is not possible to select an optimum portfolio. To solve this problem a multivariate shrinkage estimator is proposed by Okhrin and Schmid (2008) for the optimal portfolio weights in their paper. By assuming log asset returns to be i.i.d. Gaussian, explicit solutions were derived for the optimal shrinkage factors by shrinking the classical Markowitz weights to the deterministic target portfolio weights. Analytical and Monte Carlo simulations methods were used to examine the properties of the estimated shrinkage weights.

Chang et. all (2000) considered the problem of finding the efficient frontier associated with the standard mean-variance portfolio optimization model heuristically. They put an effort by expanding the standard model to include cardinality constraints which limit a portfolio to have a specified number of assets, and to impose limits on the proportion of the portfolio held in a given asset. Depending on the change in the dimensions of the constraints like number of assets present in a portfolio and the weight of the assets the effect on the shape of the efficient frontier was exhibited in their paper. To find out the cardinality constrained efficient frontier they used three heuristic algorithms based upon genetic algorithms, tabu search and simulated annealing. On the other hand, Gilli and Kellezi (2000) illustrated how a realistic non-convex portfolio

optimization problem can be solved heuristically. According to them the heuristic optimization algorithms can be easily applied to solve the said problem and the heuristic method is the only way out to solve the problem. They have applied the threshold accepting method for the purpose. It was first illustrated by solving a standard mean-variance optimization problem and the solution was also computed with the quadratic programming algorithm which was used as a benchmark and thus provided some insight into the quality of the threshold accepting heuristic, which appears very satisfactory. The problem of non-convex optimization problem also got illustrated with the help of the threshold accepting algorithm. VaR and expected shortfall constraints to show the expected maximum future return of portfolio was used for the purpose. In the conclusion they referred that the threshold accepting algorithm opens new perspectives in the practice of portfolio management as it allows to deal easily with all sorts of constraints of practical importance, it gives useful approximations of the optimal solutions, it appears to be computationally efficient and is relatively easy to implement. Alentorn, et. all (2010) proposed an application of heuristic methods for portfolio optimization in implementing a market neutral hedge fund strategy. They proposed a framework for implementing a market neutral hedge fund strategy by using VaR as the risk measure. In their research paper they demonstrated how the Geometric Nelder-Mead Algorithm (GNMA) method is suitable for tackling the problem of portfolio optimization, delivering solutions that are in line with a GA approach. Roy et. all (2010, 2011) heuristically optimized portfolio for addressing the issue of portfolio optimization. Classical Markowitz mean-variance model was considered, maintaining minimization of the risk subject to a certain level of return provided the empirical evidence that an investor can reach to optimum solution heuristically.

The most prominent value that is risk value, marked from Jauch and Glueck (1988), which affects the business decision-making process acts to be the consideration factor in the present study. Jauch and Glueck first pointed out seven values that affect the business decision making process. In case of portfolio optimization, two most important terms are risk and return. Risk taking propensity of any investor changes the choice of the portfolio. The present paper is greatly influenced by this risk taking propensity of investor. As selection of portfolio requires decision based on the risk taking propensity of the investor, the decision theory can help in this regard. The decision theory allows the investors to take decision to invest. Many models are suggested by the authors in decision theory, out of which "Hurwitz coefficient of optimism" was considered for solving the problem of portfolio optimization.

### Scope and Research Problem

Classical models of portfolio optimization seems to be highly sophisticated and ornamentally elaborative for the general investors to infer meaning out of it, some heuristic solution can act as handy substitute to help them in selecting optimum portfolio. Optimization can be done in two ways – maximization of return subject to a certain level of risk and minimization of risk subject to a certain level of return. In our

earlier interventions, minimization of risk subject to a certain level of return was considered to select an optimum portfolio heuristically. In the present work the concept is elaborated upon by maximization of return, considering a certain level of risk. The decision theory was incorporated in portfolio optimization. Hurwitz Principle as applied in decision theory field has been considered with required modifications. With the help of empirical analysis we propose heuristic solution of portfolio optimization.

**Mathematical Exposition**

We first optimize the portfolio using classical optimum technique. Optimization of portfolio can be done in two ways – (1) maximizes return subject to a certain level of risk and (2) minimize risk subject to a certain level of minimum return. An investor always wants to go for either of these above two optimizations. We have made a balance between these two approaches by maximizing the return subject to a step-by-step increase in the level of minimum risk by introducing coefficient of optimism in the maximum risk constraint. Thus, the optimization problem considered for the current work is of the type:

$$\text{Max } \sum_{i=1}^n w_i E(R_i)$$

$$\text{Subject to } \hat{w}' \sigma_i \leq \alpha \sigma_{\max} + (1-\alpha) \hat{g}_{\min}$$

$$\sigma_{\min} \leq \sigma_0 \leq \sigma_{\max}$$

where,  $\sigma_{\min}$  = Minimum risk of the security in the portfolio and  
 $\sigma_{\max}$  = Maximum risk of the security in the portfolio.

We have introduced  $\alpha$  (coefficient of optimism) in this concept and have applied mathematical programming to calculate the risk and return of optimum portfolio for different value of coefficient of optimism starting from 0 to 1. As we are maximizing the return subject to a certain level of risk, we replace  $\mu_{\min}$  and  $\mu_{\max}$  to  $\sigma_{\min}$  and  $\sigma_{\max}$ . The risk and return of the optimum portfolio have been calculated using the following equations.

$$R_p^O = \sum_{i=1}^n w_i^O R_i$$

where,  $R_p^O$  = Expected return of the optimum portfolio,  
 $w_i^O$  = Optimum weight of the 'i'th security,  
 $R_i$  = Expected return of the 'i'th security and  
 $n$  = Number of the securities in the portfolio.

$$\sigma_p^O = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i^O w_j^O \sigma_i \sigma_j r_{ij}}$$

where,  $\sigma_p^O$  = Risk of the optimum portfolio,  $r_{ij}$  = Correlation coefficient between 'i'th and 'j'th securities and  $\sigma_i$  = Standard Deviation of 'i'th security.

**Heuristically Modeled Portfolio Optimization Model**

In the next phase we have introduced a heuristic solution with the help of which we may get an alternative choice of

portfolio based on propensity to take risk. We have considered two types of investors viz optimistic, highly risk taker and pessimistic, highly risk averse is those, where no short sell is allowed. Hence, two portfolios have been constructed for these two kinds of investors. For heuristic optimistic portfolio we have assumed that an optimistic investor gives importance on expected return ignoring the risk part, thereby the security weights are considered directly proportional to expected return of the security and for pessimistic investor, as the pessimistic investor gives importance on risk of the portfolio ignoring the expected return part, we have considered security weights inversely proportional to standard deviation of the security.

For heuristic optimistic solution the following formulation has been used for calculating the weights of the securities of the portfolio.

$$W_i^{HO} = \frac{\mu_i - \mu_{\min}}{\sum_{i=1}^n (\mu_i - \mu_{\min})}$$

where,  $W_i^{HO}$  = Heuristic optimistic weight of the 'i'th security and  
 $\mu_i$  = Expected return of 'i'th security.

Given this weight function we have calculated the expected return and risk as follows:

$$R_p^{HO} = \sum_{i=1}^n \frac{(\mu_i - \mu_{\min})}{\sum_{i=1}^n (\mu_i - \mu_{\min})} R_i$$

where,  $R_p^{HO}$  = Expected return of the optimistic portfolio,  
 $\sigma_p^{HO} =$

$$\frac{1}{\sum_{i=1}^n (\mu_i - \mu_{\min})} \sqrt{\sum_{i=1}^n \sum_{j=1}^n (\mu_i - \mu_{\min}) (\mu_j - \mu_{\min}) \sigma_i \sigma_j \rho_{ij}}$$

where,  $\sigma_p^{HO}$  = Standard Deviation of the heuristic optimistic portfolio.

For heuristic pessimistic solution the following formulation has been used for calculating the weights of the securities of the portfolio.

$$W_i^{HP} = \frac{1/\sigma_i}{\sum_{i=1}^n 1/\sigma_i}$$

where,  $W_i^{HP}$  = Heuristic pessimistic weight of 'i'th security. Given this weight function we have calculated the expected return and risk as follows:

$$R_p^{HP} = \sum_{i=1}^n \frac{1/\sigma_i}{\sum_{i=1}^n 1/\sigma_i} R_i$$

where,  $R_p^{HP}$  = Expected return of the heuristic pessimistic portfolio,

$$\sigma_p^{HP} = \frac{1}{\sum_{i=1}^n 1/\sigma_i} \sqrt{\sum_{i=1}^n \sum_{j=1}^n 1/\sigma_i 1/\sigma_j \sigma_i \sigma_j \sigma_{ij}}$$

where,  $\sigma_p^{HP}$  = Standard Deviation of the heuristic pessimistic portfolio.

### Empirical Testing

In order to test the proposed model empirically and to observe whether heuristically an optimum portfolio can be constructed, closeness between the optimum portfolio and heuristic portfolio need to be taken into consideration. For this purpose many distance measures are available viz. Mahalanobish Distance, Chebyshev Distance, Euclidian distance, City Block Distance which can be applied. In Cartesian coordinates, if point p and q are two points in Euclidian space, then the distance between the said two points is given by Pythagorean formula

$$d(p, q) = d(p, p) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{\sum_{i=1}^n (q_i - p_i)^2}$$

p and q are Euclidian vectors in the Euclidian space, starting from the origin of the space and their tips indicate two points. These two points indicate the length of the vector.

The City block distance between two points,  $a(a = a_1, a_2, a_3, \dots, a_k)$  and

$b(b = b_1, b_2, b_3, \dots, b_k)$ , with k dimensions is calculated as:

$$\sum_{j=1}^k |a_j - b_j|, j=1,2,3,\dots,k$$

The City block distance is always greater than or equal to zero. The measurement would be zero for identical points and high for points that show little similarity. It is being used in plant location and other areas.

Mahalanobis distance is a distance measure introduced by P. C. Mahalanobis in 1936. According to this distance different patterns can be identified and analyzed with the help of correlation. It is a multivariate measure.

The Mahalanobis distance of a multivariate vector  $a = (a_1, a_2, a_3, \dots, a_n)^T$  from a group of values with mean  $\mu = (\mu_1, \mu_2, \mu_3, \dots, \mu_n)^T$  is defined as:

$$D_m(a) = \sqrt{(a - \mu)^T S^{-1} (a - \mu)}$$

where,  $S$  = variance-covariance matrix

Mahalanobis distance can also be shown as a dissimilarity measure between two random vectors  $\vec{a}$  and  $\vec{b}$  of the same distribution with a common covariance matrix  $S$  as

$$d(\vec{a}, \vec{b}) = \sqrt{(\vec{a} - \vec{b})^T S^{-1} (\vec{a} - \vec{b})}$$

The Mahalanobis distance reduces to the Euclidean distance when the covariance matrix is the identity matrix. In case of

diagonal covariance matrix the distance measure, called the normalized Euclidean distance is given as follows:

$$d(\vec{a}, \vec{b}) = \sqrt{\sum_{i=1}^n (a_i - b_i)^2 / s_i^2}$$

Where,  $s_i$  = the standard deviation of the  $a_i$  and  $b_i$  over the sample set.

Chebyshev distance or Tchebychev distance is developed by Pafnutiy Chebyshev. It is a metric defined on a vector space where the distance between two vectors is the greatest of their differences along any coordinate dimension. The distance is used in chess. The Chebyshev distance between two spaces on a chess board gives the minimum number of moves a king requires to move between them. For this reason it is also known as chessboard distance. The Chebyshev distance between two vectors or points p and q, with standard

coordinates  $a_i$  and  $b_i$ , respectively, is

$$\lim_{p \rightarrow \infty} \left( \sum_{i=1}^n |a_i - b_i|^p \right)^{1/p} = \max_{i=1}^n |a_i - b_i|$$

However, The City Block Distance is a standard measure of distance to show the similarities and dissimilarities between two points. As we aim to find out the similarities and dissimilarities between optimum portfolio and heuristic portfolios under different values of co-efficient of optimism, this distance measure can be applied on a two dimensional set up with the expected return and risk of the optimum and heuristic portfolios. As Mahalanobis distance is used mainly in cluster analysis and classification techniques and Chebyshev distance is mainly used in chess, for these reasons these distance measures have not been used for the current work.

### Analysis and Interpretation

For empirical purpose daily security – wise data of ten companies listed in Nifty have been considered. The duration of the data has been taken for ten years from 2005 to 2014. Data have been collected from www.nseindia.com. 10 companies have been selected by using simple random sampling for the study purpose from a pool of stocks which were consistently included in the index (Nifty) for a period from 1st January, 2005 to 31st December 2014.

Optimum weights and heuristic weights of all the ten companies have been calculated in table number 1 to observe the return and risk of the three portfolios. The return and risk of the portfolios are presented in table no. 2.

With the help of the weights at different values of co-efficient of optimism expected return and risk have been calculated. From Table number 2, it is clear that heuristic pessimistic portfolio is closer to optimum portfolio than the heuristic optimistic portfolio in return-risk space. To observe the closeness between the optimum portfolio and heuristic portfolio the City Block Distance is considered and the values of the distances as shown in Table 3.

**Table 1: Weights of heuristic optimistic and heuristic pessimistic portfolios and weights of optimum portfolios under different values of coefficient of optimism**

	GAIL	SBIN	ICICI	L&T	Bank of Baroda	PNB	ACC	LUPIN	UltraCemCo	Asian Paints
Heuristic Optimistic Weights	0.157826258	0.142287553	0.119063523	0.171547194	0.071459132	0.052276482	0.014310137	0.206251035	0.064978737	0
Heuristic Pessimistic Weights	0.101193452	0.101095476	0.091312109	0.105843728	0.084411083	0.094520689	0.104256114	0.101949654	0.097175288	0.118242408
Optimum Weight, $\alpha=0$	0.000000085665150	0.000000091162730	0.000000100850300	0.000000080191500	0.000000120899600	0.9999992	0.000000097504490	0.00000006997590	0.000000126814900	0.000000077913430
Optimum Weight, $\alpha=0.1$	0.000000085709630	0.000000091180220	0.000000100842600	0.000000080235650	0.000000120862000	0.9999992	0.000000097318720	0.000000070077220	0.000000126755900	0.000000077710920
Optimum Weight, $\alpha=0.2$	0.000000085730240	0.000000091174110	0.000000100809400	0.000000080258070	0.000000120788900	0.9999992	0.000000097099690	0.000000070135410	0.00000012666060	0.000000077481480
Optimum Weight, $\alpha=0.3$	0.000000085729810	0.000000091147170	0.000000100753700	0.000000080261380	0.000000120684100	0.9999992	0.000000096850160	0.000000070174770	0.00000012653280	0.000000077227840
Optimum Weight, $\alpha=0.4$	0.000000085710770	0.000000091101790	0.000000100677900	0.000000080247790	0.000000120550900	0.9999992	0.000000096572630	0.000000070197520	0.00000012637560	0.000000076952400
Optimum Weight, $\alpha=0.5$	0.000000085675200	0.000000091040020	0.000000100584200	0.000000080219240	0.000000120392100	0.9999992	0.000000096269380	0.000000070205550	0.00000012619190	0.000000076657320
Optimum Weight, $\alpha=0.6$	0.000000085624920	0.000000090963640	0.000000100474500	0.000000080177370	0.000000120210200	0.9999992	0.000000095942500	0.000000070200520	0.00000012598430	0.000000076344540
Optimum Weight, $\alpha=0.7$	0.000000085561490	0.000000090874180	0.000000100350500	0.000000080123610	0.000000120007500	0.9999992	0.000000095593930	0.000000070183830	0.00000012575510	0.000000076015830
Optimum Weight, $\alpha=0.8$	0.000000085486290	0.000000090773010	0.000000100213600	0.000000080059230	0.000000119785800	0.9999992	0.000000095225460	0.000000070156730	0.00000012550610	0.000000075672790
Optimum Weight, $\alpha=0.9$	0.000000085400510	0.000000090661310	0.000000100065000	0.000000079985300	0.000000119547000	0.9999992	0.000000094838770	0.000000070120290	0.00000012523940	0.000000075316900
Optimum Weight, $\alpha=1$	0.000000085305220	0.000000090540130	0.000000099905990	0.000000079902800	0.000000119292800	0.9999992	0.000000094435450	0.000000070075450	0.00000012495640	0.00000007494951

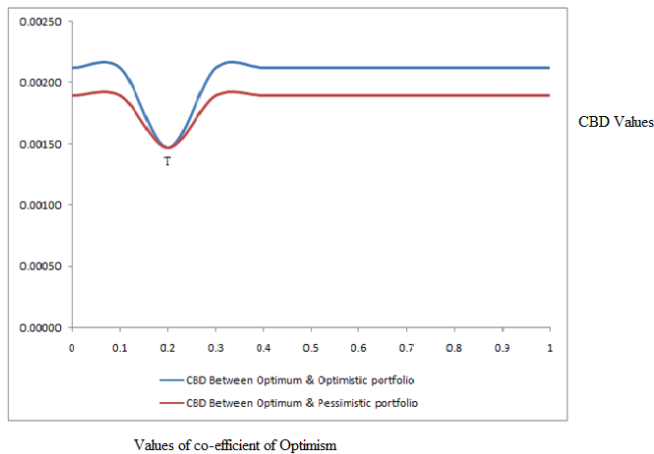
**Table 2: Expected return and risk of heuristic optimistic and heuristic pessimistic and optimum portfolio**

	Expected Return	Expected Risk
Heuristic Pessimistic Portfolio	-0.000575286	0.000242035
Heuristic Optimistic Portfolio	-0.0008644426661	0.000304604
Optimum Portfolio, $\alpha=0$	0.000263818246240	0.001295709082768
Optimum Portfolio, $\alpha=0.1$	0.000263818	0.001295709082721
Optimum Portfolio, $\alpha=0.2$	0.000263818246143	0.000647854541290
Optimum Portfolio, $\alpha=0.3$	0.000263818246304	0.001295709082356
Optimum Portfolio, $\alpha=0.4$	0.000263818246585	0.001295709082058
Optimum Portfolio, $\alpha=0.5$	0.000263818246973	0.001295709081694
Optimum Portfolio, $\alpha=0.6$	0.000263818247455	0.001295709081269
Optimum Portfolio, $\alpha=0.7$	0.000263818248023	0.001295709080791
Optimum Portfolio, $\alpha=0.8$	0.000263818248668	0.001295709080264
Optimum Portfolio, $\alpha=0.9$	0.000263818249382	0.001295709079693
Optimum Portfolio, $\alpha=1$	0.000263818250159	0.001295709079082

**Table 3: City Block Distance between optimum portfolio and heuristic optimistic portfolio and optimum portfolio and heuristic pessimistic portfolio at different values of coefficient of optimism.**

Co-efficient of Optimism	CBD Between Optimum & Optimistic portfolio	CBD Between Optimum & Pessimistic portfolio
0	0.002119365993948	0.001892777754214
0.1	0.002119365993776	0.001892777754042
0.2	0.001471511452373	0.001471511452373
0.3	0.002119365993600	0.001892777753866
0.4	0.002119365993583	0.001892777753849
0.5	0.002119365993606	0.001892777753872
0.6	0.002119365993664	0.001892777753930
0.7	0.002119365993754	0.001892777754020
0.8	0.002119365993872	0.001892777754137
0.9	0.002119365994015	0.001892777754280
1	0.002119365994180	0.001892777754446

The City Block Distance (CBD) between optimum portfolios and heuristic portfolios under different values of co-efficient of optimism is presented in a graph shown below. In the said figure the CBD values are measured along the vertical axis and values of co-efficient of optimism along the horizontal axis.



**Figure 1: the city block diagram for CBD between optimum & optimistic Portfolio and CBD between optimum & Pessimistic Portfolio.**

From the above figure it can be stated that except 0.2 co-efficient of optimism, in all the cases the heuristic pessimistic portfolio is closer to optimum portfolio than the heuristic optimistic portfolio. Only when the value of co-efficient of optimism is 0.2, the distance between both the heuristic portfolios with the optimum portfolio is same. They touched themselves at point T. For the rest values of coefficient of optimism, we can say that during the study period pessimistic investors will go near to the optimum portfolio as compared to the optimistic investors. Therefore, if an investor knows their coefficient of optimism, they can take wise decision while investing in stock market.

### Conclusion

Portfolio optimization coupled with its various dimensions and categorization has its immense contribution in shaping the context and dimensions of theoretical and practical maneuvers of financial engineering field. Decision science as an application field has its own area and sphere of influence in shaping the decision making process, its understanding and its manipulations wherever it was felt applicable by the decision makers. Drawing upon the experiences on these two fields the present intervention aimed at an effort of hybrid modeling, marrying in the concepts of portfolio optimization in to the realm of decision sciences with very basic fundamental yet very powerful mathematical exposition. The empirical results demonstrate that the mathematical exposition coupled with heuristic design of the portfolio optimization model was capable enough to generate results by which an investor can reach near to the optimum portfolio heuristically, provided they are aware of their coefficient of optimism. Rather we can

be ambitious to infer that the investor should know their propensity to take risk.

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