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Estimation of Co-Efficient of Variance For Transmission Flow Series-Parallel Reliability Systems

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Abstract

A systematic theory of reliability is based on probability theory. It is well known that the conventional reliability analysis using the probabilities has been found to be inadequate to handle the uncertainty of failure in data and modeling. The fuzzy set theory provides an useful tool to complement conventional reliability theories. This paper proposes the transmission flow system design in series-parallel structure to evaluate the maximum reliability subject to the system cost. The co-efficient of variation (CV) is introduced to estimate each membership function. This approach is to find minimum CV with high reliability estimate.

Keywords: Reliability Optimization, Multi-stage series- parallel System, Co-efficient of Variance, Estimation of uncertainty, Triangular Fuzzy number.

2010 Mathematics Subject Classification: 68M15, 03E72, 49M37, 60K10

Introduction

Reliability is an important factor in the management, planning and design of any engineering product. A systematic theory of reliability is based on probability theory. It is well known that the conventional reliability analysis using the probabilities has been found to be inadequate to handle the uncertainty of failure in data and modeling. The fuzzy set theory provides an useful tool to complement conventional reliability theories. The concept of fuzzy reliability has been introduced and formulated in Cai [2] due to uncertainties and imprecision of data. Using fuzzy theoretic approach Singer [21], Utkin [23] and Guan et al [7] attempted to define and evaluate system reliability in terms of fuzzy set theory and technique. Huang et al [10] presented a Bayesian reliability analysis for fuzzy life time data.

Using the fuzzy set theoretic approach Cai et al [3], [4], and [5] introduced the fuzzy state assumption and possibility assumption to replace binary state assumption and probability assumption. Chen [6] presented a method for fuzzy system reliability analysis using Simplified Fuzzy Arithmetic Operations of fuzzy numbers, in which the reliability of each component is represented by a triangular fuzzy number. Hong et al [9] presented fuzzy system reliability by the use of t-norm based convolution of fuzzy arithmetic operation, in which the reliability of each component is represented by L-R type fuzzy numbers. Abdul Razak at al [1] presented a new method for finding fuzzy system reliability using fuzzy profust reliability theory in which transition fuzzy model is represented by trapezoidal fuzzy number.

Practically the problem of series, parallel or series-parallel system reliability may be formed as a typical non-linear programming with cost functions in fuzzy environment. Park [17] presented a two-component series system subject to a single constraint by fuzzy non-linear programming technique. Ruan and Sun [18] presented an exact method for cost minimization problem in series reliability system with multiple component choices. Sung et al [22] presented a reliability optimization problems for a series system with multiple-choice to maximize the system reliability subject to the system budget. Mahapatra et al [13] presented fuzzy reliability problem of series system model through fuzzy parametric geometric programming using max-min and max additive operator.

The series-parallel system structure has widely used in a system design, in addition to series system structure and parallel system structure. Sardar Donighi et al [19] presented a new approach in fuzzy reliability model for series-parallel system in which beta type distribution as its membership function. Liu C.M [12] presented redundancy – reliability allocation problems in multi-stage series parallel system under uncertain environment. Mahapatra and Roy [14] presented optimal redundancy allocation problem in series-parallel system using generalized fuzzy number to find out the maximum system reliability subject to available cost and weight. Many researchers have applied different techniques and solutions using Series system, Series-Parallel system [11,15] in different environment.

In this paper we have analyzed the flow transmission system. General transmission flow system design is series-parallel structure and the flow is transmitted from left to right. This paper presents a solution for transmission flow system to evaluate the maximum reliability of the above system subject to the system cost. The cost function is taken as the interval for triangular fuzzy number. In fuzzy system reliability the variance of the component and system reliability estimate is used as a metric. An approach is introduced in the co-efficient of variation (CV) of the system reliability estimate for each interval membership function. The advantage of this approach is to find minimum CV with high reliability estimate. The system reliability estimate of CV is obtained by Hatice Tekiner and Coit [8].

This paper is segmented as the below mentioned sections. Section 2 gives the mathematical model for series-parallel systems, notations and fuzzy mathematics prerequisites. Section 3 gives the mathematical formulation in crisp model and fuzzy model. Section 4 describes the mathematical analysis and the co-efficient of variance for Series- Parallel System Models. Section 5 gives the solution procedure for the Series - Parallel system models. Section 6 illustrates the construction of the Transmission flow system. In section 7 the maximum reliability with CV has been concluded.

Mathematical Model For Series - Parallel Systems

For a series – parallel system, there are m subsystem connected in series and those subsystems consisting of n_j components in parallel for j=1,2 .... n_k Fig.1 shows the diagram for m-unit series-parallel systems.

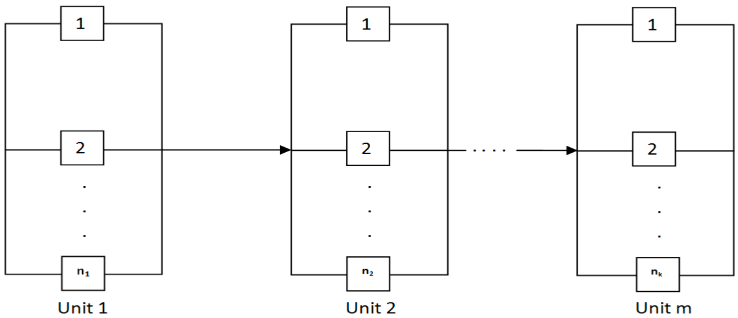

Figure 1: M-Unit Series-Parallel System

The Series-Parallel Systems model has been developed and worked out for the following notations.

Notations

In Order to formulate the problem in Series-parallel system model following notations has been developed.

R_i	-	Reliability of subsystem i, for i=1,2 ... m
r_{ij}^{L}	-	Left interval value reliability of j^{th} component in subsystem i, for i=1,2 ... m, j= 1,2,.... n_k
r_{ij}^{R}	-	Right interval value reliability of j^{th} component in subsystem i, for i=1,2 ... m, j= 1,2,.... n_k
c_{ij}^{L}	-	Left interval cost value of j^{th} component in subsystem i, for i=1,2 ...m, j= 1,2,.... n_k
c_{ij}^{R}	-	Right interval cost value of j^{th} component in subsystem i, for i=1,2 ...m, j= 1,2,.... n_k
$R_{sp}(R_1,R_2,...R_m)$	-	System reliability of m subsystem with each reliability R_i, for i=1,2 ... m
C_{SP} $(R_1,R_2 ...R_m)$	-	System cost of m sub system with each reliability R_i, for i=1,2 ...m

The system reliability for i^{th} stage parallel system is R_P (R_i) = $1-\prod_{j=1}^{n_k}(1-r_{ij})$ for i=1,2 ...m

The Series-Parallel System of reliability has m subsystem with reliability R_i , i=1,2...m is

R_{SP} (R_1,R_2 ... R_m) $=\prod_{i=1}^{m}R_P(R_i)$ $=\prod_{i=1}^{m}\left[1-\prod_{j=1}^{n_k}(1-r_{ij})\right]$

The linear cost components for i^{th} stage is $C_{sp}(R_i)$ = $\sum_{j=1}^{n_k}c_{ij}r_{ij}\le C_i, c_{ij}\ge 0$

Fuzzy Mathematics Prerequesites

Definition 2.2.1 Fuzzy set

Fuzzy Set theory was first introduced by Zadeh [24] in 1965. Let X be universe of discourse defined by X={x_1,x_2, ... x_n}. A fuzzy set $\tilde{A}$ in X is an object having the form $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$ where $\mu_{\tilde{A}}(x):X\rightarrow[0,1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x\in X$ in the fuzzy set $\tilde{A}$.

Definition 2.2.2 α-cut of a Fuzzy set or Interval of confidence at level α

An Interval of confidence at level α or α-cut of a fuzzy set $\tilde{A}$ of X is a crisp set A that contains all the elements of X that have membership value in $\tilde{A}$ greater than or equal to α. i.e. $A_\alpha=\{x:\mu_{\tilde{A}}(x)\ge\alpha, x\in X, \alpha\in[0,1]\}$.

Definition 2.2.3 Traingular Fuzzy Number

Let $a_1\le a_2\le a_3$. A Triangular Fuzzy number (TFN) $\tilde{A}$ in R is a fuzzy number with the membership function $\mu_{\tilde{A}}(x):X\rightarrow[0,1]$ is defined as follows.

$$\mu_{\tilde{A}}=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{for } a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2} & \text{for } a_2\le x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

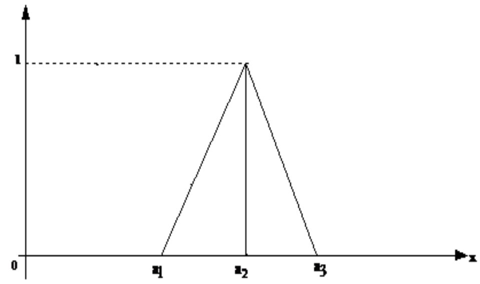

Figure 2: Triangular Fuzzy Number

Mathematical Formulation

Crisp Model

Consider the series-parallel model with m subsystem connected in series and those subsystem consisting of n_j, j = 1, 2, ..., n_k components in parallel. The maximization of reliability is found to be R_{sp} (R_1, R_2, ..., R_m) having subjected to the limited available cost C_j, j = 1,2... n_k.

The mathematical forms of i^{th} stage series- parallel system are as follows

$$\text{Max } R_{sp}(R_i) = 1 - \sum_{j=1}^{n_k}(1 - r_{ij})$$

$$\text{Sub to } C_{sp}(R_i) = \sum_{j=1}^{n_k} c_{ij} r_{ij} \le C_i \quad \text{where } 0 \le R_i < 1, 0 \le r_{ij} \le 1 \text{ and } i = 1,2\ldots m \; j = 1,2\ldots.n_k \quad \ldots 1$$

Fuzzy Model

Generally the system reliability of cost component and cost constraints can be involved in uncertain factors. So the constraint of system reliability becomes uncertain in a reliability optimization problem. Therefore it can be represented as fuzzy non-linear programming fuzzy number. The above problem can be modified as follows.

For i^{th} Stage of given fuzzy model as

$$\text{Max } R_{sp}(R_i) = 1 - \sum_{j=1}^{n_k}(1 - r_{ij})$$

$$\text{Sub to } C_{sp}(R_i) = \sum_{j=1}^{n_k} \tilde{c}_{ij} r_{ij} \le \tilde{C}_i \quad \text{where } \; 0 < R_i \le 1 \text{ and } 0 < r_{ij} \le 1 \text{ for all } i, j \quad \ldots 2$$

Mathematical Analysis

Consider a non-linear programming problem having one inequality constraint of the type Maximize Z = f (x_1, x_2 ... x_n)

Subject to g (x_1, x_2, ... x_n) ≤ b, x_1, x_2 ... $x_n \ge 0$

The above problem can be expressed as

Maximize Z = f (x_1, x_2 ... x_n)

Subject to h(x) ≤ 0 where h(x) = g(x_1,x_2 ... x_n) – b, h(x) ≥ 0 . .. 3

By Kuhn-Tucker condition the necessary conditions for the maximization in non-linear programming problem can be summarized as

$$\frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial xj} = 0 \quad . \quad ..4$$

$$\lambda h(x) = 0, \text{ where } h(x) \le 0 \text{ and } \lambda \ge 0$$

The objective function and constraint (3) into fuzzy non-linear programming problem is as follows

Max z = f (x_1, x_2 ... x_n)

Sub to $\tilde{h}(x) \le 0$ ...5

The Co-Efficient of Variance For Series-Parallels System Models

The co-efficient of variance (CV) is well-known statistical tool to characterize the estimation of uncertainty. CV is defined as the percentage of standard deviation divided by the mean or percentage of sample standard deviation divided by the average.

Let V be the variance of the system reliability estimate and R be the system reliability estimate then the co-efficient of variance for series-parallel system reliability by Teikner and coit (24) is

$$(CV)^2 = \frac{V}{R^2} = \frac{\prod_{i=1}^{m}[R_i^2 + V_i] - \prod_{i=1}^{m} R_i}{\prod_{i=1}^{m} R^2} = \prod_{i=1}^{m}\left(1 + \frac{V_i}{R_i^2}\right) - 1 .$$

Solution Procedure For Series-Parallel Models

Step: 1

Let the cost parameter be $\tilde{c}_{ij} = (c_{ij1}, c_{ij2}, c_{ij3})$ and the cost constraint be $\tilde{C}_i = (C_{i1}, C_{i2}, C_{i3})$ i = 1, 2...m, j= 1, 2... n_k are taken as Triangular fuzzy number.

Step: 2

Using α cut membership function of cost parameters and cost constraints are given by

$\tilde{c}_{ij} = [c_{ij1} + \alpha(c_{ij2} - c_{ij1}), c_{ij3} - \alpha(c_{ij3} - c_{ij2})] \quad i = 1,2,3,....m, j = 1,2,3,....n_k$ and

$\tilde{C}_i = (C_{i1} + \alpha(C_{i2} - C_{i1}), C_{i3} - \alpha(C_{i3} - C_{i2}) i = 1,2,3......m$ respectively.

Step: 3

Applying the Kuhn - Tucker condition in a fuzzy non-linear programming problem for given models with i^{th} stage i = 1, 2... m of left and right interval α-cut is expressed as

Max $R^L_{SP}(R_i) = 1 - \prod_{j=1}^{n_k}(1 - r^L_{ij})$

Subject to $\sum_{j=1}^{n_k} \tilde{c}^L_{ij} \; r_{ij} - \tilde{C}^L_i \le 0 \quad \text{where} \quad 0 < r_{ij} \le 1, C^L_{ij} \ge 0$ and

Max $R^R_{SP}(R_i) = 1 - \prod_{j=1}^{n_k}(1 - r^R_{ij})$

Subject to $\sum_{j=1}^{n_k} \tilde{c}^R_{ij} \; r_{ij} - \tilde{C}^R_i \le 0 \quad \text{where} \quad 0 < r_{ij} \le 1, C^R_{ij} \ge 0$

Step: 4
To find the optimal solution of R_i^L and R_i^R i = 1, 2... m for each membership value of α from step-2,3.

Step: 5
To calculate the system reliability $R^L = \prod_{i=1}^{m} R_i^L$ and $R^R = \prod_{i=1}^{m} R_i^R$ with their corresponding CV for each value of α.

Step: 6
To find out the maximum reliability and minimum CV from the values of α.

Numerical Examples

The proposed technique is used to find out the optimal solution of flow transmission which can be maximized for system reliability subject to available cost. The design configuration of flow transmission is presented in figure 2 as follows.

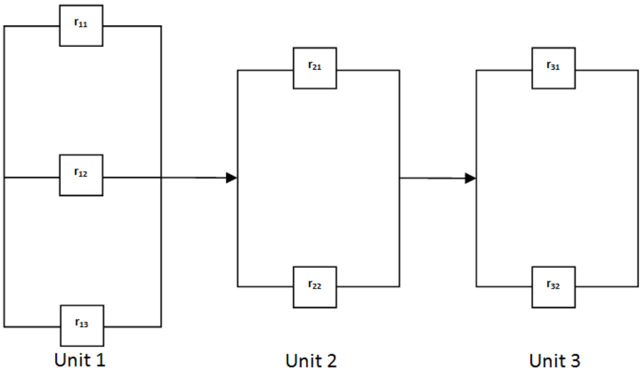

Figure 3: Design Configuration of Flow transmission system

The flow is transmitted from left to right. It consists of three main units (subsystems) connected in series. The first unit consisting of 3 components, second and third components consisting of 2 units. The cost components and cost constraints are taken as triangular fuzzy number.

For Unit 1, the cost components is c_{11} = (63, 65, 68), c_{12} = (69.5, 72, 74) and c_{13} = (83, 85, 87.5) and cost constraints is C_1 = (204.5, 208, 211)

For Unit 2, the cost components is c_{21} = (42.5, 44.5, 47), c_{22} = (38, 40.5, 43.5) a and cost constraints is C_2 = (74, 76, 79)

For Unit 3, the cost components is c_{31} = (52, 54, 56.5), c_{32} = (47.5, 50, 52) a and cost constraints is C_3 = (93, 95.5, 98.5)

The cost components and constraints of left and right interval α-cut membership function are tabulated as follows.

Unit(i)	Cost components	Cost constraints
1	$c_{11} = (c_{11}^L, c_{11}^R) = (63.0+2.0\alpha, 68.0-3.0\alpha)$ $c_{12} = (c_{12}^L, c_{12}^R) = (69.5+2.5\alpha, 74.0-2.0\alpha)$ $c_{13} = (c_{13}^L, c_{13}^R) = (83.0+2.0\alpha, 87.5-2.5\alpha)$	$C_1 = (C_1^L, C_2^R)$ $= (204.5 + 3.5\ \alpha,\ 211.0-3.0\alpha)$
2	$c_{21} = (c_{21}^L, c_{21}^R) = (42.5+2.0\alpha, 47.0-2.5\alpha)$ $c_{22} = (c_{22}^L, c_{22}^R) = (38.0+2.5\alpha, 43.5-3.0\ \alpha)$	$C_2 = (C_2^L, C_2^R)$ $= (74.0 + 2.0\ \alpha,\ 79.0- 3.0\alpha)$
3	$c_{31} = (c_{31}^L, c_{31}^R) = (52.0+2.0\alpha, 56.5-2.5\alpha)$ $c_{32} = (c_{32}^L, c_{32}^R) = (47.5+2.5\alpha, 52.0-2.0\ \alpha)$	$C_3 = (C_3^L, C_3^R)$ $= (93.0 + 2.5\ \alpha,\ 98.5- 3.0\alpha)$

The following table 1 and table 2 show the left and right interval optimal solution of reliability system through fuzzy parametric non-linear programming with co-efficient of variance.

Table 1: Left interval optimal solution of fuzzy membership value of α with CV

α	**r_{11}**	**r_{12}**	**r_{13}**	**R_1**	**r_{21}**	**r_{22}**	**R_2**	**r_{31}**	**r_{32}**	**R_3**	**R**	**CV**
0.0	0.941799	0.947242	0.955823	0.999864	0.923529	0.914474	0.993460	0.937500	0.931579	0.995724	**0.989077**	**0.007934**
0.2	0.939012	0.944762	0.953637	0.999844	0.918415	0.909091	0.992583	0.934160	0.928125	0.995268	0.987732	0.008213
0.4	0.936259	0.942317	0.951472	0.999822	0.913395	0.903846	0.991673	0.930871	0.924742	0.994798	0.986337	0.008473
0.6	0.935099	0.941066	0.950515	0.999811	0.908467	0.898734	0.990731	0.927632	0.921429	0.994314	0.984911	0.008611
0.8	0.930857	0.937529	0.947203	0.999772	0.903628	0.893750	0.989760	0.924440	0.918182	0.993818	0.983417	0.008937
1.0	0.928205	0.935185	0.945098	0.999745	0.898876	0.888889	0.988764	0.921296	0.915000	0.993310	0.981898	0.009144

Table 2: Right interval optimal solution of fuzzy membership value of α with CV

α	**r_{11}**	**r_{12}**	**r_{13}**	**R_1**	**r_{21}**	**r_{22}**	**R_2**	**r_{31}**	**r_{32}**	**R_3**	**R**	**CV**
0.0	0.909535	0.916870	0.929696	0.999471	0.877660	0.867816	0.983829	0.911504	0.903846	0.991491	0.974941	0.010461
0.2	0.912957	0.920290	0.932567	0.999532	0.881720	0.871795	0.984836	0.913393	0.906008	0.991860	0.976362	0.010235
0.4	0.916667	0.923953	0.935645	0.999592	0.885870	0.875887	0.985835	0.915315	0.908203	0.992226	0.977772	0.009982
0.6	0.920443	0.927656	0.938760	0.999648	0.890110	0.880096	0.986824	0.917273	0.910433	0.992590	0.979166	0.009717
0.8	0.924289	0.931400	0.941910	0.999698	0.894444	0.884428	0.987801	0.917708	0.913176	0.992855	**0.980447**	**0.009129**
1.0	0.928205	0.935185	0.945098	0.999745	0.898876	0.888889	0.988764	0.921296	0.915000	0.993310	0.981898	0.009144

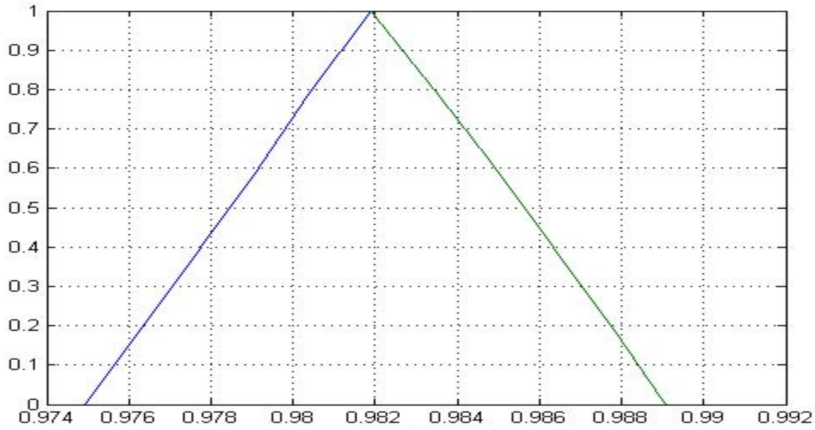

Figure 4: Interval Valued Reliability with Fuzzy Triangular Function

Conclusion

In the industrial system, the cost component is considered to be the most important factor and the reliability of system is also considered to be important. The available information about the constituent components of the system is imprecise, vague and uncertain often, in such a situation most decision makers specify the reliability evaluations are risk averse for which they want to have higher reliability and lower estimations. In General, the Series-Parallel system is a non-linear programming problem with system available cost. But cost component and cost constraint are fuzzy numbers in nature. This paper attempts to provide the evaluation of reliability in transmission flow series-parallel system and the cost component and cost constraint of each subsystem are taken as Triangular fuzzy number. Kuhn-Tucker conditions are taken into concern to solve the non-linear programming problem with fuzzy components to find out the optimal solutions for left and right interval valued membership function of α. This can be maximized for the system reliability subject to the available cost. Table 1 shows that the left interval optimal solution of fuzzy system reliability with CV, which identifies that maximum reliability and minimum CV for $\alpha = 0.0$ is 0.9890. Table 2 shows that the right interval optimal solution of fuzzy system reliability with CV, which also identifies that the maximum reliabilities and minimum CV for $\alpha = 0.8$ is 0.9851. Decision maker can make use of the strategy of system reliability where optimization is involved.

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